

MODELING KICKS FROM THE MERGER OF NONPRECESSING BLACK HOLE BINARIES

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ABSTRACT

Several groups have recently computed the gravitational radiation recoil produced by the merger of two spinning black holes. The results suggest that spin can be the dominant contributor to the kick, with reported recoil speeds of hundreds to even thousands of kilometers per second. The parameter space of spin kicks is large, however, and it is ultimately desirable to have a simple formula that gives the approximate magnitude of the kick given a mass ratio, spin magnitudes, and spin orientations. As a step toward this goal, we perform a systematic study of the recoil speeds from mergers of black holes with mass ratio $q \equiv m_1/m_2 = 2/3$ and dimensionless spin parameters of a_1/m_1 and a_2/m_2 equal to 0 or 0.2, with directions aligned or antialigned with the orbital angular momentum. We also run an equal-mass $a_1/m_1 = -a_2/m_2 = 0.2$ case, and find good agreement with previous results. We find that, for currently reported kicks from aligned or antialigned spins, a simple kick formula inspired by post-Newtonian analyses can reproduce the numerical results to better than $\sim 10\%$.

Subject headings: black hole physics — galaxies: nuclei — gravitational waves — relativity

Online material: color figures

1. INTRODUCTION

In the past two years, numerical relativity has undergone a revolution such that now multiple codes are able to stably evolve the last few cycles of the inspiral, merger, and ring-down of two black holes (Brügmann et al. 2004; Pretorius 2005; Campanelli et al. 2006a, 2006b; Baker et al. 2006a, 2006b). An important astrophysical output of such simulations is the net recoil due to asymmetric emission of gravitational radiation, because this has major implications for the growth of supermassive black holes in hierarchical merger scenarios (Merritt et al. 2004; Boylan-Kolchin et al. 2004; Haiman 2004; Madau & Quataert 2004; Yoo & Miralda-Escudé 2004; Volonteri & Perna 2005; Libeskind et al. 2006; Micic et al. 2006) as well as the evolution of seed black holes and present-day intermediate-mass black holes (Taniguchi et al. 2000; Miller & Hamilton 2002a, 2002b; Mouri & Taniguchi 2002a, 2002b; Miller & Colbert 2004; Gültekin et al. 2004, 2006; O’Leary et al. 2006, 2007). Analytical calculations (Peres 1962; Bekenstein 1973; Fitchett 1983; Fitchett & Detweiler 1984; Redmount & Rees 1989; Wiseman 1992; Favata et al. 2004; Blanchet et al. 2005; Damour & Gopakumar 2006) have now been augmented with full numerical results for nonspinning black holes with different mass ratios (Herrmann et al. 2007c; Baker et al. 2006c; Gonzalez et al. 2007a), black holes with equal masses and spins initially orthogonal to the orbital plane (Herrmann et al. 2007a; Koppitz et al. 2007), black holes with equal masses and spins initially parallel to the orbital plane (Gonzalez et al. 2007b; Campanelli et al. 2007b), black holes with equal masses and spins initially oriented at some angle between the orbital plane and the orbital angular momentum (Herrmann et al. 2007b; Tichy & Marronetti 2007), and black holes with unequal masses and spins initially either parallel to the orbital plane or oriented at some angle between the orbital plane and the orbital angular momentum (Campanelli et al. 2007a).

The parameter space for kicks with spin is large, so for astrophysical purposes it is important to have a simple parameterized formula for the kick that can be included in simulations of N -body dynamics or cluster and galaxy mergers. For nonspinning black holes, the classic Fitchett (1983) formula $v \propto \eta^2 (\delta m/M)$ does a reasonable job (although not perfect; see Gonzalez et al. 2007a for numerical results), where for black hole masses m_1 and $m_2 \geq m_1$ we define $M = m_1 + m_2$, $\delta m = m_2 - m_1$, and $\eta = m_1 m_2 / M^2$. Initial explorations of equal-mass spin kicks also show evidence of simplicity, with fits linear in spin reproducing the results of Herrmann et al. (2007a) and Koppitz et al. (2007). This encourages us to explore a more general class of kicks.

