Binary sources of gravitational radiation

For our final two lectures we will explore binary systems in general and the LIGO/Virgo detections in particular. Binaries obviously have a large and varying quadrupole moment, which we determined back in Lecture 4 is necessary for gravitational radiation, and they have the additional advantage that even before the detections we knew that gravitational radiation is emitted from them in the expected quantities (based on observations of double neutron star binaries starting in the 1970s). The characteristics of the gravitational waves from binaries, and what we could learn from them, depend on the nature of the objects in those binaries. In this lecture we will therefore start with some general concepts, then discuss individual types of binaries. In the next lecture we will wrap up with specifics about the LIGO detections, and about the future of gravitational wave detections of binaries of other masses.

For what it's worth, I have written a textbook with Nico Yunes of the University of Illinois on gravitational radiation: "Gravitational Waves in Physics and Astrophysics: An Artisan's Guide", which was an interesting and fun experience!

Suppose that the binary is well-separated, so that the components can be treated as points and we only need to incorporate the lowest order contributions to gravitational radiation in our analysis. Temporarily restricting our attention to circular binaries, how will their frequency and amplitude evolve with time?

Let the masses be m_1 and m_2, and the orbital separation be R. From the information in Lecture 4 we find that the amplitude a distance $r \gg R$ from this source is $h \sim (\mu/r)(M/R)$, where $M \equiv m_1 + m_2$ is the total mass and $\mu \equiv m_1 m_2/M$ is the reduced mass. We can rewrite the amplitude using $f \sim (M/R^3)^{1/2}$, to read

$$\begin{aligned} h &\sim \mu M^{2/3} f^{2/3}/r \\ &\sim M_{\rm ch}^{5/3} f^{2/3}/r \end{aligned} \tag{1}$$

where $M_{\rm ch}$ is the "chirp mass", defined by $M_{\rm ch}^{5/3} = \mu M^{2/3}$. The chirp mass is named that because it is this combination of μ and M that determines how fast the binary sweeps, or chirps, through a frequency band. When the constants are put in, the dimensionless gravitational wave strain amplitude (which, remember, is roughly the fractional amount by which a separation changes as a wave goes by) measured a distance r from a circular binary of masses M and m with a binary orbital frequency $f_{\rm bin}$ is (Schutz 1997)

$$h = 2(4\pi)^{1/3} \frac{G^{5/3}}{c^4} f_{\rm GW}^{2/3} M_{\rm ch}^{5/3} \frac{1}{r} \ , \tag{2}$$

where $f_{\rm GW}$ is the gravitational wave frequency. Redshifts have not been included in this formula.

The luminosity in gravitational radiation is then

$$\begin{aligned} L &\sim 4\pi r^2 f^2 h^2 \\ &\sim M_{\rm ch}^{10/3} f^{10/3} \\ &\sim \mu^2 M^3/R^5 \ . \end{aligned} \tag{3}$$

The total energy of a circular binary of radius R is $E_{\rm tot} = -G\mu M/(2R)$, so we have

$$\begin{aligned} dE/dt &\sim \mu^2 M^3/R^5 \\ \mu M/(2R^2)(dR/dt) &\sim \mu^2 M^3/R^5 \\ dR/dt &\sim \mu M^2/R^3 \ . \end{aligned} \tag{4}$$

What if the binary orbit is eccentric? The formulae are then more complicated, because one must average properly over the orbit. This was done first to lowest order by Peters and Matthews (1963) and Peters (1964) by calculating the energy and angular momentum radiated at lowest (quadrupolar) order, and then determining the change in orbital elements that would occur if the binary completed a full Keplerian ellipse in its orbit. That is, they assumed that to lowest order, they could have the binary move as if it experienced only Newtonian gravity, and integrate the losses along that path.

Before quoting the results, we can understand one qualitative aspect of the radiation when the orbits are elliptical. From our derivation for circular orbits, we see that the radiation is emitted much more strongly when the separation is small, because $L \sim R^{-5}$. Consider what this would mean for a very eccentric orbit $(1-e) \ll 1$. Most of the radiation would be emitted at pericenter, so this would have the character of an impulsive force. With such a force, the orbit will return to where the impulse was imparted. That means that the pericenter distance will remain roughly constant, while the energy losses decrease the apocenter distance. As a consequence, the eccentricity decreases. In general, gravitational radiation will decrease the eccentricity of an orbit.

