Models of Kilohertz Quasi-Periodic Brightness Oscillations

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Abstract. The remarkable discovery of kilohertz quasi-periodic brightness oscillations (QPOs) in the accretion-powered emission from some sixteen neutron-star low-mass X-ray binary systems has led to much speculation about and theoretical modeling of the origin of these oscillations. It has also led to intense study of the implications that these QPOs have for the properties of neutron stars and of the accretion flow onto them. In this review we describe the strengths and weaknesses of the models that have been proposed for the kilohertz QPOs observed in the accretion-powered emission. We conclude that beat-frequency models, and in particular the sonic-point model, are currently the most promising. If these models are correct, the kilohertz QPOs provide the strongest constraints to date on the masses and radii of neutron stars in low-mass X-ray binaries, and on the equation of state of the dense matter in all neutron stars.

INTRODUCTION

It has long been expected (see, e.g., [1,2]) that significant information about neutron stars and black holes, as well as accretion onto them, could be extracted from high-frequency variability of the X-ray brightness of these systems. The *Rossi* X-ray Timing Explorer (RXTE) was specifically designed [3,4] to have the large area, microsecond time resolution, and high telemetry rate necessary to probe the high-frequency regime. Within two months of the launch of RXTE, the value of access to high frequencies was dramatically confirmed with the discovery of high-frequency quasi-periodic brightness oscillations (QPOs). These QPOs, some of which have frequencies in excess of 1200 Hz, are the fastest astrophysical oscillations ever discovered. Since their initial discovery, kilohertz QPOs have been revealed to be nearly ubiquitous in neutron-star low-mass X-ray binaries (LMXBs).

Considerable theoretical effort has been devoted to understanding the mechanisms that produce the kilohertz QPOs. In this review we discuss the different types of models that have been suggested, and compare them to the observations. We conclude that beat-frequency models, and in particular the sonic-point model, are the most promising at present. If this interpretation is correct, then observations of kilohertz QPOs allow us to constrain simultaneously the masses and radii of the neutron stars in LMXBs, and yield the strongest astrophysical constraints to date on the equation of state of the dense matter in neutron stars. We begin by summarizing the major observational constraints on the models of the QPOs. We then review specific models, comparing them to the data and discussing their strengths, weaknesses, and implications. We conclude by examining the current observations, and by discussing which future observations would help distinguish between models.

OBSERVATIONAL CONSTRAINTS ON MODELS

As of this writing, there are sixteen neutron-star LMXBs which have been observed with RXTE to have kilohertz brightness oscillations either during type I (thermonuclear) X-ray bursts or during the persistent emission between bursts. Here we will concentrate on the QPOs observed during persistent emission. Observation of many sources has revealed a strikingly consistent set of phenomenological trends, including (see [5] for more details):

1. High QPO frequencies ($\sim$ 300–1200 Hz has been observed).
2. Variable QPO frequencies. In several sources the centroid of a QPO peak has been observed to change by several hundred Hertz. In a given source the QPO frequency tends to increase steeply with increasing countrate, although the frequency-countrate relation can change for a given source between different observations just a few days apart (see, e.g., [6,7]).
3. Common occurrence of pairs of QPO peaks, with a frequency separation that usually remains constant even as the centroids of the peaks shift by hundreds of Hertz (except in Sco X-1 and possibly 4U 1608–52; see [5,8]). In the four sources with kilohertz QPO pairs in which brightness oscillations have been observed during X-ray bursts, the burst oscillation frequency, which is thought to be the stellar spin frequency or its first overtone, is consistent with either one or two times the frequency separation of the pair of kilohertz QPOs.
4. High relative amplitudes. Fractional rms amplitudes of up to 15% over the 2–60 keV band of the Proportional Counter Array on RXTE have been detected. Moreover, the relative amplitude of the brightness oscillation increases with increasing photon energy in many sources.
5. High coherences. Quality factors $Q \equiv \nu_{\rm QPO}/\Delta\nu_{\rm QPO}$ up to $\sim$200 have been observed, and $Q \sim 50$–100 is common.

The neutron-star LMXBs in which kilohertz QPOs appear were studied extensively prior to the launch of RXTE, and a comprehensive physical picture was produced based on the 2–20 keV energy spectra and 1–100 Hz power spectra of neutron-star LMXBs that were collected using satellites such as EXOSAT and Ginga [9,10]. In this picture, neutron-star LMXBs can be naturally divided into two subclasses, both named after the path they make, over time, on color-color diagrams. The six "Z" sources have accretion rates comparable to the Eddington critical rate, and inferred surface magnetic fields $B \sim 10^9$–10^{10} G. The ~15 "atoll" sources are both less luminous and more weakly magnetized than the Z sources, with accretion rates ~1%–10% of the Eddington rate and $B \sim 10^7$–10^9 G. Modeling of the continuum spectrum [11,12] indicates that both types of source are surrounded by a hot central corona with a scattering optical depth $\tau \sim 3$–10, and that the Z sources also have a radial inflow.

A successful model of the kilohertz QPOs must be able to explain all of the observational trends for those QPOs, and it must also fit in naturally with the pre-existing physical picture described above. We now evaluate the proposed models for kilohertz QPOs in this light. Beat-frequency models are by far the most promising, so we will concentrate on them.