Spins that are aligned (prograde) with the orbital angular momentum may be of particular interest, because it has been argued that interaction with accretion disks will tend to align spins during “wet” mergers (Bogdanovic et al. 2007). The only previous numerical studies for a black hole with an aligned spin were carried out by Herrmann et al. (2007a) and Koppitz et al. (2007), which considered only cases where the spin of the second black hole was antialigned (retrograde) with the angular momentum, and the masses of the black holes were equal. Here we compute the kick speeds from a $q \equiv m_1/m_2 = 2/3$ mass ratio set of mergers, with spin parameters of 0 or 0.2, and directions either aligned (prograde) or antialigned (retrograde) with the orbital angular momentum. The symmetry of the configuration therefore guarantees that the kick direction is in the orbital plane. We find a formula that matches all of our kick speeds, and those of Herrmann et al. (2007a) and Koppitz et al. (2007) to within $\sim 10\%$. If the in-plane kicks can be generalized straightforwardly to more general orientations, and added to kicks perpendicular to the orbital plane, there is the prospect of simple astrophysical modeling of the gravitational rocket effect for arbitrary black hole mergers. In § 2 we describe our initial data and methodology. In § 3 we present our results, and discuss the implications of these simulations.

2. INITIAL DATA AND METHODOLOGY

In the following, we use geometrized units where Newton’s gravitational constant G and the speed of light c are set to unity

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TABLE 1
INITIAL DATA PARAMETERS

Run	$J_1(M^2)$	$J_2(M^2)$	$m_{p1}(M)$	$m_{p2}(M)$	$P(M)$	$L(M)$
NE ₋₋	-0.032	-0.072	0.374	0.586	0.119	7.05
NE ₋₊	-0.032	0.072	0.374	0.586	0.119	7.05
NE ₀₋	0.000	-0.072	0.374	0.586	0.119	7.05
NE ₀₀	0.000	0.000	0.382	0.584	0.119	7.05
NE ₀₊	0.000	0.072	0.374	0.586	0.119	7.05
NE _{+ -}	0.032	-0.072	0.374	0.586	0.119	7.05
NE ₊₊	0.032	0.072	0.374	0.586	0.119	7.05
EQ _{+ -}	0.050	-0.050	0.480	0.480	0.124	7.00

NOTES.—Runs are labeled “EQ” for equal mass and “NE” for unequal mass. J_1 and J_2 are the spin angular momenta of the two holes, either aligned (positive) or anti-aligned (negative) with the orbital angular momentum. The parameters m_{p1} and m_{p2} are the directly specified puncture masses of the holes. The parameter P is the initial transverse momentum on each hole, while L is the initial coordinate separation of the punctures.

so that all relevant quantities can be represented in terms of their mass scaling. For example, $1 M_\odot$ is equivalent to a distance of 1.4771×10^5 cm and a time of 4.9272×10^{-6} s. Accordingly we express distances in terms of M , the initial (ADM) mass of the system.

We simulated inspiraling black hole binaries of various mass ratios and spins, with the same initial coordinate separation in each case. In these cases our initial mass ratio approximated either 2/3 or unity. Our simulations were performed with our finite differencing code Hahndol (Imbiriba et al. 2004), which solves a $3 + 1$ formulation of Einstein’s equations. Adaptive mesh refinement and most parallelization was handled by the software package PARAMESH (MacNeice et al. 2000). For initial data we used the Brandt & Brüggmann (1997) Cauchy surface for black hole punctures, as computed by the second-order accurate, multigrid elliptic solver AMRMG (Brown & Lowe 2005). We evolved this data with the standard Baumgarte-Shapiro-Shibata-Nakamura (Nakamura et al. 1987; Shibata & Nakamura 1995; Baumgarte & Shapiro 1999; Imbiriba et al. 2004) evolution equations, modified only slightly with dissipation terms as in Hübner (1999) and constraint-damping terms as in Duez et al. (2004). Our gauge conditions were chosen according to the “moving puncture” method, as in van Meter et al. (2006). Time integration was performed with a fourth-order Runge-Kutta algorithm, and spatial differencing with fourth-order accurate mesh-adapted differencing (Baker & van Meter 2005). Interpolation between refinement regions was fifth-order accurate.

To explore the parameter space of nonprecessing spin configurations with some mass ratio dependence, we have performed simulations for seven different data sets of black holes with unequal masses, as well as one equal-mass case. The initial-data parameters for these eight data sets are given in Table 1. The spins in these simulations were always orthogonal to the orbital plane. Our choice of initial tangential momenta was informed by a quasi-circular post-Newtonian (PN) approximation (Damour et al. 2000).