The Peters formulae bear this out. If the orbit has semimajor axis a and eccentricity e, their lowest-order rates of change of the orbital parameters are

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3 \mu M^2}{c^5 a^3 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) \tag{5}$$

and

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304}{15} e \frac{G^3 \mu M^2}{c^5 a^4 (1-e^2)^{5/2}} \left(1 + \frac{121}{304} e^2\right) \ , \tag{6}$$

where the angle brackets indicate an average over an orbit. One can show that these rates imply that the quantity

$$ae^{-12/19}(1-e^2)\left(1+\frac{121}{304}e^2\right)^{-870/2299} \tag{7}$$

is constant throughout the inspiral. If we ignore the final factor (which is always between 0.88 and 1), we can write this as $a(1-e)(1+e)e^{-12/19} \approx \text{const}$. For high eccentricities such that $1-e \ll 1$, $1+e$ and $e^{-12/19}$ are roughly constant, so $a(1-e) = r_p \approx \text{const}$, which means that the pericenter distance r_p is roughly constant as promised. For low eccentricities such that $1-e^2 \approx 1$, we get $ae^{-12/19} \approx \text{const}$. The orbital frequency (which is half the dominant gravitational wave frequency when $e \ll 1$) is $f \propto a^{-3/2}$, which means that $f \propto e^{-18/19}$, or roughly $e \propto f^{-1}$. Thus for low eccentricities, the eccentricity roughly scales as the reciprocal of the frequency. This means that binary sources at the high frequencies detectable using LIGO can usually be considered to be effectively circular.

How compact binaries can merge

The basic ways that compact binaries can come together break down to two major categories:

1. Evolution of an isolated massive binary. That is, we start with a pair of massive stars that both evolve into black holes, and merge, without any other stars coming close enough to do anything.

2. Dynamical processes. Examples include single-binary interactions, the Kozai-Lidov resonance, and direct dynamical capture.

Let's talk first about isolated massive binaries. The fine line that must be walked to result in a compact object merger is that the stars must begin far enough apart that they do not merge before both are compact objects, but close enough together that the final double compact object binary can then merge within a few billion years under the influence of gravitational radiation alone. As we now discuss, the study of the evolution of massive binaries is particularly difficult because observational evidence is tough to obtain: massive stars are rare and short-lived, and the most critical evolutionary phases for compact object mergers occupy very small fractions of the short lives of these systems.

We are therefore largely dependent on theory to tell us what is likely to happen. Massive stars will under most circumstances expand into giants after they run out of hydrogen in their cores (an exception might be if they rotate rapidly enough that they are fully convective, which would allow the stars to continue to cycle hydrogen into the core). Thus it is possible that a pair of massive stars that are initially much too far separated to spiral in via gravitational wave emission can, in the "common envelope" phase (where the envelope of a giant encompasses its companion), be dragged much closer together. If the pair begins too close together, it might merge; if one of the stars was already a compact object, it could then reside in the center of the other star and thus form a hypothesized "Thorne-Żytkow object", but it will not produce a compact binary. Thus binaries need to start their lives far enough apart to avoid merger, but not so far apart that common envelope drag is insufficient to reduce the separation to a few tenths of an astronomical unit (which is needed for the inspiral to take a few billion years or less).

Unfortunately, the common envelope phase is *very* difficult to understand from a purely theoretical point of view, and given that no binary has ever been definitively seen *in* a common envelope state, the uncertainties are huge. In fact, over the last decade plus there have been times when different treatments of common envelopes have given rate estimates (say, for double black hole binaries) that differ by more than two orders of magnitude!

That's not the only problem, either. For example, both neutron stars and black holes are produced by core-collapse supernovae. When we look at neutron stars it is clear that many of them have received kicks (i.e., net linear momentum) because of the core collapse. There is also evidence of supernova kicks for some black holes. However, the origin of these kicks is not known, and neither are the kick direction or the kick magnitudes as a function of the compact object mass (perhaps neutron stars are often kicked at hundreds of km s^{-1}, with some exceptions, and black holes are kicked at tens of km s^{-1}).

The best that can be done is to look at the final systems of two compact objects and tune the parameters of binary evolution models to agree with those systems as well as possible. The problem at this stage is that there are only ~ 10 double neutron star systems known, and no known binaries in our Galaxy with two black holes or a black hole and a neutron star. Binary evolution models aren't simple, and the interpretation of the aftermath systems is far from easy, so these models are very underdetermined.

