See Shapiro and Teukolsky, Chapter 10

Rotation-Powered Pulsars

As we discussed last time, the extreme properties of neutron stars (gravity, density, magnetic field) make them excellent objects to study to see how physics operates in unusual environments. However, until about thirty years ago it was not clear that they would ever be observed. It is useful to backtrack a bit and go over some of the early history of thought about neutron stars.

The neutron was discovered in 1932 by Chadwick (evidence had been seen earlier by Irene and Frederic Joliot-Curie, but they thought it was a high-energy photon). Even earlier, Lev Landau apparently had a dinner meeting in which Landau proposed something like the neutron and suggested that, just as white dwarfs are supported by electron degeneracy pressure, "neutron stars" might exist that are supported by neutron degeneracy pressure. Just two years later, Baade and Zwicky suggested that supernovae represent the transition from normal stars to neutron stars (and, incidentally, that supernovae are the source of cosmic rays). However, how could these be seen? Such objects would be extremely dim, even though hot, because they would have such small areas. There was some work in the early 1960s on whether they might be observable in the X-rays (since X-ray sounding rockets were just then being launched), but it didn't look really feasible.

Into this milieu in 1967 came Jocelyn Bell, a graduate student at Cambridge University working with Antony Hewish. Her thesis project had to do with scintillation observations of quasars, which had just been discovered in 1963. She and the other members of Hewish's team constructed a radio array, then she sat down to take the data. In the fall of 1967 she saw a strange item on one of the recording charts: a bit of "scruff" that was distinct from the usual types of noise that she had seen before. At the suggestion of Hewish, she put a high-speed recorder on the instrument, and incredibly the scruff was resolved as a series of very regular pulses, about 4/3 seconds apart. After eliminating artificial sources, it was clear that this source (and a couple others like it) were natural, coming from somewhere in the heavens. But what could cause this?

The Nature of Pulsars

We know now that they are rotation-powered pulsars, but it is very instructive to go through the chain of logic leading to that conclusion. By 1968, there were three important facts established about pulsars:

1. They were fast, with periods between 0.033 s and 3 s. The pulses themselves could be even shorter, <0.01 s in some cases.

2. They were extremely stable, with $P/\dot{P}$ equal to thousands to millions of years.
3. They always *increased* in period slightly, never decreased.

Let's consider what this implies. **Ask class:** what does the first fact imply about the nature of the object? The fast periods and short pulses imply something small. Barring beaming, the size of the object would have to be $< ct$, if t is the time of the pulse. That gives 3,000 km for 0.01 s. Barring relativistic beaming with $\gamma \sim 100$ or more, this demands a compact object (white dwarf, neutron star, black hole) instead of a normal star. The next question is **Ask class:** what are ways in which one can get a periodic signal from a star or stars? Generically one might think of rotation, vibration, or a binary orbit (indeed, all three are seen in various circumstances). Let's focus first on white dwarfs. Can they, by rotating, vibrating, or pulsating, give a 0.033 s period? No: for all three processes, the minimum period is of the order $2\pi(G\rho)^{-1/2}$, which is a minimum of $\sim$seconds for $\rho \sim 10^8$ g cm^{-3} maximum densities of white dwarfs. Therefore, white dwarfs can't do it. **Ask class:** what could one say about black holes? Can they produce periodic pulsations? No, but the answer isn't as obvious as it used to be. Getting a periodic signal out implies some fixed structure, and there just isn't any such place on a black hole. Of course, *quasiperiodic* oscillations are seen, just nothing as coherent as from a pulsar. That means we're down to neutron stars. Is it rotation, vibration, or a binary orbit? **Ask class:** what could one say against vibration? Here the ultrahigh density means that the vibration period, of order $2\pi(G\rho)^{-1/2}$, is a few milliseconds, much too fast for the majority of pulsars. **Ask class:** what can be said against binary orbits? Two things. First, inspiral due to gravitational waves would cause the period to decrease, not increase. Second, a binary orbit with a period of seconds would cause the two to spiral into each other in at most a few hours, whereas many pulsars have now been observed for thirty years.

