Evolution of Binary Stars

We now turn our attention to the modifications of evolution caused by the presence of a close binary. **Ask class:** why can we consider only close binaries, and not close trinaries or other multiple systems? The answer is that with more than two objects in close orbit around each other, there is a fairly rapid gravitational ejection of at least one of them. One can have hierarchical systems, in which there is a close binary, and at a much larger distance is a third star (or another binary). The Alpha Centauri triple system is an example of a hierarchical triple, and Mizar+Alcor in the Big Dipper form a sextuple system (two binaries close to each other but well-separated compared with their individual orbital sizes [Mizar], orbited by a more distant binary [Alcor]). Generally, an object that is at a distance at least 5x the separation of the binary can orbit stably.

On a side note, in the Galaxy the number density of stars is low enough that, of course, star-star hyperbolic collisions are rare, and binaries are essentially undisturbed as well. In the dense core of a globular cluster, however, it's another story. The effective cross section of a binary for interactions (not collisions, but gravitational interactions) is $\sim \pi R_{\rm orb}^2$, where $R_{\rm orb}$ is the binary orbital radius. Numerical simulations show that for encounters with distant third stars when all three stars are of comparable mass, if the binary orbital speed is greater than the linear speed of the third star then the binary tends to "harden" (meaning become more compact), whereas if the orbital speed is less then the binary softens (becomes wider). In three-body interactions, the most likely result is for the two most massive objects to end up in a final binary, while the lightest object is ejected.

But now let's think about isolated binaries. **Ask class:** at what approximate separation would they expect the presence of a companion to have a significant effect on evolution? When the separation is comparable to or less than the maximum size the star would have in isolated evolution, there is an influence. The main new feature compared to the evolution of single stars is that the expansion of the star is curtailed by the presence of the companion. This means that there can be mass transfer between the stars.

So, let's think first about the distribution of binary parameters. There are three parameters that are introduced. One common choice of these parameters is the mass of the primary (M_p), the mass ratio of the secondary M_s to the primary ($q = M_s/M_p$), and the binary period P. We would like to know, observationally or theoretically, the initial distribution function for these parameters

$$F(M_p, q, P)d\ln M_p\, d\ln q\, d\ln P\ . \tag{1}$$

As we discussed before, there are plenty of selection effects that operate. More massive stars are easier to detect, and in a binary if there is an extreme mass ratio ($q \ll 1$) then the secondary will be hard to see. However, Population I binaries appear to have a distribution function that can be split into a product of three independent functions:

$$F(M_p, q, P) d\ln M_p \, d\ln q \, d\ln P = F(M_p) d\ln M_p V(q) \, d\ln q W(P) \, d\ln P \; . \tag{2}$$

The distribution of primary masses is about the same as that for single stars: $F(M_p) d\ln M_p \propto M_p^{-2.35} d\ln M_p$. The mass ratio is tougher to characterize because of selection effects, but it seems to have a peak at $q = 1$ (as in, $> 60\%$ of unevolved systems have $q > 0.8$). The period distribution is roughly flat with logarithmic period interval, and is probably close to $W(P) d\ln P = 0.06 d\ln P$. More massive stars are more likely to have binary companions; less than half of M stars appear to have stellar companions, whereas for O stars the fraction is at least 80-90%. Indeed, O stars have lots of companions; some estimates are that the *average* number of companions to an O star is 2.1!

For well-separated binaries, there is little effect on the evolution of the component stars. However, since stars expand greatly during their evolution (the factor can be >100 between the radius at helium ignition and the ZAMS (Zero Age Main Sequence) radius, and another factor of 10 to the radius at CO ignition), the presence of a binary within 1–10 AU can matter.

Let us now consider orbital angular momentum of the system. In general, if we have stars of mass M_1 and M_2 in a circular orbit around each other, where the binary separation is a, then the orbital angular momentum J is given by

$$J^2 = \frac{Ga(M_1 M_2)^2}{M_1 + M_2} \; , \tag{3}$$

$$J = a^2 \frac{M_1 M_2}{M_1 + M_2} \omega \tag{4}$$

where $\omega = 2\pi/P$ is the orbital frequency. If there is mass transfer, with a net change per time in M_1 of $\dot{M}_1$, and if the mass transfer is *conservative*, meaning that no mass leaves the system, then the rate of change of the separation is

$$\frac{\dot{a}}{a} = -2 \left(1 - \frac{M_1}{M_2} \right) \frac{\dot{M}_1}{M_1} \; . \tag{5}$$