An interesting side point is that when double neutron star merger rates are computed or used to calibrate model parameters, one such system, which will merge in a few billion years, is not included in the calculations. That poor, unappreciated system is ignored because it is in a globular cluster. I love globular clusters, so I am outraged, outraged I tell you, at this blatant discrimination!

But should I be? The estimated rate of mergers in the body of our Galaxy is about 10–

100 per million years, with large uncertainties. What rate might we expect from globulars? Suppose that every one of the ~ 100 globular clusters around our Galaxy had, initially, 200 neutron stars (probably a large overestimate) and they all merge within 10 billion years (certainly overly optimistic!). Then there are $100 \times 100 = 10^4$ mergers in 10^{10} years, for a rate of 1 per million years. We can ignore this because the disk contribution is much greater. This is *not* true of estimates of double black hole or BH-NS mergers, because as indicated above we don't know any examples of such systems in our Galaxy and thus possibly formation channels in globulars dominate.

It is thus worth taking a brief diversion to discuss what might be different about globulars, and indeed this will lead us into the second general path to mergers: dynamical processes.

The main difference between the main part of our Galaxy and globular clusters (or systems such as nuclear star clusters) is stellar number density; in the Solar vicinity there are roughly 0.15 stars per cubic parsec, but in the center of the densest globulars the density can be 10^6 per cubic parsec. This still isn't enough to have stars collide directly with each other very often, but it does mean that binary systems, which act as if their collision cross sections are the sizes of the orbits, can have collisionless three- and four-body encounters. That can be significant. For example, in a standard semi-rich globular with a velocity dispersion of 10 km s^{-1}, stars pass within 1 AU of each other (and hence binaries with radii of 1 AU have strong encounters) once per few hundred million years on average in a cluster of number density 10^5 per cubic parsec. Thus over the 10^{10} year lifetimes of these clusters, binaries can undergo tens of such interactions.

The interactions are chaotic, but computer simulations show that when a binary and single interact, the binary that emerges from the interaction tends to contain the two most massive of the three original objects. Thus neutron stars and black holes, which are more massive than the average star in a globular, can swap into binaries and eventually find compact objects as companions. Therefore, per stellar mass, globulars are expected to have far more of these systems than the low number density bulk of their host galaxies. For the same reason, there is a high rate per mass in globulars of low-mass X-ray binaries and millisecond pulsars (which are thought to have been spun up by accretion from a companion).

Another popular process that can happen with multiple stars is the Kozai (or Kozai-Lidov, or more recently von Zeipel - Kozai - Lidov; it turns out that determining who had an idea first isn't trivial!) resonance. Kozai and Lidov discovered independently in the early 1960s (and von Zeipel found in the early 1900s) that if a binary is orbited by a third object in a hierarchical triple (such that the system has long term stability), and the binary orbital axis is strongly tilted with respect to the orbit of the tertiary, then over many orbits of both the binary and the tertiary, the relative inclination of the binary to the tertiary cycles between low and high values, while conserving the semimajor axis. Most importantly for merger possibilities, when the inclination goes down the eccentricity goes up and vice versa. Thus in the right range of orientations the binary could be driven to such a high eccentricity that gravitational radiation grinds it down to merger. Careful observations of massive binaries in our Galaxy suggest that 10% or more of them could actually be triples, so in principle such systems could evolve naturally to high-eccentricity states (although if the system is susceptible to such evolution it is likely that it would be driven to collisions on the main sequence or giant branch rather than when the objects become compact). In dense stellar systems such as globular clusters, hierarchical Kozai-susceptible triples can be created *after* evolution to compact objects, for example as an outcome of binary-binary interactions.

Yet another possibility, although one with much smaller probability, is that two initially unbound compact objects could pass close enough to each other that the gravitational radiation they emit during their closest passage carries away enough energy to bind the objects together. They would then coalesce quickly. The reason that this is a very low-probability event is that the objects would have to come very close to each other to radiate the required energy, and thus the cross section for the process is tiny. If this happens, it seems most likely to happen during the many chaotic interactions that occur during a binary-single interaction than during a random encounter between two single objects. In any case, high stellar densities are obviously important for this mechanism to have any chance.

All this means that globulars, and the nuclear star clusters in the centers of galaxies, could be ripe breeding grounds for the BH-BH and BH-NS systems that we have not yet seen in our Galaxy.