Three-Body Encounters of Black Holes in Globular Clusters

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Abstract. Evidence has been mounting for the existence of black holes with masses from 10^2 to $10^4\ M_\odot$ associated with stellar clusters. Such intermediate-mass black holes (IMBHs) will encounter other black holes in the dense cores of these clusters. The binaries produced in these interactions will be perturbed by other objects as well thus changing the orbital characteristics of the binaries. These binaries and their subsequent mergers due to gravitational radiation are important sources of gravitational waves. We present the results of numerical simulations of high mass ratio encounters, which help clarify the interactions of intermediate-mass black holes in globular clusters and help determine what types of detectable gravitational wave signatures are likely.

INTRODUCTION

Recent observations have given rise to the possibility of large black holes located in the centers of stellar clusters. Optical observations of velocity profiles of M15 and G1 are consistent with 2.5×10^3 and $2.0 \times 10^4\ M_\odot$ black holes (BHs) (Gebhardt et al. 2000; Gerssen et al. 2002; van der Marel et al. 2002; Gebhardt, Rich, & Ho 2002; for a review see van der Marel, this volume and for other interpretations see Baumgardt et al. 2003). X-ray observations show unresolved, non-nuclear sources associated with both young and globular clusters with $L \approx 10^{39}$ to 10^{41} erg s^{-1} in multiple galaxies; for a review see Mushotzky, this volume. The observed variability and fluxes, if neither beamed nor super-Eddington, indicate BHs with M$\sim 10^3\ M_\odot$.

The existence of IMBHs suggests a formation mechanism different from those of stellar-mass BHs and supermassive BHs. Several models have been proposed to account for the origin of IMBHs including formation from population III stars (Madau & Rees 2001; Schneider et al. 2002), interactions of stars in young clusters, and interactions of compact objects in old clusters (Miller & Hamilton 2002; Portegies Zwart & McMillan 2002; for a review see Miller, this volume).

Wherever and however IMBHs formed, the best candidates are found in clusters where three-body encounters are important. Any IMBH in the center of a stellar cluster will pick up companions and undergo three-body encounters. These binaries and their mergers are important sources of gravitational waves. Advanced LIGO is expected to detect the merger, and LISA is expected to detect the inspiral phase. In order to predict the gravitational wave signature of the merger, the expected separations and eccentricities of the binaries must be known. As three-body encounters alter these quantities, simulations of these encounters are needed to predict their distributions. In addition, the simulations are useful for estimating the source population and event rates.

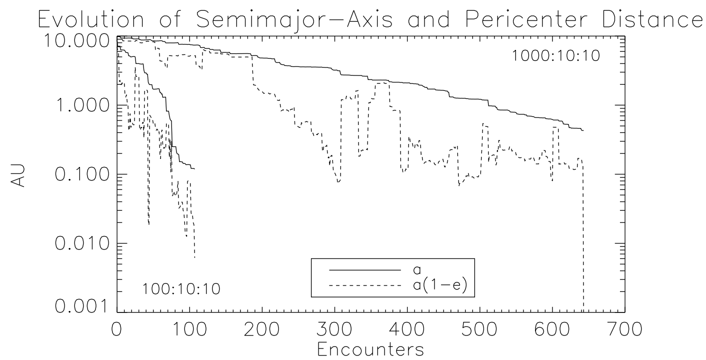

FIGURE 1. Semimajor axis (solid) and pericenter distance (dashed) as a function of number of encounters for 100:10:10 mass-ratio (left) and 1000:10:10 (right). As the binary undergoes a sequence of three-body interactions, its semimajor axis decreases steadily, but the eccentricity, and thus the pericenter distance, jumps from low to high values in a single encounter. The 100:10:10 mass-ratio decreases in many fewer interactions because the interactions are stronger.

NUMERICAL SIMULATIONS

We perform numerical simulations of a series of three-body encounters between high mass-ratio binary point masses and an interloping point mass in Newtonian gravity. We simulate an encounter between a hard binary and an interloper and then use the resulting binary for the next encounter. This is repeated until the binary merges due to gravitational radiation before its next encounter. This study of a sequence of encounters of high mass-ratio binaries differs from previous studies of three-body encounters, which have focused on nearly equal masses and single encounters.

Simulations were integrated using HNBody (Rauch & Hamilton 2003). The results presented here include pure Newtonian integrations of two mass-ratios: 10000 sequences of 100:10:10 (dominant:companion:interloper) and 3000 sequences of 1000:10:10. Both have initially circular orbits with a separation of $a = 10$ AU. A Monte Carlo initial condition generator samples all incoming directions and orientations for significant encounters. We also present simulations, described below, with general relativistic effects.

RESULTS OF NEWTONIAN SIMULATIONS

Figure 1 shows the changes in a and $a(1-e)$ with each encounter for two typical sequences. For both mass-ratios, the semimajor axis changes steadily, but the eccentricity and, thus, the pericenter distance change drastically. Both binaries merge at high e (see Fig. 2a) because the time to merge by gravitational radiation is

$$\tau \approx 3 \times 10^8 \left(M_\odot^3/\mu_{\rm bin} M_{\rm bin}^2\right) (a/R_\odot)^4 \left(1-e^2\right)^{7/2} \ {\rm yr}, \quad (1)$$

where $M_{\rm bin}$ and $\mu_{\rm bin}$ are the mass and reduced mass of the binary (Peter 1964). The lower mass-ratio merges with fewer encounters because the energy that the interloper

