See Shapiro and Teukolsky, Chapter 15

Magnetic Accretion onto Neutron Stars

A crucial difference between neutron stars and black holes is that neutron stars can have an intrinsic magnetic field. As discussed before, that field can be extremely strong, so strong that the flow of ionized gas is channeled by the field. This produces the phenomenon of accretion-powered pulsars, and is also critical in the evolution of millisecond pulsars. We'll start by looking at the interaction of the field with the inflowing matter, then investigate its consequences.

Alfvén radius

As a first calculation, let's see if we can figure out the characteristic radius at which magnetic stresses dominate the flow in the accretion disk. The region inside this radius is called the *magnetosphere*. This involves comparing quantities. **Ask class:** what quantities should we compare to see if the magnetic field can channel the flow? As a first guess, we could try energy density. The magnetic energy density is $B^2/8\pi$, and the kinetic energy density of the matter is $\frac{1}{2}\rho v^2$, where ρ is the density and v is the typical velocity. Specifically, suppose that the magnetic field is dipolar, so that $B = \mu/r^3$, and that the matter moves in spherical radial free fall, so that $v = v_{ff} = \sqrt{2GM/r}$. By continuity, $\rho = \dot{M}/(4\pi v_{ff} r^2)$.

Before solving this equation, note the radial dependences. The magnetic energy density goes as r^{-6}, whereas the material energy density goes as $r^{-5/2}$. The magnetic stresses thus increase much more steeply with decreasing radius than the material stresses do. Therefore, generically one expects that far from the star, material stresses must dominate. Close to the star, magnetic stresses will dominate if the field is strong enough; for $B = 10^{12}$ G, the magnetic stresses at the stellar surface are orders of magnitude stronger than the material stresses, so there is some radius where the two balance approximately. This radius is sometimes called the Alfvén radius, and is

$$r_A = \left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/7} = 3.2 \times 10^8 \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} \left(\frac{M}{M_\odot}\right)^{-1/7} \text{ cm} . \quad (1)$$

A magnetic moment of $\mu = 10^{30}$ G cm^3 gives a surface field of about 10^{12} G, so this is typical of neutron stars in high-mass X-ray binaries. Since the radius of a neutron star is $R \approx 10^6$ cm, the accretion flow onto a strongly magnetized neutron star is dominated by the magnetic field.

Caveat: the preceeding derivation gives an approximate value for this stress balance radius, not an exact one. For example, one would really be interested in nearly circular flow, so the numbers would change a bit. Also, to be careful one should really compare the $r\phi$

components of the stress, because this is relevant for nearly circular flow that is moving in slowly. However, the ultimate answer for the radius is close to what we derived, for the simple reason that the radial dependence of the magnetic stress is much stronger than that of the material stress, so a little change in the radius and the ratio of stresses changes dramatically. Another subtlety is that there is, of course, not a sharp transition from matter-dominated to field-dominated. Still, this is good enough for a start.

Ask class: suppose that the field is quadrupolar instead of dipolar. Would you expect the transition to be narrower or broader than in the dipolar case? Narrower, because the radial dependence of the field stresses is steeper.

The details of how the plasma in the disk actually hooks onto the magnetosphere are complicated. It may be that magnetic Rayleigh-Taylor instabilities play a role, or it could be magnetic reconnection. It's sketchy, and a problem is that observation of the relevant sources (accretion-powered pulsars) can't tell us about the specific plasma physics.

Spinup by magnetic accretion

Just as with any accretion from a disk, angular momentum is accreted. Let's start by imagining that the star is not rotating. Then, to a reasonable approximation, the angular momentum accreted per time (i.e., the torque) is just the accretion rate times the specific angular momentum at r_A, or $N \approx \dot{M}\sqrt{GMr_A}$. Therefore, the star spins up. Now, suppose that the star has spun up to a frequency equal to the orbital frequency at r_A: $\omega_s = \sqrt{GM/r_A^3}$. **Ask class:** what effect does further accretion have on the star, given that the specific angular momentum at r_A is unchanged (by assumption)? The star's spin frequency is not changed, but its angular momentum goes up (this is possible because the moment of inertia increases). To understand this, let's define the corotation radius $r_{\rm co}$. At $r_{\rm co}$, the Keplerian angular velocity equals the spin angular velocity of the star: $\omega_{\rm spin} = \sqrt{GM/r_{\rm co}^3}$. Material in Keplerian orbits outside $r_{\rm co}$ that interacts with the star via the magnetic field exerts a braking torque on the star, whereas material in Keplerian orbits inside $r_{\rm co}$ that interacts with the star speeds the star up. When $r_{\rm co} \approx r_A$, the two roughly cancel each other (in reality the radius at which the torques balance is slightly different from r_A).

