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CONSTRAINTS ON NEUTRON STAR MATTER FROM KILOHERTZ QPOs

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Abstract

One of the most dramatic discoveries made so far with the *Rossi X-Ray Timing Explorer* is that many accreting neutron stars with weak magnetic fields generate strong, remarkably coherent, high-frequency X-ray brightness oscillations. The ∼325–1200 Hz quasi-periodic oscillations (QPOs) observed in the accretion-powered emission are almost certainly produced by gas orbiting very close to the stellar surface and have frequencies related to the orbital frequencies of the gas. The ∼360–600 Hz brightness oscillations seen during thermonuclear X-ray bursts are produced by one or two hotter regions on the stellar surface and have frequencies equal to the stellar spin frequency or its first overtone. Measurements of these oscillations are providing tight upper bounds on the masses and radii of neutron stars, and important new constraints on the equation of state of neutron star matter.

1 Introduction

Since the birth of X-ray astronomy 35 years ago, scientists have sought to use the X-radiation that comes from near the event horizons of black holes and the surfaces of neutron stars to probe quantitatively the strong gravitational fields near these objects and to determine the fundamental properties of dense matter (see, e.g., [2, 6]). The *Rossi X-Ray Timing Explorer* (*RXTE*), which was launched on December 30, 1995, was specially designed to have the large area, microsecond time resolution, high telemetry bandwidth, and pointing flexibility needed to address these questions (see [21]). With *RXTE*, strong, high-frequency X-ray brightness oscillations have been discovered from at least

two black holes (see [7]) and sixteen neutron stars (see [13, 23]). As a result, we appear to be on the threshold of achieving this decades-old goal of X-ray astronomy. Here we focus on the oscillations discovered in neutron stars and their implications for the properties of these stars and for neutron star matter.

2 High-Frequency Brightness Oscillations

2.1 Observed Properties

High-frequency X-ray brightness oscillations are observed both in the transient X-ray emission produced during type I (thermonuclear) X-ray bursts (see [18]) and in the persistent, accretion-powered X-ray emission (see [23]).

The high-frequency oscillations observed during X-ray bursts have frequencies in the range 360–600 Hz and rms amplitudes at least as high as $\sim$35% [19]. Only a single oscillation is observed during a burst, the oscillation appears to be highly coherent during the burst decay (see, e.g., [16]), and the frequency of the oscillations produced by a given star is always the same (measurements of the burst oscillations in 4U 1728$-$34 over about a year show that the timescale of any variation in the oscillation frequency is $\gtrsim 3000$ yr [17]). The evidence is compelling that the oscillations seen during X-ray bursts are produced by regions of brighter X-ray emission that rotate with the star, and that the frequency of the burst oscillations is the stellar spin frequency or its first overtone (see [13, 19]). The spin frequencies of these neutron stars appear to be in the range 250–350 Hz [13].

The kilohertz quasi-periodic oscillations (QPOs) observed in the persistent X-ray emission have frequencies in the range 325–1200 Hz, rms amplitudes as high as $\sim$15%, and quality factors $\nu/\delta\nu$ as high as $\sim$200 [23]. Two kilohertz QPOs are commonly observed simultaneously in a given source (see Fig. 1). Although the frequencies of the two QPOs vary by hundreds of Hertz, the frequency separation $\Delta\nu$ between them appears to be nearly constant in almost all cases (see [23]). The frequency separation of the two kilohertz QPOs observed in a given star is closely equal to the spin frequency of the star inferred from its burst oscillations (see [13, 20]).

2.2 Origin of Kilohertz QPOs

Although other types of models have been suggested [4, 22], the fact that the separation $\Delta\nu$ between the frequencies of the two kilohertz QPOs observed from a given star is closely equal to the spin frequency of the star inferred from its burst oscillations is very strong evidence in favor of beat-frequency models