Numerically, we can only directly specify the “puncture” masses m_{1p} and m_{2p} of the two holes; to determine each hole’s physical (horizon) mass m , we first locate its apparent horizon (using an adapted version of the AHFinderDirect code; see Thornburg 2003) and then apply the Christodoulou (1970) formula

$$m^2 = m_{\text{irr}}^2 + J^2/4m_{\text{irr}}^2, \quad (1)$$

where J is the magnitude of the spin angular momentum of the hole, $m_{\text{irr}} = \sqrt{(A_{\text{AH}}/16\pi)}$, and A_{AH} is the area of the apparent horizon.

TABLE 2
PREDICTED VERSUS COMPUTED KICK SPEED

Run	q	\hat{a}_1	\hat{a}_2	v_{num}	v_{pred}	$ \Delta v /v_{\text{num}}$ (%)
NE ₋₋	0.654	-0.201	-0.194	116.3	119.5	2.7
NE ₋₊	0.653	-0.201	0.193	58.5	58.2	0.5
NE ₀₋	0.645	0.000	-0.195	167.7	153.1	8.7
NE ₀₀	0.677	0.000	0.000	95.8	98.6	2.9
NE ₀₊	0.645	0.000	0.194	76.9	71.7	6.8
NE _{+ -}	0.655	0.201	-0.194	188.6	181.9	3.6
NE ₊₊	0.654	0.201	0.194	83.4	92.4	10.8
EQ _{+ -}	1.001	0.198	-0.198	89.8	92.6	3.2
S0.05.....	1.000	0.200	-0.200	96.0	93.8	2.3
S0.10.....	1.000	0.400	-0.400	190.0	187.6	1.2
S0.15.....	1.000	0.600	-0.600	285.0	281.5	1.2
S0.20.....	1.000	0.800	-0.800	392.0	375.3	4.3
r0.....	1.000	-0.584	0.584	260.0	274.0	5.4
r1.....	0.917	-0.438	0.584	220.0	220.8	0.3
r2.....	0.872	-0.292	0.584	190.0	178.1	6.3
r3.....	0.848	-0.146	0.584	140.0	141.9	1.4
r4.....	0.841	0.000	0.584	105.0	110.4	5.1

NOTES.—Runs labeled “S0.##” are taken from Herrmann et al. (2007a), while runs labeled “r#” are taken from Koppitz et al. (2007).

The parameters relevant to our discussion, the mass ratio $q \equiv m_1/m_2$ and the dimensionless spin parameters $\hat{a}_1 \equiv a_1/m_1$ and $\hat{a}_2 \equiv a_2/m_2$, are listed in the first eight rows of Table 2. We have striven to maintain $q = 2/3$ in all of our unequal-mass simulations.

The grid spacing in the finest refinement region, around each black hole, was $h_f = 3M/160$, with the exception of our non-spinning unequal-mass case, which was one of a set of runs described previously (Baker et al. 2006c), and for which we used $h_f = M/40$. The extraction radius was at $R = 45M$ in every case except for the nonspinning case, where it was at $R = 50M$. For one of our new physical configurations (NE₊₊) we also extracted at $R = 40M$, finding a final kick within 0.8 km s^{-1} of that extracted at $R = 45M$. Assuming a radially dependent error that falls off as $1/R$, as found in a similar kick computation (Gonzalez et al. 2007b), this implies that the kick extracted at $R = 45M$ is within 8% of what would be computed at infinite radius. Also for this