Now suppose that the star is spinning much faster than the Keplerian frequency at r_A. **Ask class:** qualitatively, what should happen? In a rough sense, one expects that the star will be slowed down by the coupling with the matter, because of a "drag" exerted at the interaction radius. It may also be (and this is a topic of current debate!) that the matter is flung out by this interaction, as if the field was like a propeller (hence this is called the "propeller effect" or, more generally, a centrifugal barrier). If so, one expects that the mass accretion rate would drop drastically if the propeller phase were entered. **Ask class:** in what circumstance might one imagine that such a phase would be entered in a real system? Many sources are transients, and from the above equations it is clear that if $\dot{M}$ changes rapidly, so will r_A. There are some cases in which evidence for a propeller phase has been claimed, due to sharp dropoff in luminosity, but this is difficult in practice because the luminosity is low as such a phase is approached.

That means that, if there is time and $\dot{M}$ and B are constant, one expects that magnetic accretion will tend to make the star spin at the Keplerian frequency at r_A. **Ask class:** how can we find out how long it will take until the star spins at roughly this equilibrium frequency? Figure out the angular momentum that needs to be accreted, then determine the time necessary from the specific angular momentum at r_A and the mass accretion rate. Suppose you have an equilibrium frequency of 1 rad s^{-1}, which is typical for stars in these systems. The angular momentum is $J = I\Omega \approx 10^{45} \cdot 1 = 10^{45}$ cgs. The Alfvén radius is determined by $\Omega = \sqrt{GM/r_A^3}$, or $r_A \approx 6 \times 10^8$ cm for $M = 1.5\,M_\odot$. The specific angular momentum is then $\ell = \sqrt{GMr_A} \approx 10^{17}$ cgs. Therefore, the amount of mass ΔM that must be accreted to spin the star up from nonrotating to close to the equilibrium frequency is given by $\Delta M \approx J/\ell = 10^{28}$ g. If the neutron star accretes at roughly 10% of the Eddington rate, or about 10^{17} g s^{-1} (common for these systems) then this takes only $\sim 10^{11}$ s $\sim 10^{3-4}$ years. In reality, once a star has spun up close to its equilibrium spin frequency, the timescale for further change is increased.

Types of Sources

As with accreting black holes, there are two types of accreting neutron star systems: low-mass X-ray binaries (LMXB) and high-mass X-ray binaries (HMXB), where "low-mass" and "high-mass" refer to the mass of the donor star. As with black holes, LMXB accretion tends to be via Roche lobe overflow, whereas HMXB accretion is from a stellar wind. But for neutron stars, there is another major difference between NS in LMXB and NS in HMXB: neutron stars in HMXB have surface magnetic fields on the order of 10^{11-13} G, similar to normal rotation-powered pulsars such as the Crab. However, NS in LMXB have much weaker surface fields, inferred typically to be in the 10^{8-10} G range. The cause of this difference is not well understood, although there has been some speculation. Let's talk first, though, about the properties of each source.

High-mass X-ray binaries.—since NS in high-mass binaries have strong fields, the field is able to capture and channel matter. From the Alfvén radius derived earlier, the capture radius should be of order few$\times 10^8$ cm, compared with only 10^6 cm for the radius of the star. Therefore, matter flows along field lines that connect to the magnetic polar regions: the equation of a dipole field line is $r/\sin^2\theta$=const, so when $\theta \ll 1$ we have $\theta \sim (R_s/r_A)^{1/2} \sim 10^{-1}$ for a typical HMXB system. Therefore, most of the accreting matter falls on a region which is a fraction $(10^{-1})^2/4\pi \approx 10^{-3}$ or often less of the whole surface area of the star. As a result, almost all of the accretion energy is released in two "hot spots" near the two magnetic poles. If the magnetic axis is not aligned with the rotational axis, then as the star rotates we see more or less of the hot spot, and hence see pulsations in the X-rays. This is very similar to how we see pulsed radiation from rotation-powered pulsars. One sometimes sees these two classes of pulsars referred to as "X-ray pulsars" and "radio pulsars", but this is not as descriptive as "accretion-powered" and "rotation-powered", respectively.

Ask class: on thinking about this more deeply, don't we have a problem? The accretion rate of, say, $0.1\,\dot{M}_E$ on a surface area only 10^{-3} of the star means that the local flux generated can be 100 times Eddington or more! What does that mean for this system? It means that for such accretion to persist, the radiation cannot escape back up the accretion funnel. Instead, it has to come out the sides. This is a reminder that the Eddington flux is a limit only for spherically symmetric systems, and in this case we have a system that is very aspherical! It also means that the radiation pattern can be a "fan beam", so that we might get two peaks per cycle from the funnel (one from one side, one from the other) as opposed to the one peak we would expect if this were just a thermally glowing hot spot.

