Practice Problems Related to Line Broadening

Line broadening is related in some ways to signal analysis. For the two problems below, we assume that you have written or have found a code that can perform a Fourier transform of a time series of intensities and that yields a power spectrum (the complex square of the amplitude of the Fourier transform).

For both your time series, let the underlying intensity be a constant plus a sinusoid: $A + B \cos(\omega t + \phi)$, where $A > B$. Sample the time series regularly (Δt between samples is much less than $2\pi/\omega$, and it is the same Δt between any two adjacent samples).

1. At random times, reset ϕ. Thus, with an average time T between phase changes, re-select ϕ randomly between 0 and 2π. What power spectrum do you expect? How does this relate to the shape of a line that would be produced by collisions?

2. Now keep ϕ fixed, but let B range from 0 to a maximum to 0 again (perhaps as a Gaussian, with a time width much larger than $2\pi/\omega$). What do you expect for the power spectrum? This could represent a wave with a beginning and an end.