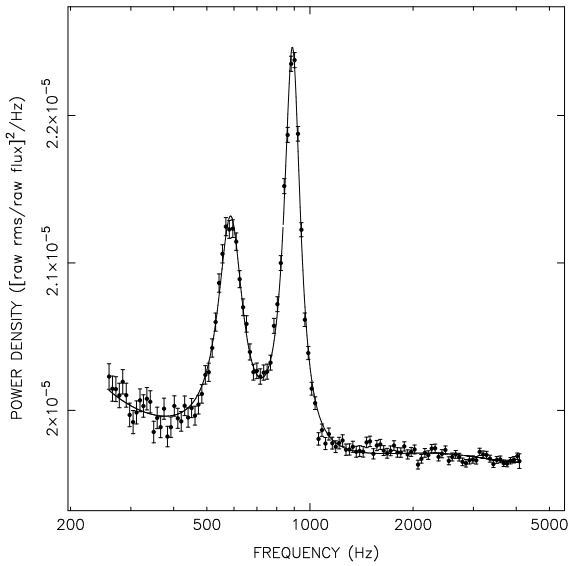

Fig. 1. Power spectrum of Sco X-1 X-ray brightness variations, showing the two simultaneous kilohertz QPOs that are characteristic of the kilohertz QPO sources. The QPOs observed in Sco X-1 are two of the weakest kilohertz QPOs detected, with rms amplitudes ∼1%. The continuum power density is consistent with that expected from photon counting noise alone. From [24].

[23]. In these models, the frequency of the higher-frequency QPO is the Keplerian frequency at a special radius and the frequency of the lower-frequency QPO is the difference between the higher frequency and the spin frequency $\nu_{\rm spin}$ of the neutron star. The magnetospheric beat-frequency model, which was developed to explain the single, ∼15–60 Hz X-ray brightness oscillations discovered in the Z sources a decade ago (see [5]), has been discussed [20] as a possible explanation for the kilohertz QPOs. However, it is difficult to explain many basic features of the kilohertz QPOs using this model, including why there are *two* such QPOs (see [13]).

The most fully developed and successful model of the kilohertz QPOs is the so-called sonic-point beat-frequency model [13]. This model is based on previous work [9, 10] which showed that the drag force produced by radiation from a central star can terminate a Keplerian disk flow near the star. In the sonic-point model, some accreting gas spirals inward in nearly circular orbits until it is close to the neutron star, where radiation forces or general relativistic effects cause a sudden increase in the inward radial velocity. The radius at which this occurs is conveniently referred to as the sonic radius, even though the transition to supersonic flow is not directly relevant in this model.

The sharp increase in the inward velocity is usually caused by the drag exerted on the orbiting gas by radiation from the star, but may instead be caused by general relativistic corrections to Newtonian gravity, if the gas in

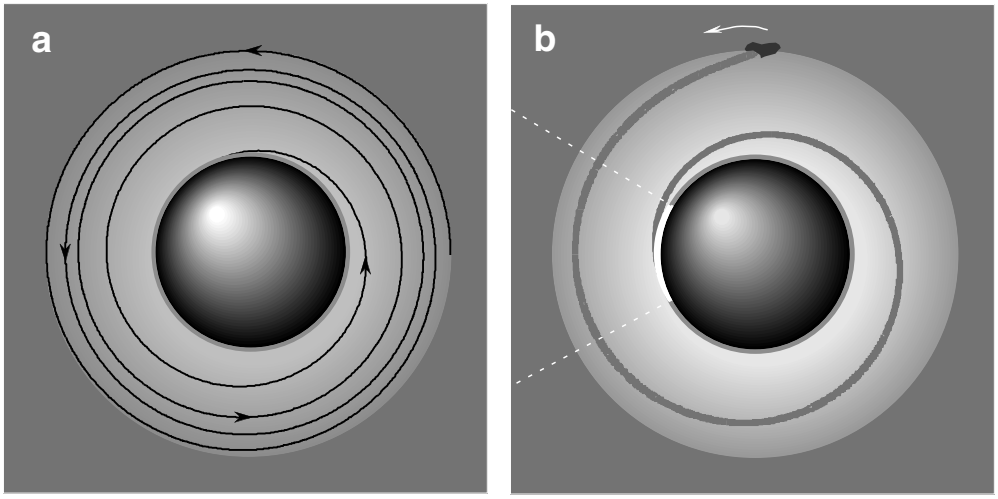

Fig. 2. View of the neutron star and accretion disk along the rotation axis of the disk, which is rotating counterclockwise in this view. (a) Spiral trajectory followed by a single element of gas as it falls supersonically from the sonic radius to the stellar surface. (b) Spiral pattern of higher gas density formed by gas streaming inward along spiral trajectories with the shape shown in (a), from a clump orbiting near the sonic radius. The trajectory and density pattern are from general relativistic gas dynamical calculations [13].

