Neutrinos, nonzero rest mass particles, and production of high energy photons

Particle interactions

Previously we considered interactions from the standpoint of photons: a photon travels along, what happens to it? Now, we'll think about interactions of particles: an electron, proton, or nucleus zips along, what happens to it?

Ask class: generically, what could, say, an electron interact with? Photons, protons or nuclei, magnetic fields, neutrinos. Let's first consider interactions of electrons with photons. **Ask class:** for a low-energy electron interacting with low-energy photons, what is the cross section (not a trick question)? Just the Thomson cross section. In fact, since this is exactly the same process as we considered before, the cross section for general energies is again the Klein-Nishina value.

Compton scattering

In the absence of a magnetic field, the cross section for the interaction of a photon with an electron is just the Thomson cross section ($\sigma_T = 6.65 \times 10^{-25}\text{cm }^2$) for low energies, but becomes more complicated at higher energies. The general Klein-Nishina cross section is in Rybicki and Lightman and other standard references. Let us define $x \equiv \frac{\hbar\omega}{mc^2}$, where $\hbar\omega$ is the energy of the photon in the rest frame of the electron. For $x \ll 1$ this reduces to the Thomson value, whereas for $x \gg 1$

$$\sigma \approx \frac{3}{8}\sigma_T x^{-1}(\ln 2x + 0.5)\ . \tag{1}$$

Radiation can exert a force on matter, via scattering or other interactions. Radiation force is often referred to as radiation pressure in the literature. However, let's give some thought to this. Suppose that an electron is in an isotropic bath of radiation. The radiation pressure is nonzero; $P_r = aT^4$, in fact. **Ask class:** is there any net radiation force on the electron? No, because the bath is isotropic. In this situation it would be more accurate to say that the force is due to a pressure *gradient*. This is the same reason why we're not currently being crushed by the atmosphere, despite the pressure of about 1 kg per square centimeter (many tons over the whole body). Even more accurately, it's the net radiation flux that matters.

By balancing radiation force with gravitational force we can define the *Eddington* luminosity, which is very important for high-energy astrophysics. For a flux F on a particle of mass m and scattering cross section σ around a star of mass M, the balance implies

$$\frac{GMm}{r^2} = \frac{\sigma}{c}F = \frac{\sigma}{c}\frac{L}{4\pi r^2}\ , \tag{2}$$

where L is the luminosity. The r^{-2} factors cancel, leaving us with the Eddington luminosity L_E:

$$L_E = \frac{4\pi GcMm}{\sigma}\ , \tag{3}$$

For fully ionized hydrogen, we assume that the electrons and protons are electrically coupled (otherwise a huge electric field would be generated), so the light scatters off the electrons with cross section σ_T and the protons provide the mass m_p. Then $L_E = 4\pi GcMm_p/\sigma_T = 1.3\times10^{38}(M/M_\odot)$ erg s^{-1}. If the luminosity of the star is greater than this, radiation will drive matter away. This is also the maximum luminosity for steady spherical accretion.

Ask class: does the dependence on m and σ make sense, that is, should m be in the numerator and σ in the denominator? Yes, because if gravity is stronger (m is higher) then more luminosity is needed; if σ is greater, radiation couples more effectively and the critical luminosity is less. This means that for a fixed density, large things are less affected by radiation than small things (because for a size a, the mass goes like a^3 whereas the area goes like a^2. So, asteroids are not affected significantly by radiation!

Curvature radiation

We know that any accelerated charge radiates. If there is a magnetic field around, there are two ways the charge can be acclerated. One is if it moves perpendicular to the field (synchrotron radiation). The other is if it moves along the field, but the field is curved (curvature radiation). We'll just state a couple of results here.

The power emitted by curvature radiation is (see, e.g., Jackson 1975)

$$P \approx \frac{2}{3}\frac{e^2c}{R^2}\gamma^4 \tag{4}$$

where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = |\mathbf{v}|/c$. **Ask class:** is the dependence on R reasonable? Yes, because for smaller radius of curvature and a fixed energy, the acceleration is greater and hence so is the radiation.

Synchrotron radiation

If a particle of charge e and energy E is moving perpendicular to a static magnetic field of strength B, the frequency of its orbit around the field is

$$\omega_c = \frac{eBc}{E}\ . \tag{5}$$

If the particle has velocity v, this means that its orbital radius is $d = v/\omega_c$, so for highly relativistic particles with $v \approx c$, $d = E/eB$.

