Evolution of intermediate-mass stars

Intermediate-mass stars are defined as those which are nondegenerate when helium burning begins, but degenerate when CO burning begins. This turns out to imply that the helium core mass is greater than $0.31\,M_\odot$ and the mass of the CO core is less than $1.06\,M_\odot$. These stars therefore burn helium quiescently, but might burn CO explosively. These stars have initial masses $\sim 3-9\,M_\odot$.

Key points:

(1) All these stars have central temperatures $T > 2\times10^7$ K, so they burn hydrogen primarily via the CNO cycle rather than the pp chain. Therefore, the energy generation rate is proportional to X (not X^2 like for pp), and there is thus less contraction required to counterbalance the drop in pressure resulting from transformation of H to He.

(2) For lower-mass stars in this range, the opacity is well-represented by the Kramers opacity law; for higher-mass stars, electron scattering dominates. There is a difference because for higher-mass stars the temperature in the center is larger, and since a Kramers opacity $\kappa \propto T^{-3.5}$, at higher temperatures electron scattering is more important.

(3) There is a limit to the fractional mass of the star in the helium core if ideal gas pressure dominates, above which hydrostatic equilibrium cannot be maintained (this is called the Schönberg-Chandrasekhar limit, and is about 0.1). Intermediate mass stars exceed this limit, with the result that core contraction is accelerated greatly in the red giant stage (where there is already an inert helium core and energy generation is from hydrogen shell burning around that core).

(4) With one shell source and core contraction, the envelope expands greatly.

(5) Most of the energy released is used to expand the envelope, so the luminosity does not increase; the star therefore moves to the red part of the H-R diagram. It does this very quickly (~1% of the star's lifetime, or $\sim 10^5$ yr for a $5\,M_\odot$ star), so there are few stars in this intermediate region between the high-mass main sequence and high-mass red giants. This is called the Hertzsprung gap.

(6) For stars of initial mass $M > 2.3\,M_\odot$, helium burning starts before the core is degenerate, so there is no helium flash.

(7) The temperature of the core increases with increasing core mass, so the burning rate goes up (and the duration of the quiescent He burning phase goes down) with increasing stellar mass.

(8) Practically all stars in this range go through an instability phase, driven by the thermodynamic and opacity properties of the ionization zones. Low-mass stars become RR Lyrae variables, higher mass ($> 2.3\,M_\odot$) stars become Cepheids. The general cause is that

when the envelope is small, the atomic opacity is larger and thus radiation drives the envelope outward. However, this process ionizes a lot of atoms, which decreases the opacity and thus the envelope falls back. These oscillations can occur millions of times, which means that they are adiabatic (i.e., they conserve entropy); otherwise, they could not continue as cycles. As you know, the period-luminosity relation for Cepheids allows them to be used as standard candles and thus as cosmological distance indicators.

(9) One Cepheid is among the most famous stars in the sky. The North Star, Polaris, is a weak Cepheid (0.05 mag variations), which means that when Shakespeare had Julius Caesar say "But I am constant as the northern star, of whose true-fix'd and resting quality there is no fellow in the firmament" (just before Brutus et al. stabbed him to death), he was doubly wrong. Polaris was a good half degree from the North Pole at the time, and it isn't quite constant in brightness either!

(10) All stars with initial masses $M \sim 1 - 9\,M_\odot$ go through the asymptotic giant branch phase (see above as well). The cores of these stars are mainly CO, with degenerate electrons; this means that outward energy diffusion by electron conductivity is very efficient, so the temperature can't get high enough to ignite the CO.

(11) As before, thermal pulses occur. Convection occurs through the whole envelope when the helium layer burns (the helium shell burning can be explosive, and provides another example of thermonuclear propagation).

(12) After the maximum of a pulse, convection in the shell stops and the base of the convective layer moves inward, entering the carbon-rich layer which was produced by the previous episode of helium burning. Carbon and oxygen can be transferred to the surface in this way. This is called "dredge-up", and it means that by observing the surface spectra of such stars we get clues about their composition and about burning processes.

Oscillations of stars

We might go into a little more detail near the end of the class, but since we've mentioned Cepheid variables and their ilk, now is a good time to go into the very basics of stellar oscillations. The reason we're interested is that the characteristic frequencies and modes of vibration of stars, whose study is called asteroseismology, can tell us a lot about stars including their interiors.

So what are those basics? Imagine that you take something (e.g., a wire, or a bell, a star, a planet, or something else) and you thwack it. It will start oscillating in response. In general, if your thwack was arbitrary, the oscillations will be a complicated mess. But if you pose your problem mathematically, with differential equations and boundary conditions, you will find that the oscillations can be broken up into *normal modes*, which in the simplest approximation can evolve independently of each other.