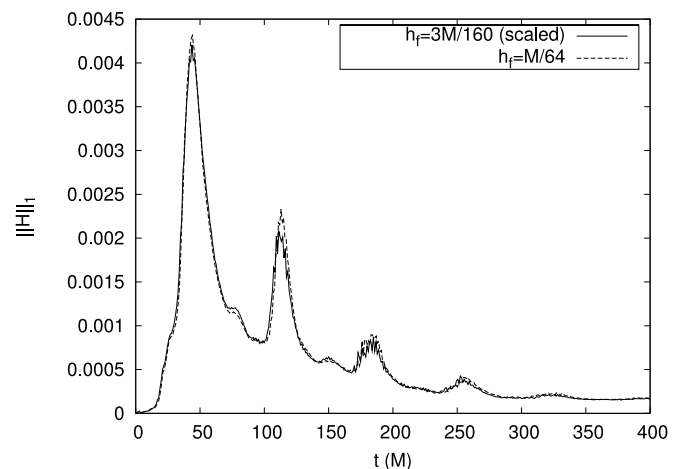


FIG. 1.—L1 norm of the Hamiltonian constraint for the case of unequal masses with prograde spins (NE₊₊) at two different resolutions, computed on a domain extending to $|x_i| = 48M$, which includes our wave-extraction surface. The finest level, containing nonconvergent points near the puncture, has been excluded. The lower resolution, $h_f = 3M/160$, has been scaled such that for third-order convergence it should superpose with the $h_f = M/64$ curve. [See the electronic edition of the Journal for a color version of this figure.]

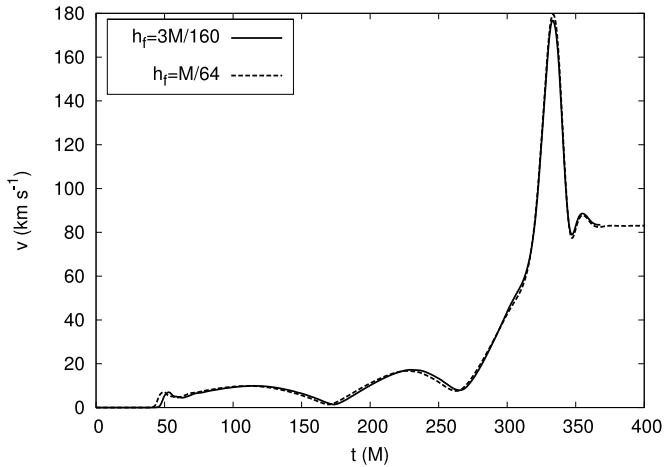


FIG. 2.—Total radiated momentum for the case of unequal masses with prograde spins (NE_{++}) at two different resolutions. The curves have been shifted slightly in time to line up the peaks, for comparison of the merger dynamics. The agreement is better than 5% for most of the evolution and better than 2% through the merger and final kick ($t > 300M$). [See the electronic edition of the *Journal* for a color version of this figure.]

physical configurations we ran a higher resolution, $h_f = M/64$, to verify that the lower resolution of $h_f = 3M/160$ would be sufficient. We found satisfactory convergence of the Hamiltonian constraint (Fig. 1) and consistency of the radiated momentum (Fig. 2).

The thrust dP^i/dt imparted by the radiation was derived by Newman & Tod (1981) and is computed from a surface integral of the squared time integral of the radiative Weyl scalar ψ_4 times the unit radial vector, as given explicitly by Campanelli & Lousto (1999):

$$\frac{dP^i}{dt} = \lim_{R \rightarrow \infty} \left\{ \frac{R^2}{4\pi} \oint d\Omega \frac{x^i}{R} \left| \int_{-\infty}^t dt \psi_4 \right|^2 \right\}. \quad (2)$$

To perform the angular integration in equation (2), we use the second-order Misner algorithm described in Misner (2004) and Fiske et al. (2005). This result is then integrated numerically to give the total radiated momentum P^i ; to obtain the final velocity of the merged remnant black hole, we divide P^i by the final black hole mass, as computed from the difference of the initial ADM mass and the total radiated energy.

3. RESULTS AND DISCUSSION

In Figure 3 we present the aggregated recoil kick from each of our simulations. The kicks obtained range from $\sim 60 \text{ km s}^{-1}$, in the case where the larger hole's spin is aligned and the smaller anti-aligned with the orbital angular momentum, to $\sim 190 \text{ km s}^{-1}$, when the alignments are reversed. The kicks possess a common profile, with the bulk of the momentum radiated over $\sim 50M$ before merger. In all but the equal-mass case (EQ_{+-}), we observe a sharp monotonic rise in kick over $40M$, followed by a substantial “unkick.” That is, around the time of merger and ring-down, we often observe a sudden thrust in momentum that is directed counter to the momentum that had accumulated during inspiral. In the EQ_{+-} case, this unkick is absent. We summarize the final kicks for each of our configurations in the first eight rows of Table 2.

It has been noted by Campanelli et al. (2006c) that the presence of spins on black holes in an inspiraling binary can significantly extend or reduce the time to merger, depending on whether the spins are aligned or anti-aligned with the orbital angular momentum.

