Coding in advance of the Feb 19, 2018 class

The task here is to do parameter estimation based on an effectively continuous data set. That set, which is in the file data4_1.txt on the website, has photon energies that are drawn from a blackbody with a temperature T that you are to estimate from the data set. We assume that the detector is perfect, i.e., it has 100% efficiency at any energy. The basic principles will be the same as are given in the Gaussian example in Lecture 4.

To set this problem up, we note that the number of photons in a blackbody spectrum of temperature T, from energy E to $E + dE$, is proportional to

$$N(E)dE \propto \frac{E^2}{e^{E/kT} - 1} dE \ . \tag{1}$$

We begin by normalizing the spectrum so that the total number of photons from $E = 0$ to $E = \infty$ is the number of photons in the data, $N_{\rm dat}$ (please note that although this is a common procedure, the most general approach is to leave the normalization as a free parameter):

$$\int_0^\infty N_0 \frac{E^2}{e^{E/kT} - 1} dE = N_{\rm dat} \ . \tag{2}$$

Looks nasty, but what we're really like to know is how the normalizing constant depends on the temperature. We can learn this, without doing the integral, by making the substitution $x \equiv E/(kT)$, so that $dx = dE/(kT)$. Then the integral becomes

$$N_0(kT)^3 \int_0^\infty \frac{x^2}{e^x - 1} dx = N_{\rm dat} \ . \tag{3}$$

This is now a universal integral, which is independent of T. In fact, the value of the integral is 2.40411. Thus

$$N_0 = \frac{N_{\rm dat}}{2.40411(kT)^3} \ . \tag{4}$$

As in the Gaussian example, we don't need to worry about the dE, because we will keep its value the same for any T, and thus the expected number of events per some (unspecified but small) dE near E at a temperature T is

$$N(E|T) = \frac{N_{\rm dat}}{2.40411(kT)^3} \frac{E^2}{e^{E/kT} - 1} \ . \tag{5}$$

This plays the same role that $N(v)$ did in our Gaussian radial velocity example. Then the log likelihood of the whole data set is the sum of $\ln(N(E_i|T))$ at each energy E_i in the data set.

Using the Bayesian Poisson likelihood approach:

1. What is the T that gives the highest log likelihood?

2. If you use Wilks' Theorem, for which $\Delta \ln \mathcal{L} = -0.5$ from the peak to get the 68.3% credible region, what are $T_{\rm min}$ and $T_{\rm max}$ for that region?
3. If instead you normalize the likelihood (by integrating it from $kT = 0$ to $kT = 1$ keV and then dividing by that total; note that this is the correct normalization if our prior has zero probability for $kT > 1$ keV and a constant probability for $kT = 0$ to $kT = 1$ keV), what is the minimum range of kT that integrates to 68.3% of the probability?

To compare our result with what we would get from χ^2, note that some people advocate having at least 20 points per bin when using χ^2. Thus put the data into three equal bins of 20 photon energies each, where the first bin contains the 20 lowest energies, the second contains the 20 middle energies, and the third contains the 20 highest energies. You will then need to integrate the normalized blackbody number density in each of your energy ranges, for a given temperature, to determine the expected number. Then:

1. Find the value of T that minimizes χ^2.
2. Find the range of T within $\Delta\chi^2 = 1$, which is the prescription for the 68.3% region.

If you have a general enough code that you can easily switch the probabilities, how do the 2σ (95.45%, $\Delta\chi^2 = 4$, $\Delta \ln \mathcal{L} = -2$) and 3σ (99.73%, $\Delta\chi^2 = 9$, $\Delta \ln \mathcal{L} = -4.5$) regions compare between the three methods?