A particle may acquire a nonnegligible motion perpendicular to the magnetic field if, e.g., it is created from a photon which was moving with some angle to the magnetic field. If $\gamma\frac{B}{B_c} \ll 1$, then a classical treatment of synchrotron radiation is approximately valid. **Ask class:** recalling that synchrotron radiation is due to acceleration of a charge, suppose you have an electron and a proton with the same Lorentz factor. Which would you expect to lose energy on a faster time scale? The electron, because the proton is not as easily accelerated, hence the proton does not lose energy as rapidly. That's why circular accelerators accelerate protons or ions instead of electrons, and why electron accelerators are straight: the radiation losses are too significant otherwise. However, when the magnetic fields are weak, relativistic electrons have a long synchrotron cooling time. In fact, radio emission from many AGN is dominated by synchrotron radiation.

Pair annihilation, $e^-e^+ \rightarrow \gamma\gamma$

In the extreme relativistic limit, the cross section for two-photon annihilation is

$$\sigma_{\rm an} \approx \frac{3}{8}\sigma_T \frac{\ln 2\gamma}{\gamma} \ . \tag{6}$$

We will consider one-photon annihilation in neutron star section.

Bremsstrahlung

The energy lost per unit length to bremsstrahlung radiation by an electron or positron traversing a region of number density n fixed charges per unit volume is approximately

$$\frac{d(\ln E)}{dx} \approx n\frac{160}{3}\left(\frac{e^2}{\hbar c}\right)\left(\frac{e^2}{m_e c^2}\right)^2 \ . \tag{7}$$

This expression contains two combinations of symbols that are useful to remember. $e^2/\hbar c = 1/137$ is the fine structure constant, and is dimensionless. $e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm is the classical radius of the electron. Evaluating this, we find the column depth for significant interaction, where $d(\ln E) \approx 1$, is about 3×10^{25} cm^{-2}, roughly constant for large E.

Electron-neutrino interactions

Neutrinos interact very weakly; in fact, their existence is the hallmark of the weak force. Typically, a neutrino of energy E_ν has an electron scattering cross section of

$$\sigma_\nu \approx 10^{-44}\left(\frac{E_\nu}{m_e c^2}\right)^2 \ {\rm cm}^2 \ . \tag{8}$$

This is what is technically known as an itsy bitsy cross section. Now, particle physicists have a lot of time and a fondness for alcohol, leading to interesting terminology and names for units. In this case, they've dubbed 10^{-24} cm^2 a "barn" and 10^{-48} cm^2 a "shed", so a typical neutrino cross section is some ten thousand sheds! This compares with the Thomson cross section, which is close to one barn; indeed, hitting an electron with a photon is like hitting the broad side of a barn compared to hitting an electron with a neutrino. For people without a sense of humor, 10^{-44} cm^2=10^{-48} m^2 is one square yoctometer. Pretty small, no matter how you slice it.

Let's figure out the fraction of neutrinos interacting in certain circumstances. First, the Sun. **Ask class:** to order of magnitude, what is the density of the Sun? About 1 g cm^{-3}. That means that the number density is about 10^{24} cm^{-3}. **Ask class:** so, what is the mean free path of ~ 1 MeV neutrinos? About 10^{20} cm. The Sun is about 10^{11} cm in radius, so only about 10^{-9} of the neutrinos interact.

Now let's think about the dense core in the center of a star just prior to a supernova. **Ask class:** if you crush the Sun down to a radius 1000 times less than it actually has, what happens to the optical depth to neutrinos? Density is $1000^3 = 10^9$ times greater, but the length traveled is 1000 times less, so optical depth is 10^6 times greater. That suggests an optical depth of about 10^{-3}. The neutrinos in supernova are actually somewhat more energetic as well, about 3–5 MeV, so a fraction $\sim 10^{-2}$ of the energy is absorbed. This seems to be enough to be the crucial driver of the supernova, since a good 10^{53} erg is released in neutrinos.

Proton-proton interactions

Because of their relatively large mass, protons do not interact significantly in the ways discussed above. However, at high energies proton-proton collisions may produce photons, neutrinos or other products through strong interactions. At TeV energies or higher, more than 99% of the interactions are of the form

$$p + p \rightarrow \pi + X \ , \tag{9}$$

where π indicates a pion (charged or neutral), and X indicates the other products. At a few TeV, the interaction length is roughly 20g/cm^2, or a column depth of $\approx 10^{25}$cm^{-2}. The pions can decay to produce photons or neutrinos. Slane and Fry (1988) found the optimum column depth for photon production is ≈ 50g/cm^2. At this column depth, a proton will typically produce about 10 photons of average energy $\approx \frac{1}{30}$ that of the proton.