This line of argument, due originally to Tommy Gold, indicates that pulsars must be rotating neutron stars. Therefore, remarkably, neutron stars are *very* observable.

Pulsar Physics: Rotating Dipole

Interestingly, just *before* the discovery of pulsars Franco Pacini had suggested that a rotating pulsar might emit radio waves. This was a promising start, but our understanding of why pulsars emit radio waves is very incomplete even now, decades later.

Let's start with the overall energetics. Suppose that we have a neutron star of radius R with a centered dipolar magnetic field of strength B_p at the magnetic pole, and that the axis of the dipole is offset by an angle α from the rotational axis. The magnitude of the magnetic moment is then

$$|\mathbf{m}| = B_p R^3/2 \ . \quad (1)$$

The nonalignment of the magnetic moment with the rotation axis means that the magnetic moment changes with time. Just as an accelerated charge radiates, so does an accelerated magnetic moment, and the rate is

$$\dot{E} = -\frac{2}{3c^3}|\ddot{\mathbf{m}}|^2 = -\frac{B_p^2 R^6 \Omega^4 \sin^2\alpha}{6c^3} \ . \tag{2}$$

This radiation is (at least initially) emitted at the rotational frequency Ω. This energy comes from the rotational energy of the star, $E = \frac{1}{2}I\Omega^2$, where $I \sim 10^{45}$ g cm^2 is the moment of inertia. Therefore we also have $\dot{E} = I\Omega\dot{\Omega}$. The pulsar slows down as a result of the torque exerted by the radiation. If it starts with spin frequency Ω_i, then if we consider the current slowdown time $T = -\Omega/\dot{\Omega} = P/\dot{P}$ and say that the current spin frequency is Ω_0, then the spin frequency at time t is

$$\Omega = \Omega_i \left(1 + \frac{2\Omega_i^2}{\Omega_0^2}\frac{t}{T}\right)^{-1/2} \ , \tag{3}$$

so the present age of the pulsar is ($\Omega = \Omega_0$)

$$t = \frac{T}{2}\left(1 - \Omega_0^2/\Omega_i^2\right) \ . \tag{4}$$

If the current spin frequency is much less than the initial spin frequency, then $t \approx T/2$. This leads to the definition of the "characteristic age" $T_c = P/2\dot{P}$ of the pulsar. In addition, the orthogonal component of the pulsar's magnetic field, $B_p \sin\alpha$, is given in this model by

$$B_p \sin\alpha(\mathrm{G}) = 3 \times 10^{19}\sqrt{P\dot{P}} \ , \tag{5}$$

where P is the period in seconds and $\dot{P}$ is dimensionless. For example, these formulae imply that the pulsar in the Crab Nebula has a magnetic field $B_p \approx 5 \times 10^{12}$ G (for $\sin\alpha = 1$) and a characteristic age of $\sim$ 1200 yr. The actual event that produced the neutron star occurred in 1054 AD, our time, so this is remarkably close. The spindown energy of the Crab pulsar is $\approx 5 \times 10^{38}$ erg. This is a lot of energy. It is thought, in fact, that although the initial electromagnetic energy might be in long-wavelength waves, the energy is primarily carried out in the form of ultrarelativistic particles. These pervade the nebula and give rise to characteristic synchrotron emission from electrons.

This model is a decent general model that explains many of the broad characteristics of pulsars. Aspects of it are, in fact, more general than one would suspect. For example, as Goldreich and Julian (1969; see Shapiro and Teukolsky 10.7) showed, even if the pulsar is an *aligned* rotator, the scaling of the energy loss is the same as in the nonaligned rotator model. The basic reason is that you have a magnetic field spinning through a very good conductor. This produces electric fields, which pluck charges from the surface and accelerate them to high energy, leading to energy loss and rotational torques. In fact, GJ showed that there is a characteristic charge density (called the Goldreich-Julian density) that is produced,

$$\rho_{\rm GJ} = -\frac{1}{2\pi c}\mathbf{\Omega}\cdot\mathbf{B}\ . \tag{6}$$

As is often the case, this charge density is set up because electromagnetic forces are overwhelmingly larger than gravitational (even near a neutron star!), so this density is formed to compensate. Note that the *number* density of particles could greatly exceed $\rho_{\rm GJ}/e$, because the G-J density is simply the *uncompensated* charge.