the Keplerian flow reaches the innermost stable circular orbit without being significantly affected by radiation. Gas streams inward from density fluctuations (clumps) orbiting near the sonic radius along tightly spiraling trajectories like that shown in Figure 2a, generating a more open spiral density pattern like that shown in Figure 2b. This pattern rotates around the star with the Keplerian orbital frequency at the sonic point, $\nu_{\rm Ks}$. Collision of the denser gas from the clumps with the stellar surface creates beams of brighter X-ray emission, like that indicated by the white dashed lines in Figure 2b. These beams move around the star's equator, generating a quasi-periodic brightness oscillation with frequency $\nu_{\rm Ks}$. Accreting gas is funneled to certain parts of the stellar surface by the star's weak magnetic field, producing weak X-ray beams that rotate *with the star*. These beams are overtaken by a given orbiting clump once each beat period, so the inward mass flux from the clumps, and hence the accretion luminosity, varies at the sonic-point beat frequency $\nu_{\rm Ks} - \nu_{\rm spin}$.

The sonic-point model explains naturally why the separation between the frequencies of the two kilohertz QPOs is nearly constant and equal to the burst oscillation frequency or half this frequency. It is also consistent with

the accretion rates and stellar magnetic fields inferred previously and accounts for the main features of the kilohertz QPOs, including their high and variable frequencies, their high amplitudes and coherences, and the common occurrence of two simultaneous kilohertz QPOs [13].

3 Constraints from Kilohertz QPOs

3.1 Nonrotating Stars

In order to see how constraints on the equation of state of neutron star matter can be derived, suppose first that the star is not rotating and assume that, for the star in question, $\nu^*_{\rm QPO2}$—the highest observed value of the frequency of the higher-frequency (Keplerian-frequency) QPO in the kilohertz QPO pair—is 1220 Hz (this is the highest QPO frequency detected so far from any neutron star; see [13]). Obviously, the orbital radius $R_{\rm orb}$ of the clumps producing the QPO must be greater than the stellar radius; $R_{\rm orb}$ must also be greater than the radius $R_{\rm ms}$ of the innermost stable circular orbit in order for the clumps to produce a wave train that lasts tens of oscillation periods, as observed. These requirements constrain the representative point of the star to lie in a pie-slice shaped region of the radius-mass plane (see Fig. 3a). This bounds the mass and radius of the star from above. In terms of $\nu^*_{\rm QPO2}$, these bounds are [13]

$$M^0_{\rm max} = 2.2\,(1000\,{\rm Hz}/\nu^*_{\rm QPO2})\,M_\odot \quad \text{and} \quad R^0_{\rm max} = 19.5\,(1000\,{\rm Hz}/\nu^*_{\rm QPO2})\ {\rm km}. \quad (1)$$

Figure 3b compares the regions of the radius-mass plane allowed for three values of $\nu^*_{\rm QPO2}$ with the mass-radius relations for nonrotating stars given by five representative equations of state.

3.2 Rotating Stars

Rotation affects the structure of the star and the spacetime, altering the region of the radius-mass plane allowed by a given value of $\nu^*_{\rm QPO2}$. Slow rotation expands the allowed region whereas rapid rotation shrinks it greatly. The parameter that characterizes the importance of rotational effects is the dimensionless quantity $j \equiv cJ/GM^2$, where J and M are the angular momentum and gravitational mass of the star. For the spin frequencies ∼300 Hz inferred for the neutron stars in the kilohertz QPO sources, j is ~ 0.1–0.3, depending on the mass of the star and the equation of state. For such small values of j, the structure of the star is almost unaffected and a treatment that is first-order in j is adequate. To this order, the existence of upper bounds on the mass and

