Practice Problems Related to Radiative Transitions

Shortly before the appendix in the notes we discuss dipole selection rules. Let's phrase this in terms of parity, where we'll consider one dimension for simplicity. A function f has *positive* parity if $f(-x) = f(x)$ for any x, and *negative* parity if $f(-x) = -f(x)$ for any x.

1. Show that whether f has positive or negative parity, $\int_{-\infty}^{\infty} f(x)xf(x)dx = 0$.

2. More generally, show that for any two functions f and g that have definite parity (meaning either negative or positive parity), $\int_{-\infty}^{\infty} f(x)xg(x)dx$ is nonzero only if f and g have opposite parity.

3. Even more generally, show that for any two functions f and g with definite parity, $\int_{-\infty}^{\infty} f(x)x^n g(x)dx$ is nonzero only if the product of the parities of f and g equals $(-1)^n$. This underlies many selection rules.