In these models, there is another important radius that enters the problem. This is called the light cylinder, or the speed-of-light cylinder. Plasma near the neutron star must rotate with it, because of the amazingly strong magnetic fields. However, at a large enough radius the plasma cannot rotate with the star, because this would force it to go faster than the speed of light. Therefore, the light cylinder radius is $r_L = c/\Omega$. Well inside this, at least for most pulsars, the magnetic field is probably primarily dipolar (recall that higher multipoles decrease with radius faster than the dipole, so the farther away one gets the more dipolar the field configuration is). Outside it, magnetic flux is conserved (all field lines are open), so the field strength scales such that Br^2=const, so $B \sim r^{-1}$. This means that the field configuration is azimuthal, with tighter and tighter windings.

One last comment about magnetic dipole models has to do with the "braking index". This is defined as $n = \Omega\ddot{\Omega}/\dot{\Omega}^2$, which is 3 for the magnetic dipole model. Another way of saying this is that this implies a power-law deceleration model in which $\dot{\Omega} \propto -\Omega^n$. This is not easy to measure for most pulsars, since $\ddot{\Omega}$ is a tough quantity to determine accurately, but it is somewhat disturbing that existing measurements almost always give n significantly less than 3.

Pulsar Emission Mechanisms

That's all very well, you may say, but exactly how does a pulsar emit radio waves? Well, er, no one really knows in detail. There are some observational clues. The radiation has to be substantially beamed in many cases (<10% duty cycle in many pulsars), and it has to be coherent (i.e., maser-like) because the brightness temperatures are often up to $T = 10^{25-30}$ K! One big problem in figuring this out is that the fraction of energy released in radio waves is a tiny, tiny fraction of the total energy: $< 10^{-9}$ for the Crab pulsar, for example. So, figuring out what causes the radio emission may be like figuring out how an airplane works by watching the seatbelt light go on and off. Some pulsars have gamma-ray emission that can be a few tens of percent of the spindown energy, so there may be more hope in that direction.

Still, let's plow onwards anyway. As an example of one such model, let's talk about a general idea proposed a few years after the discovery of pulsars, that is probably close to correct. The idea is that since you have a magnetic field rotating through a conductor, there will be an electric field set up, and hence a potential drop that can be of the order of 10^{15-16} V (remember 1 sV=300 V). This is extremely large. Imagine that a stray electron gets into this region. It is accelerated to really huge energies almost immediately. But, the strength of the magnetic field forces the electron to move along it; therefore, the electron emits curvature radiation. When the electron has an energy of $\sim 10^{12}$ eV, the curvature photons themselves have energies of several times $m_e c^2$. They travel across the field lines, but in doing so will single-photon pair produce almost immediately. This produces more electrons, and also positrons that go in the opposite direction but with the same effect.

All this leads to a "pair cascade", in which a single seed electron can generate huge numbers of charges. The effect of all these charges is to short out the electric field that started the ball rolling. The charges then sail out (relativistically) into space. Their high density makes maser activity favorable, so radio waves are generated. After the charges have moved away the cycle begins again.

This model probably has kernels of truth in it. One interesting prediction is that if the electric field is too weak, curvature radiation will not produce pairs, so there will be no pair cascade and no radiation. This predicts that there will be a "death line" at too high a period or too low a magnetic field. Indeed, few pulsars are seen beyond this death line, so this probably is close to the real situation. On the other hand, there are a number of bizarre behaviors of pulsars that aren't easily explained in this model or any other current model. For example, there are subpulses seen in the emission, and these drift systematically from pulse to pulse. For some pulsars, "nulling" occurs, such that for minutes to hours there are no pulses at all. When the pulsar becomes active again, though, the subpulses are at the same phase that they had before the nulling! Somehow, there is a memory of phase.