As the simplest example, consider a taut wire which is clamped at both ends. The boundary conditions mean that the ends can't move. If you say that one end is at position $x = 0$ and the other is at $x = a$, then a function such as $f = C_n \sin(\pi n x/a)$, where $n > 0$ is an integer and f represents the deviation from the equilibrium straight line, satisfies those boundary conditions. If you now multiply that function by a sinusoid $\sin(\omega_n t)$ (or, because it's easier to work with, $e^{i\omega_n t}$), then that represents the oscillation of that function, with an angular frequency ω_n, where you would need to find ω_n based on the physical conditions of the problem (e.g., involving the forces that try to restore the perturbed wire shape). You would find that any perturbation of the wire could be represented by the sum of such functions; in this case, that's just a Fourier expansion of the function. In the simplest (linear!) approximation, you could find how any general perturbation would evolve by looking at the independent evolutions of each mode n (in more detail, one mode can excite each other, but that excitation is often weak). In general, ω_n can be complex. This means that in addition to an oscillatory component (coming from the real part of ω_n), there can be exponential damping or exponential growth of the oscillation (depending, respectively, on whether i times the imaginary part of ω_n is negative or positive).

For more complicated systems, such as a drum head or a star, some of the details change (to two or three dimensions, respectively). However, the basic idea is the same: the generalization is that you have to find, typically, three-dimensional normal modes (or "eigenfunctions") whose sum can produce general oscillations. Each eigenfunction will have a characteristic oscillation frequency and damping time (or it will blow up or collapse!). If we can identify particular oscillations from a star (typically from the overall modulation of the flux that we see), then we can model the star and figure out aspects of its interior. This has been one of the great and yet largely unsung contributions of the Kepler mission, which is mainly famous for its transit discoveries of exoplanets: Kepler's outstanding photometric precision has allowed researchers to model stellar oscillations with high precision and thus to refine stellar models.

For the Sun, where it is called helioseismology, this analysis led indirectly to a Nobel Prize! Starting in the 1960s, various researchers performed long-term detections of neutrinos from the Sun. The number, however, was only about 1/3 of the prediction using the standard solar model. There were lots of uncertainties in that model, but those uncertainties were reduced dramatically by fits to helioseismic modes (aided by the fact that for the Sun, we can actually get excellent angular resolution of the disk and thus resolve modes spatially as well as in frequency). As a result, it was determined that the neutrino deficit had to do with neutrino flavor changes (electron to muon to tau and back) rather than to a fault of the model, and those flavor changes were deemed worthy of a Nobel.

Evolution of massive stars

These have initial masses greater than about $9\,M_\odot$.

(1) The dominant new feature of these stars is their enormous losses of mass to stellar winds. The mass loss rate is correlated with luminosity and can be up to $3 \times 10^{-5}\,M_\odot\ \mathrm{yr}^{-1}$ for Wolf-Rayet stars (although the wind loss rates are tough to measure and W-R stars are rare and thus typically distant).

(2) The speed of the winds is comparable to the surface escape speed, and is thought to be caused by radiation forces off of UV lines. **Ask class:** why physical process might mean that the wind speed is comparable to the escape speed?

(3) The stellar mass therefore decreases, and the ISM is enriched with metals as a result. We also get to see what the star had for dinner, in the sense that its digestive (thermonuclear) products are brought to the surface.

(4) The effects of mass loss on the hydrogen burning phase are: (a) the decrease of mass means that the central temperature increases less rapidly, so the mass of the convective core increases less rapidly. However, (b) the total mass drops even faster, so the fractional mass of the core is higher. (c) The luminosity is lower. However, the star is undermassive for a given luminosity, i.e., overluminous for a given mass. (d) The main sequence lifetime is increased: there is less nuclear fuel in the burning regions, but the lower luminosity more than makes up for it. (e) The increase in the fractional core mass widens the main sequence band. (f) At the end of H core burning the mass of the helium core is smaller than for evolution at constant mass.

(5) The effects of mass loss on the He burning phase are: the position of He burning stars in the HR diagram depends on initial mass and mass loss rates.

(6) Stars above $9\,M_\odot$ evolve towards a phase of core collapse (the boundary might be as low as $\sim 8\ M_\odot$). Mass loss determines where in the HR diagram they are prior to the collapse and explosion. For $M > 30 - 40\,M_\odot$, the progenitor is a blue supergiant, whereas below that range the supernova progenitor is a red supergiant.