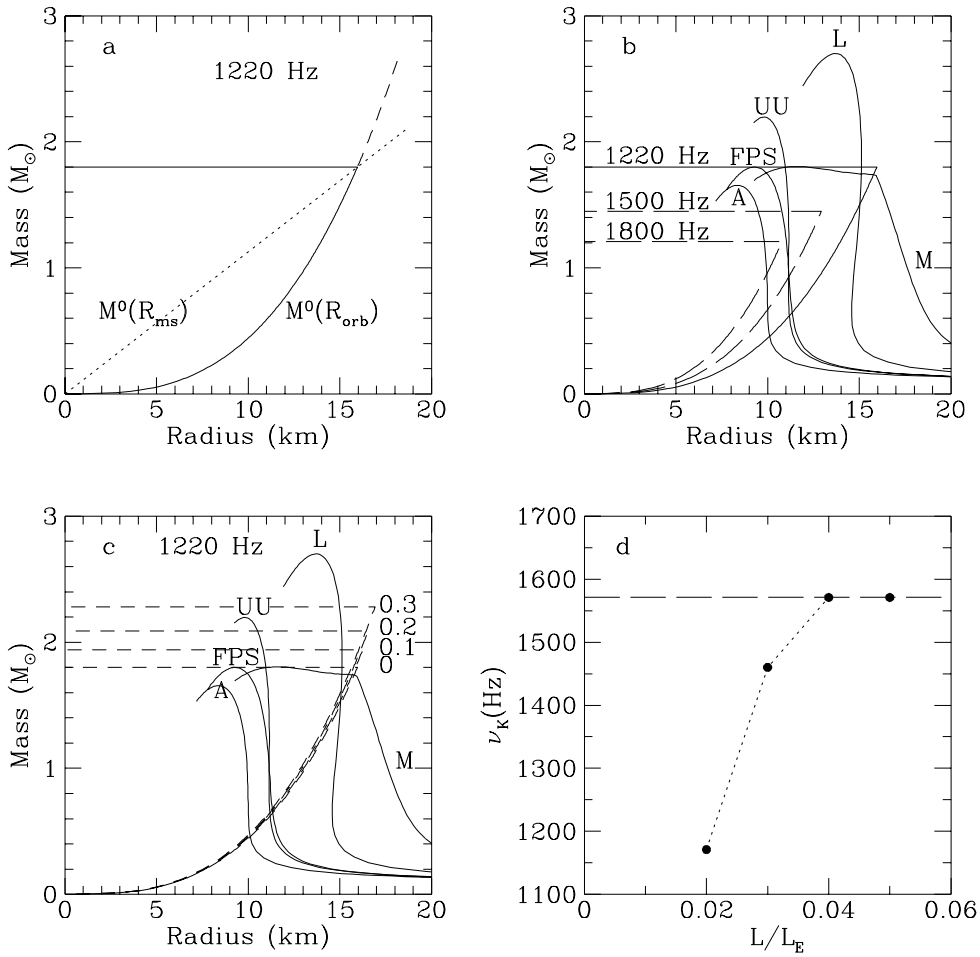

Fig. 3. (a) Radius-mass plane, showing how to construct the region allowed for a nonrotating neutron star with $\nu^*_{\rm QPO2} = 1220$ Hz. $R_{\rm orb}$ must be greater than the stellar radius, so the star's representative point must lie to the left of the (dashed) cubic curve $M^0(R_{\rm orb})$ that relates the star's mass to the radius of orbits with frequency 1220 Hz. The high coherence of the oscillations constrains $R_{\rm orb}$ to be greater than $R_{\rm ms}$, the radius of the innermost stable orbit, which means that the radius of the actual orbit must lie on the $M^0(R_{\rm orb})$ curve below its intersection with the (dotted) straight line $M^0(R_{\rm ms})$ that relates the star's mass to $R_{\rm ms}$. These requirements constrain the star's representative point to lie in the pie-slice shaped region enclosed by the solid line. (b) Comparison of the regions allowed for nonrotating stars with three different QPO frequencies with the mass-radius relations for nonrotating neutron stars given by five representative equations of state. (c) Regions allowed for rotating neutron stars with four values of j and $\nu^*_{\rm QPO2} = 1220$ Hz, when effects of the stellar spin are included to first-order (see text). (d) Illustrative Keplerian QPO frequency vs. accretion luminosity curve predicted by the sonic-point model.