Ask class: so what can we say about the change in the orbital separation, depending on the masses? What does this suggest about the stability of mass transfer? If the lower-mass star loses mass then this results in a larger separation, which stabilizes the transfer; if instead the higher-mass star donates mass then this results in a smaller separation, which accelerates and destabilizes the mass transfer. Said another way, if star 1 is losing mass ($\dot{M}_1 < 0$) and is the less massive star ($M_1 < M_2$), then the orbit expands ($\dot{a} > 0$). Mass transfer from the less massive to the more massive star expands the binary; mass transfer the other way contracts the binary. **Ask class:** what are ways in which the mass transfer might not be conservative? Jets or winds could cause the system to lose mass. If the mass transfer is not conservative, and a fraction α of the transferred matter leaves the system, then

$$\frac{\dot{a}}{a} = -2\left[1 + (\alpha - 1)\frac{M_1}{M_2} - \frac{1}{2}\alpha\left(\frac{M_1}{M_1 + M_2}\right)\right]\frac{\dot{M}_1}{M_1} + 2\frac{\dot{J}_{\rm orb}}{J_{\rm orb}} \tag{6}$$

where $\dot{J}_{\rm orb}$ is the rate of change in the orbital angular momentum. **Ask class:** what ways can they think of in which angular momentum might leave the entire system? The actual orbital angular momentum can be decreased in a number of ways. For orbital periods $P > 10$ hr, one of the most important ways is by a stellar wind: spin-orbit interaction keeps the stars tidally locked to each other, and a stellar wind slows down the rotation, which removes angular momentum from the system. For $P < 10$ hr, gravitational radiation can be important.

But how is the mass transferred in the first place? In a rotating system, the total potential of a test particle in synchronous orbit ($\Omega = \omega$) at a cylindrical distance s from the center of mass is

$$\Psi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\omega^2 s^2}{2}\ . \tag{7}$$

An analysis of the equipotential surfaces gives the Lagrange points, the most important of which (for our purposes) is the first Lagrange point, which is between the two stars and is where there is no net radial force toward either star; a little closer to M_1 and it belongs to star 1, and the same for M_2. This surface is called the *Roche surface*, and the two regions are Roche lobes. The equipotential surfaces are essentially spherical close to either of the stars, and although at L_1 the surface isn't quite spherical, it is fairly close. It is therefore convenient to define the Roche radius as being the radius of a sphere with the same volume as the Roche lobe: $\frac{4}{3}\pi R_R^3$=volume of Roche lobe. A reasonably accurate expression for $q < 0.8$ is due to Paczynski:

$$\frac{R_R}{a} \approx 0.46\left(\frac{1}{1 + 1/q}\right)^{1/3}\ . \tag{8}$$

In planetary science there is a generalized concept of the Roche radius called a *Hill sphere*. If one has a large mass ratio ($q \ll 1$), then the Hill sphere is the region around the smaller object (planet) such that orbits are bound to the planet. Outside of that radius, orbits of satellites escape.

Let's see if we can get a quick derivation of the $q^{1/3}$ dependence for $q \ll 1$. Our first try might be to assume a static force balance, with no orbit. Let $M \gg m$. Clearly, the balance point is very close to the lower-mass object, so if the separation is a, the Roche radius is $R \ll a$. The force balance would then be $GM/a^2 \approx Gm/R^2$, which would imply that $R \approx a\sqrt{m/M}$. This is wrong; it implies a $q^{1/2}$ dependence instead of the correct $q^{1/3}$ dependence. To do it right, we need to include the effects of the orbit. This shows a case in which we can't be *too* careless in the derivation!

Mass transfer time scales

When mass transfer actually occurs, the structure of both stars is modified. The star tries to maintain its volume within the Roche lobe, and whether it does so successfully depends on the rate of mass transfer and the type of transfer. Two possibilities are that the mass is transferred on a dynamical time scale (essentially the time required for a sound wave to propagate all the way across the star) or that it is transferred on a thermal time scale (so that the envelope has time to adjust thermally). This is to be compared with the time scale of evolution when the binary is not in contact; this is a nuclear time scale, and is usually much longer than the thermal or dynamical time scales.