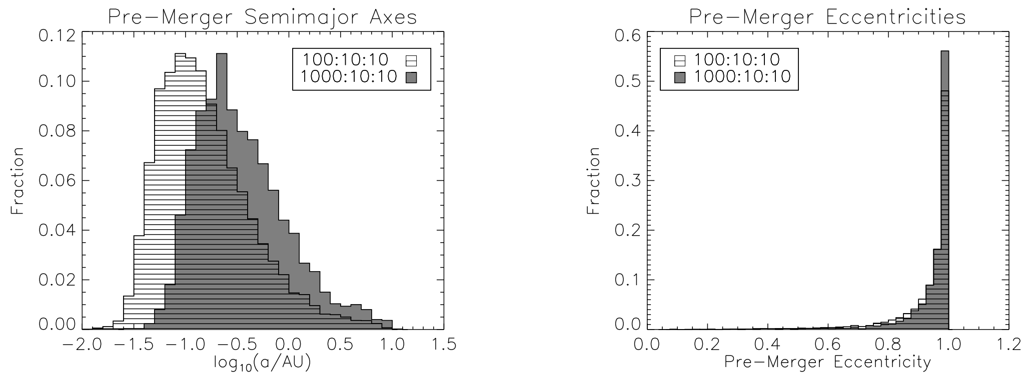

FIGURE 2. The left panel shows a histogram of the binary's semimajor axis after its last encounter and before it begins to merge. The 100:10:10 sequence (hatched) is shifted to a lower a than the 1000:10:10 sequence (solid) because the binaries with small a will tend to merge before undergoing another encounter. The right panel shows a histogram of the binary's pre-merger eccentricity. The 100:10:10 sequence (hatched) and 1000:10:10 sequence (solid) are very similar in shape, and they are both peaked at $e \approx 1$ because of merger time's strong dependence on e (see Eqn. 1) and because e can change from a low value to nearly unity in one strong interaction (see Fig. 1).

can carry away scales as $\Delta E \sim \left(m_{\rm comp}/M_{\rm bin}\right)$, where $m_{\rm comp}$ is the less massive binary member (Quinlan 1996).

Figure 2a shows a histogram of the binary's semimajor axis after its last encounter. The binary will then merge before it encounters another object. The mean semimajor axis is 0.32 AU for 100:10:10 and 0.64 AU for 1000:10:10. Both mass-ratios have a similar shape whose drop off at low a is because the binary tends to merge before its semimajor axis can decrease further. The lower mass-ratio's histogram is shifted to lower a because the less massive binary will emit less gravitational radiation, and hence shrink less, for a given orbit.

Figure 2b shows a histogram of the binary's eccentricity after the last encounter. The mean value is 0.930 for 100:10:10 and 0.950 for 1000:10:10. The histograms of both mass ratios have very similar shapes and are strongly peaked near $e = 1$. The high eccentricity right before merger is due to the strong dependence of merger time on eccentricity and because the eccentricity can change drastically in one encounter while the semimajor axis tends to decrease at a roughly constant rate. A high eccentricity is important from a gravitational wave detection standpoint because the waveform of the gravitational radiation emitted by high eccentricity binaries at inspiral is significantly different from that of circular binaries in LISA's band.

ADDING GENERAL RELATIVISTIC EFFECTS

To augment our Newtonian treatment of this problem, we modified the integrations to incorporate the effects of gravitational radiation. In between encounters the binary emits gravitational radiation causing the orbit to shrink and circularize. We add this effect to our simulations for several mass ratios to test how it changes our results. We compare 300 pure Newtonian and 300 runs with this general relativistic effect for each of three

TABLE 1. Simulations of binary evolution with two models: (1) pure Newtonian gravity and (2) Newtonian gravity with gravitational radiation between encounters. Typical variation in the average numbers in a run of 100 sequences is between 1% (for pre-merger *e*) and 10% (for pre-merger *a*).

Mass Ratio	Number of Encounters		Pre-merger a/AU		Pre-merger e		Interloper Ejections	
	Newt.	GR	Newt.	GR	Newt.	GR	Newt.	GR
10:10:10	52	48	0.17	0.17	0.92	0.89	8.7	7.2
100:10:10	103	94	0.34	0.37	0.94	0.87	25.2	20.8
1000:10:10	554	484	0.63	0.53	0.95	0.84	114.5	88.4

mass-ratios considered: 10:10:10, 100:10:10, and 1000:10:10 $M_\odot$. We are also beginning simulations that include gravitational radiation during the encounter.

The main results are summarized in Table 1. Gravitational radiation shrinks and circularizes orbits, hence the pre-merger a and e, as well as the number of encounters and number of interlopers ejected during the hardening, are all decreased because of the radiation. Thus general relativistic effects are important in determining the orbital characteristics of the typical binary inspiral.

We monitored the simulations for encounters that resulted in an ejection of one of the less massive BHs from the globular cluster, assuming an escape velocity of 50 km s^{-1}. This is important in determining whether IMBHs can be built up from mergers of stellar-mass BHs as proposed by Miller & Hamilton (2002). If the build-up to the inferred masses requires more BHs than are available in a cluster, the model cannot explain the formation of IMBHs. Miller & Hamilton (2002) estimate $100\,(M_{\rm bin}/50M_\odot) \approx 2000$ ejections for a mass ratio of 1000:10:10, assuming an eccentricity at merger of $e = 0.7$. This is far in excess of the ~ 100 ejections we find because the binaries will merge with an eccentricity much higher than the average of a thermal distribution. Thus merging stellar-mass BHs in a globular cluster may be more efficient than previously expected.

Acknowledgements

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