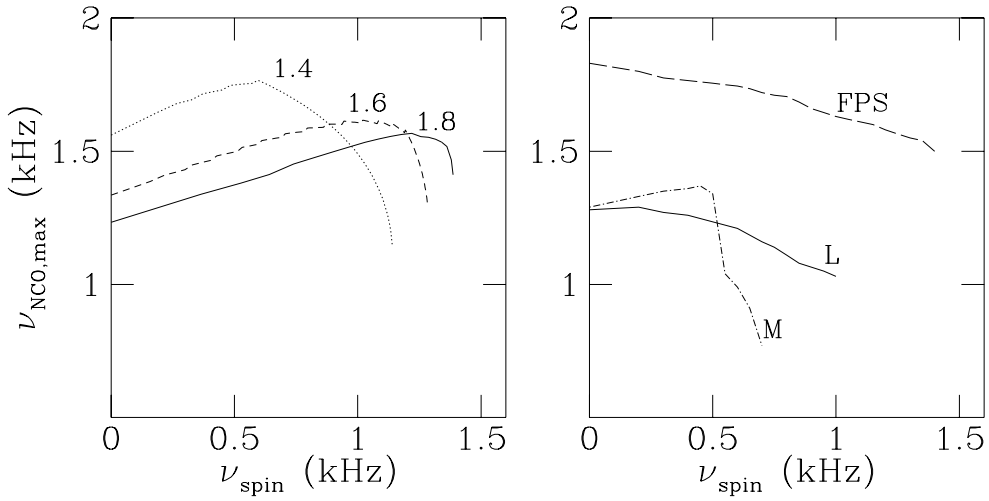

Fig. 4. (a) Maximum frequency of a nearly circular orbit as a function of stellar spin frequency for equation of state FPS and stars with $1.4\,M_\odot$ (dotted line), $1.6\,M_\odot$ (dashed line), and $1.8\,M_\odot$ (solid line) gravitational masses. (b) Maximum frequency of a nearly circular orbit for a star of *any* mass, for the three equations of state indicated. See [12] for details and further discussion.

radius can be proved analytically [13]. The bounds for prograde orbits are

$$M_{\rm max} \approx [1 + 0.75 j(\nu_{\rm spin})] M^0_{\rm max} \quad \text{and} \quad R_{\rm max} \approx [1 + 0.20 j(\nu_{\rm spin})] R^0_{\rm max} \ , \qquad (2)$$

where $j(\nu_{\rm spin})$ is the value of j for the observed stellar spin rate *at the maximum allowed mass for the equation of state being considered* and $M^0_{\rm max}$ and $R^0_{\rm max}$ are the bounds on the mass and radius for a nonrotating star (see eqs. [1]). Equations (2) show that the region allowed for a slowly rotating star is always larger than the region allowed for a nonrotating star, regardless of the equation of state. Figure 1c illustrates the effects of slow stellar rotation on the allowed region of the radius-mass plane. Detailed calculations show that the mass of the neutron star in 4U 1636−536 must be less than $2.2\,M_\odot$ and its radius must be less than 17 km. The upper bounds may be smaller, depending on the equation of state assumed [13, 14].

For spin frequencies $\gtrsim 400$ Hz, the structure of the star can be significantly affected, as well as the exterior spacetime. In this case both the structure of the star and the spacetime must be computed numerically for each assumed equation of state. Determination of the bounds on the mass and radius of a star for a given QPO frequency and equation of state therefore requires construction of a sequence of stellar models and spacetimes for different masses

using the equation of state, with $\nu_{\rm spin}$ as measured at infinity held fixed [13]. Such sequences have been constructed [12] and show that if the neutron star is spinning rapidly, the constraints on the equation of state are tightened dramatically. For example, if the ∼580 Hz burst oscillation frequency observed in 4U 1636−536 is its spin frequency (rather than twice its spin frequency as indicated by the frequency separation of its two kilohertz QPOs), then very stiff equations of state like the tensor-interaction equation of state [15] are excluded by the 1220 Hz QPO already observed from this star. Regardless of the star's spin rate, a 1500 Hz QPO frequency would constrain the mass and radius of the neutron star to be less than $\sim 1.7\,M_\odot$ and ∼13 km, ruling out several equations of state that are currently astrophysically viable [12].