Particle acceleration and generation of high-energy photons

Ask class: suppose we observe a photon with an energy of 1 TeV. How could it have been produced? In particular, could it have been produced thermally? No, because the temperature equivalent is $E/k \approx 10^{16}$ K, and nothing in the universe is that hot. So, it must have been produced nonthermally. Ultimately, that means that the photon must have been produced by a high-energy particle. Let's narrow down how it could have been accelerated. **Ask class:** could the particle have been accelerated via the strong force? No, because the distance over which the strong force acts is too small. It would need to have an unbelievable acceleration over the $\sim 10^{-13}$ cm distances. Same with the weak force. **Ask class:** could the particle have been accelerated via gravity? No, but this takes a little more argument. **Ask class:** how does something get accelerated by gravity? Think in particular of boosts given to planetary probes. The boost actually comes from the velocity of the planet in its orbit, not the gravity per se. In fact, gravity is a conservative field, so in an isolated system a particle can't get any net energy from gravity alone. This means that only electromagnetism can accelerate particles.

Now let's think about how electromagnetism can accelerate particles. **Ask class:** what kind of particles are accelerated? In particular, suppose a proton, a neutron, and an iron nucleus are all put in a regions with electric fields. Which one gains the most energy, assuming no losses? The iron nucleus, being most charged, will. The proton is next in line. The neutron is uncharged, so it gains the least.

Ask class: what happens if a proton goes into a region with static magnetic fields but zero electric field? What happens to its speed and direction? Direction can change, but speed won't. **Ask class:** what happens to the energy of the proton in a reference frame in which the magnetic field is moving? Since the proton energy in the field frame is unchanged, you get a Doppler shift in the laboratory frame. Therefore, moving magnetic fields can also increase the energy of protons. Another way to get high-energy particles is to have acceleration of some type along an electric field. **Ask class:** how could this happen? It could be a potential drop or it could be acceleration by Compton scattering (which is indeed a form of electric field).

Therefore, to summarize, the only way to get very high energy photons is to produce them using high-energy charged particles, and to get high-energy particles either moving magnetic fields or electric fields are required. Moving magnetic fields are thought to play an important role in producing high-energy cosmic rays. We will discuss them more in that section. For now, let's focus on potential drops.

Ask class: if you have an electric field $\mathcal{E}$ over a region of size d and put a particle of charge e in it, how much energy does the particle gain by going across the region? $E = e\mathcal{E}d$. If the potential drop $\Phi = \mathcal{E}d$ is large enough, particles can be accelerated to extremely high energies. The highest energy cosmic rays have $E \approx 10^{20}$ eV, so the potential drop needs to be $\Phi \approx 10^{20}$ V. It turns out that a good way to get large potential drops is to have a magnetic field in the same region as a spinning conductor: for example, an accretion disk or rotating neutron star. Lenz's law says that if a magnetic field of strength B is moving with a velocity v through conducting matter, it generates an electric field $E = (v/c)B$. Over a closed circuit of length $\sim d$, this produces an electromotive force $\Phi \approx Ed = (v/c)Bd$. The net result is that a potential drop of magnitude Φ opens up along the field lines, and acceleration can take place there. Physically, what can happen is that charges are drawn apart from each other and separate, leaving a low-density region surrounded by mostly positive charges on one side and negative charges on the other.

We can calculate potential drops in some representative situations. When making conversions, we have to be a little careful: sadly, the cgs unit of potential is a "statvolt", which is 300 Volts(!) You can also look in a book to find that $e = 4.8 \times 10^{-10}$ in cgs units and do it that way. Anyway, let's think first of a strongly magnetized, rapidly rotating neutron star. If $B = 10^{13}$ G on the surface, $d = 10^6$ cm (the size of the star) and $v = 0.1c$ (about a 3 ms spin period) then $\Phi = 10^{17} sV = 3 \times 10^{20}$ V. If we instead are interested in a supermassive black hole, we might have $d = 10^{14}$ cm (radius near the inner edge of the disk), $B = 10^4$ G, and $v = 0.5$c. Then $\Phi = 5 \times 10^{17} sV \approx 2 \times 10^{20} V$.

To usual astrophysical accuracy, both of these seem plenty high to get particles to the $\sim 10^{20}$ eV observed for the highest-energy cosmic rays. However, as we'll see now, it's actually extremely tough: loss processes of many kinds come in like gangbusters at high energies. Still, for really high energies this is the best current bet.