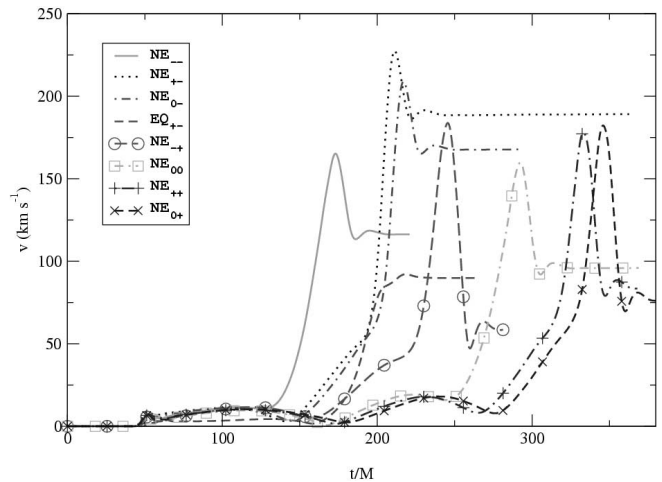


FIG. 3.—Aggregated recoil kicks from all runs listed in Table 1. The merger time for each binary matches the peak in its kick profile; the relative delay in merger times between data sets differing in initial spins is consistent with the results of Campanelli et al. (2006c). All configurations show a marked “unkick” after the peak, with the exception of the equal-mass case, EQ_{+-} . [See the electronic edition of the *Journal* for a color version of this figure.]

We observe a similar trend in merger times, as illustrated in the peak of the aggregated recoil kicks. This tendency had also been expected based on PN calculations, which show that the last stable orbit is pushed to a smaller radius, implying a later merger, for aligned spins (Damour 2001). Note that, although resolution can also affect merger time, for these short runs we have sufficiently resolved the dynamics that numerical error in merger time appears negligible compared to the effect of spin, as demonstrated in Figure 2.

Our simulation results, together with data reported by other groups, allow us to consider a simple description of the total kick for arbitrary mass ratio when the two black holes spins are aligned or anti-aligned with the system's orbital angular momentum.

Several recent papers (Herrmann et al. 2007a; Koppitz et al. 2007; Choi et al. 2007) have suggested that the kick velocity resulting from comparable-mass binary black hole mergers may be approximately described in terms of a simple scaling dependence consistent with the scaling in the leading-order post-Newtonian approximation treatment (Kidder 1995). For kicks generated by nonspinning black holes of unequal masses, this produces the Fitchett scaling, which provides a reasonable approximation to recent numerical simulation results (Gonzalez et al. 2007b). Recent papers on spinning black hole mergers suggest that the kicks from nearly equal-mass mergers may scale linearly, through the quantity $\Delta = q\hat{a}_1 - \hat{a}_2$, with spins that are either aligned or anti-aligned with the orbital angular momentum (Herrmann et al. 2007a; Koppitz et al. 2007). For head-on collisions, Choi et al. (2007) have shown that the PN expressions describe the scaling of the spin-asymmetry kick and its relation to the kick induced by mass asymmetry, correctly predicting the relative directions of the two kick contributions and supporting the idea that the effects of asymmetries in spins and masses can be considered independently.

For inspiraling mergers, with the assumption that the radial velocity is small compared to the tangential velocity, the PN prediction for the cases we consider is that the instantaneous thrust generated by the spin asymmetry will be aligned with that of the mass asymmetry, although the relative size of these effects may vary as the merger proceeds. Unlike the head-on collision case, the direction of thrust should vary as the system revolves in inspiraling mergers, so that we cannot infer that the net effects of spin and

mass asymmetry will produce collinear contributions to the overall kick. Motivated by these observations we will consider our data with the hypothesis that the magnitudes of the kicks induced by spin and mass asymmetries each scale independently with the PN-predicted scaling, but that the directional alignment of these two contributions to the kick may differ by some angle θ . The total kick may then be of the form

$$v = V_0 [32q^2 / (1 + q)^5] \times \sqrt{(1 - q)^2 + 2(1 - q)K \cos \theta + K^2}, \quad (3)$$

where $K = k(q\hat{a}_1 - \hat{a}_2)$. The parameter V_0 gives the overall scaling of the kick (note that the factor in brackets becomes unity for $q = 1$), while k gives the relative scaling of the kick contributions from spin and mass asymmetries. This expression amounts to a generalization of the post-Newtonian-inspired kick formulae discussed by Favata et al. (2004) and Koppitz et al. (2007), which was consistent with colinearity of the two kick contributions, $\theta = 0$.