Dynamical time scale.—If the star loses mass but not enough to remain within its Roche lobe, the only limit on mass loss is the rate of sonic expansion through L_1. This can't happen for stars with radiative envelopes, but can happen if the star has a deep surface convection zone or is degenerate. This means that if the mass donor is on or near the giant branch or is on the lower main sequence (convection) or is degenerate (white dwarf) it can be dynamically unstable. It does not *have* to be dynamically unstable, however. The mass loss rate for stars with convective envelopes of thickness ΔR is

$$\dot{M} \approx -10\frac{M}{P}\left(\frac{\Delta R}{R}\right)^3 . \tag{9}$$

Thermal time scale.—The star loses mass but is not in thermal equilibrium (if it were, it would be larger than the Roche lobe). These stars remain just filling the Roche lobe. **Ask class:** whenever we've discussed a thermal time scale, what time scale is it? The Kelvin-Helmholz time scale $t_{\rm KH} = GM^2/RL$, so the mass loss rate is

$$\dot{M} \approx -\frac{M}{t_{\rm KH}} . \tag{10}$$

Nuclear time scale.—Here the mass transfer is entirely driven by evolutionary processes (i.e., not enhanced by other mechanisms). The radius equals the Roche radius, and the star remains in thermal equilibrium. This takes a long time and is therefore favorable to observe.

Again, if the less massive star donates, the separation increases and transfer stabilizes; if the more massive star donates, the separation decreases and transfer is destabilized.

If the matter being donated has a much larger mean molecular weight than the μ of the accreting star, then you have an unstable composition stratification (heavy stuff on top of light stuff). This instability is the same as that of "salt fingers": salt water on top of fresh water produces long "fingers" of salt that go into the fresh water. More generally this is called "thermohaline convection" and is an essentially instantaneous mixing process.

Types of mass transfer binaries

Algol variables.—These are semi-detached systems, where the less massive component is transferring mass. **Ask class:** if the less massive component is more evolved than the more massive, what does that say about the initial relative masses? The less massive component, being more evolved, was originally the more massive component. The mass ratios of these binaries are relatively small, with a peak at about $0.2-0.3$. Algol itself is an eclipsing binary, with an observable variation in the light from the system. Systems like this can in principle have transfer back and forth, so that first one, then the other, then the original, then... can be the more massive of the two.

Cataclysmic variables.—These are binaries consisting of a white dwarf and a secondary (which can be a giant, main sequence star, or another white dwarf). These undergo several types of bursting behavior:

(1) Novae. The accreting matter on the surface of the white dwarf undergoes a thermonuclear burst. This tends to lift matter off the surface and reduce the mass, if $M > 0.6\,M_\odot$, and increase the mass if $M < 0.6\,M_\odot$.

(2) Recurrent novae. Same as regular novae, but weaker and more common.

(3) Dwarf novae. These are accretion instabilities. Matter builds up in an accretion disk, but the angular momentum transfer is insufficient to transfer matter unless enough matter has built up, at which point the matter dumps in a high mass transfer episode. This is something which may also explain some observed effects in black hole binaries.

Low-mass X-ray binaries.—These consist of a low-mass ($M < M_\odot$) star with a neutron star or black hole. Mass transfer onto the compact object produces X-rays. The companion to the compact object can be a normal star, a giant, or a compact object. They last a good 10^8 yr and are highly luminous $0.01-1\,L_{\rm Edd}$ objects. The NS LMXBs are thought to be the progenitors of millisecond pulsars, which are the most stable natural clocks in the universe. For some odd reason that is not yet understood (although there are ideas), the neutron stars in these systems are very weakly magnetic compared with ordinary pulsar, with surface field strengths more like 10^{8-9} G than the $\sim 10^{12}$ G of rotation-powered solitary radio pulsars. Having a source like this requires special circumstances. If more than half the mass of a system is lost isotropically and abruptly, then the system is unbound. A supernova can do that easily, so only a kick to the NS or BH toward the companion can save the system from disruption.

High-mass X-ray binaries.—These consist of a high-mass ($M > M_\odot$) star with a NS or BH. Mass transfer here is not usually via Roche lobe overflow (which would be unstable), but is instead by the wind of the companion. An example is Cyg X-1, which was the first convincing stellar-mass black hole candidate. These systems only persist for a few million years (because of the rapid rate of evolution of the companion). They are the progenitors of binary pulsars. Getting a binary NS or BH pair is not easy, because the system has to survive two supernovae. The first one typically does not remove more than half the mass of the system, but the second does. Studies of these systems are very important for the purpose of understanding the gravitational wave sources that have been detected.