A last comment about HMXBs is that their magnetic fields can be estimated in two ways. A direct measure is the detection of a cyclotron scattering or emission feature. Recall that the electron cyclotron energy is $\hbar\omega_c = \hbar eB/m_e c = 11.6 B_{12}$ keV, where $B = 10^{12} B_{12}$ G. Therefore, if $B \sim 10^{12-13}$ G, a feature may be visible in the spectrum (which usually has a power-law component extending to tens to hundreds of keV). From the central energy of this feature one can estimate the field. The other way is by spindown: from the luminosity one can estimate the mass accretion rate, and from the way the spin frequency changes (or an equilibrium frequency) one can estimate the field using the magnetic torque arguments we discussed earlier. This is a much less certain method, but it gives order of magnitude agreement with the field derived from cyclotron features, in the cases where both methods can be used.

Low-mass X-ray binaries.—With a weak field, the situation is similar in some ways but quite different in others. **Ask class:** suppose that at some accretion rate, $r_A = 3\times 10^8$ cm if $\mu = 10^{30}$ G cm^3 ($B_s = 10^{12}$ G). Approximately what would r_A be if $\mu = 10^{26}$ G cm^3 instead? Scaling is as $\mu^{4/7}$, so it would be about 10^6 cm, or roughly the radius of the neutron star itself! Therefore, accretion by a weakly magnetized neutron star can have the matter flow very close to the stellar surface before it is captured by the magnetic field, and even then the field may not control the flow fully. Among other things, this means that radiation from the accretion disk in this case will contain a lot of information about the strongly curved spacetime near the star. This is not as true for strongly magnetized neutron stars, since the flow is channeled by the field from far out.

Another difference is a potential complication. Higher multipoles (quadrupole, octopole, etc.) die away with radius faster than the dipolar component of the field. Therefore, when one is at radii hundreds of times the radius of the star, it is probably a good approximation to assume that only the dipolar component survives. However, close to the star this may not be the case. Unfortunately, there is no easy way to model such higher multipole components, so usually they are ignored for simplicity.

In any case, if matter is captured very close to the star, it lands on an area comparable to the area of the star. **Ask class:** what would this mean in terms of pulsations? It means that they will be a lot weaker than they are in the case of HMXBs. In fact, of the dozens of NS LMXBs known, only a few have pulsations. It is a mystery why others don't show such pulsations. There are other reasons why they should be weak (e.g., scattering from gas in a surrounding corona), but it is fair to say that no one really has a quantitative reason why, particularly because the ones with pulsations have very strong, easily visible pulsations!

Neutron-star LMXBs are thought to be the progenitors of millisecond rotation-powered pulsars. MSPs were first discovered in 1983, and have weak fields ($B = 10^{8-10}$ G) and rapid spins (P is typically 1.5-10 ms, as you'd expect from the name). It is thought that these are "recycled" pulsars, and are therefore actually very old (10^{8-10} yr, perhaps). The scenario is that a neutron star in a low-mass binary accretes from a companion, and if the NS has a weak enough field then r_A is so small that the equilibrium frequency is hundreds of Hertz. The star is then spun up by this accretion, and after accretion stops it is left as a millisecond rotation-powered pulsars. Strong evidence for this is that in the Galactic disk about 80% of millisecond pulsars are in binaries, compared to about 1% of normal pulsars! Some millisecond pulsars are isolated; it could be that this is because they are born in a different way, or perhaps radiation from the MSP destroys the companion (this is the "black widow" scenario, although it is far from clear that destruction proceeds quickly enough to remove the companion entirely).

But there is still a major puzzle: what is it that makes the field so weak relative to the field in other neutron stars? Decay of neutron star magnetic fields has been suggested in one form or another since 1969, when Gunn and Ostriker produced evidence that the field of normal pulsars decays over a 10^{6-7} yr timescale. However, this evidence is no longer believed. It is currently more fashionable to think that accretion itself weakens the field from an initial value of 10^{12} G, typically, to a final value of 10^{8-9} G. However, this is very tough to do when one looks at the details. In addition, one must remember that HMXBs, which accrete actively (near Eddington in many cases) over expected 10^{6-7} yr timescales, show no evidence at all of a weakening field. It is true that LMXBs have few$\times 10^8$ yr accretion lifetimes, so maybe that's the difference, but it isn't all that satisfying to demand that the field decays only when we're not looking, so to speak! This is another major unresolved issue.