Figure 4a shows why slow rotation typically loosens the constraints on mass and radius implied by a given QPO frequency, whereas rapid rotation tightens them. At low spin rates the equatorial radius of the star is smaller than the radius of the innermost stable orbit and hence the maximum orbital frequency increases linearly with the star's spin rate (see eqs. [2]). In contrast, at high spin rates the equatorial radius becomes larger than the radius of the circular orbit that would be marginally stable, so no marginally stable orbit exists. The highest frequency orbit for a star of given mass is then the one just above the stellar surface, which increases in radius as the star spins faster, causing the highest possible orbital frequency to decrease with increasing spin rate. Figure 4b displays the maximum frequency of a nearly circular orbit for a star of *any* mass, for three different equations of state. Hence, if *any* neutron star is found to have spin and QPO frequencies that place its representative point above one of these curves, that equation of state is excluded.

3.3 Innermost Stable Circular Orbit

Establishing that an observed QPO frequency is the orbital frequency of the innermost stable circular orbit in an X-ray source would be an important step forward in our understanding of strong-field gravity and the properties of dense matter, because it would be the first confirmation of a prediction of general relativity in the strong-field regime and would also fix (for each equation of state) the mass of the neutron star involved.

Given the fundamental significance of the detection of an innermost stable orbit, it is very important to establish what would constitute strong, rather than merely suggestive, evidence that the innermost stable orbit has been detected. Probably the most convincing signature would be a kilohertz QPO with a frequency that reproducibly increases steeply with increasing accretion

rate but then becomes constant and remains nearly constant as the accretion rate increases further. This behavior emerges naturally from general relativistic calculations of the gas dynamics and radiation transport in the sonic-point model (see Fig. 3d). The constant frequency should always be the same in a given source. Two other possible signatures of the innermost stable orbit are discussed in [13].

Several authors have recently suggested that innermost stable orbits have already been observed. Zhang et al. [27] suggested that the similarity of the highest QPO frequencies seen so far indicates that innermost stable orbits are being detected and that, based on the equations (1) for nonrotating stars, the neutron stars in *all* the kilohertz QPO sources therefore have masses close to $2.0\,M_\odot$. Kaaret, Ford, & Chen [3] suggested that the ∼800–900 Hz QPOs discovered in 4U 1608−52 [1] and 4U 1636−536 [26], which were initially observed to have roughly constant frequencies, are generated by the beat of the spin frequency against the frequency of innermost stable orbits in these sources. However, *no clear signature of an innermost stable circular orbit has so far been seen in any source.*

Indeed, subsequent observations of both 4U 1608−52 [8] and 4U 1636−536 [25] are inconsistent with the interpretation that the frequencies of the QPOs seen initially are related to the frequencies of innermost stable orbits around these stars. The 1171 Hz QPO seen by Zhang et al. [26], which was assumed by Kaaret et al. [3] to be at the frequency of the innermost stable orbit in 4U 1636−536 in order to estimate the mass of the star, was later seen at 1193 Hz [25] and still later at 1220 Hz (W. Zhang, personal communication). Hence, there is as yet no evidence for a maximum QPO frequency in 4U 1636−536 and hence there is no basis for the suggestion that an innermost stable orbit has been seen in this source. A recent analysis of 4U 1608−52 data by Méndez et al. [8] shows that this source has two kilohertz QPOs that vary with countrate just like the other sources. There is as yet no evidence for a maximum QPO frequency in 4U 1608−52 and hence there is no basis for the suggestion that an innermost stable orbit has been seen in this source, either.

4 Constraints from Burst Oscillations

As noted in §2, the strong (rms amplitudes up to at least 35%), high-frequency oscillations seen during X-ray bursts are thought to be caused by emission from a single or two nearly antipodal bright regions on the stellar surface, which produce large amplitude brightness oscillations at the stellar spin frequency or its first overtone as the star turns. The anisotropy at infinity of the radiation