Core-collapse supernovae

The nomenclature of supernovae is observationally driven, and unfortunately does not divide the phenomena as we now know it. Type I supernovae have no hydrogen in their spectra; Type II do. Type I SNe are further divided depending on whether their early-time spectra show strong Si II (Type Ia), prominent He I (Type Ib) or neither Si II or He I (Type Ic). It is believed, however, that everything except Type Ia are produced by the collapse of a core of a high-mass star. These are the ones that concern us. Type Ia, which thought to be due to a thermonuclear runaway on an accreting white dwarf (or, with growing evidence, from the merger of two white dwarfs), are remarkably homogeneous in their luminosities (especially when the so-called Phillips relation is used, such that a longer decay time of the light curve corresponds to a higher-luminosity peak), and are therefore used as cosmic "standard bombs" to probe the curvature and acceleration of the universe. But that's off topic, at least for now. We will focus on what is thought to cause core-collapse in massive stars, which is a topic that is very far from being resolved!

(1) The iron core doesn't generate energy (it's at the peak of the binding energy curve), and is therefore supported by semidegenerate electrons. **Ask class:** why does this follow from the lack of energy generation? Because residual heat will leak out of core, and the only remaining support is degeneracy pressure.

(2) When the core exceeds the mass that can be supported by these electrons, which is approximately the Chandrasekhar mass (around $1.4\,M_\odot$), it begins to collapse. The actual mass at collapse can be somewhat higher than this, because the core isn't ice-cold and thus there is thermal support as well as degeneracy support.

(3) Two major instabilities are involved, often simultaneously: (a) electron capture onto iron group elements (i.e., the combination of an electron with a proton in a nucleus to make a neutron) removes electrons from supporting the core, and produces neutrinos that leave immediately without helping support the core, and (b) photodisintegration happens, putting energy into dissociating nuclei instead of raising the pressure. These instabilities cause the core to contract rapidly, essentially at free-fall once the collapse starts.

(4) The gravitational energy of collapse to $\sim$NS densities is about 10^{53} erg, which is a hundred times the kinetic and photon luminosity in an explosion. But how should this energy be coupled to the outer layers?

(5) In 1966, Colgate & White suggested a neutrino transport model: most of the energy is converted to neutrinos, which interact with the mantle and heat+expand it. At the time, this appeared not to work, because the coupling was insufficient.

(6) In the mid-1970's, the discovery of neutral weak currents suggested a twist: neutrinos interact more strongly with high-A nuclei (like A^2) than with individual nucleons, so the neutrinos might bypass the dissociated core and deposit their energy in the iron-group elements in the mantle. But again, detailed calculations make this less promising because too high a fraction of the neutrinos remain in the core and are advected in.

(7) Core bounce. When the core reaches nuclear density (around 2×10^{14} g cm^{-3}), there is a bounce and a shock driven outwards. The bounce occurs because when neutron degeneracy is reached, there is a huge amount of energy required to squeeze the core more. If the infalling matter does not have the required energy, then the bounce is reversed.

(8) The shock has plenty of energy to drive the explosion (around 5×10^{51} erg), but many losses occur as it propagates outward (neutrino emission and photodisintegration, for example; note that $\sim 1.5 \times 10^{51}$ erg is required for each disintegration of $0.1\,M_\odot$ of iron

into free nucleons). The explosion can only continue to propagate if it gets to a point where the postshock temperatures aren't high enough to emit neutrinos or disintegrate nuclei, if prompt explosion is to be possible.

(9) The shock may be refreshed by input of energy or momentum from neutrinos, which at the $\sim$5 MeV energies are at optical depths greater than unity for most of the core.

(10) One suggestion for this is that convection brings neutrino-emitting matter out farther, where it can release its energy and help power the explosion at later times ($\sim$ 100 ms). However, 2-D simulations of rotating cores show that rotation inhibits convection and decreases dramatically the efficiency with which neutrino energy can be converted into kinetic energy. The polarization of supernova remnants suggests that many of them have axes, naturally suggesting rotation, so it may not be possible to get away from rotation. However, even more recent 3-D simulations (which are phenomenally computationally expensive) suggest that convection might be just fine and thus these stars can explode after all.

(11) So, there is lots and lots of uncertainty left! Supernovae happen, and happen frequently, so nature finds a way. Neutrinos, one way or the other, seem to be the best bet, since neutronization of matter generates them at the 10^{53} erg level and only $\sim$1% needs to be tapped. 3-D numerical calculations over the last decade or so are fairly good at getting explosions, but there are a lot of knobs to turn.