We have tested the simple equation (3) with our own kick speeds, together with published kicks from Koppitz et al. (2007) and Herrmann et al. (2007a) for a total of 17 independent data points. Without more precise knowledge of the uncertainties for each measurement it is not possible to do a true statistical fit. As a substitute, however, we have obtained values for the three free parameters V_0 , θ , and k by minimizing the total of a χ^2 -like quantity $\propto \sum_i (v_{\text{pred},i} - v_{\text{num},i})^2 / v_{\text{num},i}$, where $v_{\text{pred},i}$ and $v_{\text{num},i}$ are the predicted and measured kick speeds for the i th combination of parameters, respectively. The proportionality constant is chosen so that the minimum of “ χ^2 ” is 14, equal to the number of degrees of freedom.

With this procedure, our best fit to all simulations currently reported gives $V_0 = 276 \text{ km s}^{-1}$, $\theta = 0.58 \text{ rad}$, and $k = 0.85$. The minimal regions containing 95% of the probability for each parameter are $V_0 = 267\text{--}294 \text{ km s}^{-1}$, $\theta = 0.45\text{--}0.65 \text{ rad}$, and $k = 0.8\text{--}0.89$. In Table 2 we compare the predictions of this formula to our results and those of other groups who have explored prograde or retrograde spins. We see that this simple formula performs well, with errors less than $\sim 10\%$ in all cases.

We can estimate the uncertainty in θ , and judge how strongly we can rule out a constant $\theta = 0$, by making the conservative assumption that all the numerical kick results have 10% statistical error. A χ^2 analysis then indicates that at one standard deviation $\theta = 0.58 \pm 0.8 \text{ rad}$. We also find that, formally, $\theta = 0$ is ruled out at $>5 \sigma$ and $\theta = \pi/2$ (sum in quadrature) is ruled out at $>12 \sigma$, but this far from our best values the error contours are clearly non-Gaussian. It is possible that a more complicated model (e.g., one in which θ depends on the mass ratio) is a better representation

than $\theta = \text{constant}$, but no motivation for this exists in the current data.

The success of our fit in describing the existing data suggests that this simple expression may, for many astrophysical simulations, adequately describe the dependence of the kicks on the portion of mass ratio and spin parameter space that we have studied. However, recent simulations have suggested that the dominant component of the kick may be out of the orbital plane, deriving from spins that lie in the orbital plane (Campanelli et al. 2007a), a configuration outside the parameter space we have studied. Gonzalez et al. (2007b) have shown that such a configuration can produce kicks that may exceed 2500 km s^{-1} directed out of the orbital plane. These results apparently confirm the early predictions of Redmount & Rees (1989) that out-of-plane kicks would be particularly significant.

Since initial submission of this paper, Campanelli et al. (2007a) have suggested a combined formula, which generalizes our equation (3) to include out-of-plane kicks (v_{\parallel} in their notation) of the kind discussed above. If kick speeds $>1000 \text{ km s}^{-1}$ are common in comparable-mass mergers of black holes with substantial spin, this comes into apparent conflict with the observation that essentially all galaxies with bulges appear to have supermassive black holes in their cores, since galactic escape speeds tend to be $<1000 \text{ km s}^{-1}$ (see Ferrarese & Ford 2005 for a review of supermassive black holes and their correlation with galactic properties). It seems unlikely that most supermassive black holes have low enough spins to guarantee small kicks, given evidence such as broad Fe K α lines from a number of black holes (Iwasawa et al. 1996; Fabian et al. 2002; Miller et al. 2002; Reynolds & Nowak 2003; Reynolds et al. 2005; Brenneman & Reynolds 2006) as well as overall arguments from the inferred high average radiation efficiency of supermassive black holes (Soltan 1982; Yu & Tremaine 2002). Bogdanovic et al. (2007) suggest that torques from gas-rich mergers tend to align the spins of the holes, and thus lead to small kicks, but no known preferred alignment exists for gas-poor mergers. More exploration of the parameter space of spin kicks is clearly necessary.

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