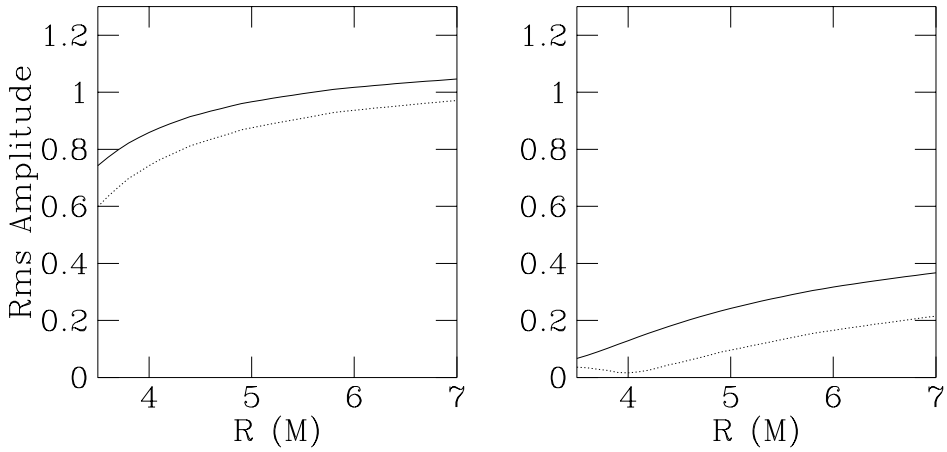

Fig. 5. Upper bounds on the observed fractional rms amplitude of oscillations in the photon number flux during bursts, as a function of neutron star radius. The observer's line of sight is assumed to be in the rotational equator and the aberration and Doppler shifts caused by rotation are neglected. (a) Amplitude as a function of neutron star radius for isotropic emission (dotted line) or the peaked emission expected for a scattering atmosphere (solid line) from a single point. (b) Amplitude as a function of radius for the same intensity distributions as in (a), but for emission from two identical, antipodal emitting points. See [11] for details of the computational method and further discussion.

emitted from such regions, and hence the amplitude of the burst oscillation, typically decreases with increasing gravitational light deflection by the star. Hence, in addition to any constraints on the radius of the star that may be derived from the X-ray spectra of the bursts, constraints on the compactness of the star (and hence the softness of neutron star matter) can be derived from the observed amplitudes of the burst oscillations [11, 17]. Such constraints are particularly useful because they complement the constraints derived from the kilohertz QPOs, which constrain the stiffness of neutron star matter.

Figure 5 shows results typical of general relativistic calculations of the maximum amplitudes of burst oscillations from one or two bright regions, as a function of stellar radius. *Photon counting rate* oscillations with 2–60 keV rms amplitudes as high as $\sim$35% have been reported in 4U 1636−536 with a frequency equal to twice the apparent spin frequency of this neutron star. Figure 5b shows that an oscillation this large in the *photon number flux* would constrain the radius of this star to be greater than $5.5\,M$. However, the amplitude of the countrate oscillation measured at infinity can be larger or smaller than the amplitude of the oscillation in the photon flux at the star, depending

on the detector response as well as the angular dependence and spectrum of the emission from the stellar surface, the stellar spin rate, and whether there is any scattering material surrounding the star, so more detailed modeling will be required to extract the actual constraints on the compactness (see [11]).

5 Concluding Remarks

The discovery using the *Rossi X-Ray Timing Explorer* that many neutron stars with weak magnetic fields produce strong ∼300–1200 Hz X-ray brightness oscillations is a spectacular achievement that validates both the scientific expectations that led to the mission and the long years of hard work that were needed to bring it to fruition. The kilohertz QPOs discovered in the accretion-powered emission are already providing interesting new upper bounds on the masses and radii of neutron stars, and on the stiffness of neutron star matter. The high-frequency oscillations discovered in the emission during thermonuclear X-ray bursts are likely to provide interesting new bounds on the compactness of neutron stars and hence on the softness of neutron star matter. Observation of a QPO with a frequency just 100 Hz higher than the highest frequency so far seen would exclude the stiffest proposed neutron star matter equations of state.

Observation of innermost stable circular orbits would be the first confirmation of a strong-field prediction of general relativity and would fix the mass of the star involved, for each equation of state considered. Although there is currently no strong evidence that an innermost stable circular orbit has been discovered around any of these neutron stars, there is reason to hope that such evidence may be forthcoming. Given the rapid pace of discoveries with *RXTE*, the prospects for obtaining compelling evidence of an innermost stable circular orbit appear good.

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