SONIC-POINT BEAT-FREQUENCY MODEL

In the sonic-point model [13]:

1. The frequency of the higher-frequency QPO in a pair is the orbital frequency near the radius where the inward radial velocity begins to increase rapidly. For convenience, we label this radius the "sonic point", even though the sonic point itself is not especially significant in this model.

2. The frequency of the lower-frequency QPO in a pair is the beat between this orbital frequency and the stellar spin frequency.

3. The rapid transition to a high inward radial velocity near the star is usually caused by the drag exerted on the accreting gas by radiation from the star, but may instead be caused by general relativistic corrections to Newtonian gravity if the gas moves inside the innermost stable orbit without being significantly affected by radiation.

The nature of the transition to supersonic radial inflow is illustrated by the simplified but fully general relativistic calculation of the gas dynamics and radiation transport in the innermost part of the accretion disk flow described in [13]. In this calculation the azimuthal velocity of the gas in the disk is assumed to be nearly Keplerian far from the star. Internal shear stress is assumed to create a constant inward radial velocity $v^{\hat{r}}$ in the disk, as measured in the local static frame, of 10^{-5}. The half-height $h(r)$ of the disk flow at radius r is assumed to be ϵr at all radii, where ϵ is a constant and r is the radius, and the kinetic energy of the gas that collides with the surface of the star is assumed to be converted to radiation that emerges from a band around the star's equator with a half-height equal to ϵR.

Once the drag force exerted by the radiation from the stellar surface begins to remove angular momentum from the gas in the Keplerian disk, centrifugal support is lost and the gas falls inward, accelerating rapidly. Radiation that comes from

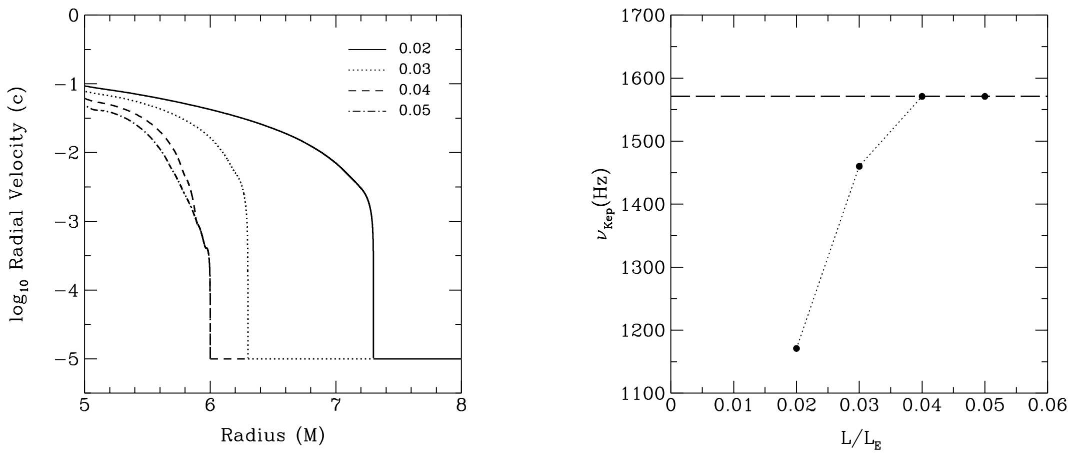
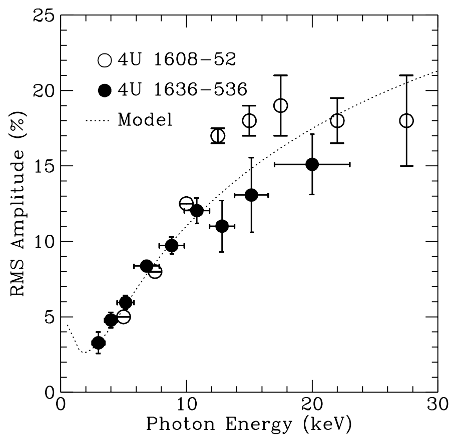


FIGURE 1. (Left panel) The inward radial velocity $v^{\hat{r}}$ of the gas in the disk measured by a local static observer, as computed in the simplified but fully general relativistic model of gas dynamics and radiation transport in the inner disk described in [13]. The four curves are labeled with the assumed accretion rate measured in units of the accretion rate that would produce an accretion luminosity at infinity equal to the Eddington critical luminosity. (Right panel) Sonic-point Keplerian frequency in Hertz versus accretion luminosity in units of the Eddington critical luminosity, calculated using the same accretion flow model. The sonic-point Keplerian frequency increases steeply with increasing accretion luminosity until it reaches $\nu_{\rm K}(R_{\rm ms})$, at which point it stops changing.

near the star and is scattered by the gas in the disk is usually scattered out of the disk plane and hence does not interact further with the gas in the disk. We therefore treat the interaction of the radiation with the gas in the disk by assuming that the intensity of the radiation coming from the star is attenuated as it passes through the gas in the disk, diminishing as $\exp(-\tau_r)$, where $\tau_r(r)$ is the Thomson scattering optical depth radially outward from the stellar surface to radius r. In calculating the radiation drag force, we assume for simplicity that the differential scattering cross section is isotropic in the frame comoving with the accreting gas [14]. The radiation field and the motion of the gas are computed in full general relativity. Calculations in this model indicate that the transition to rapid radial inflow is extremely sharp, occurring over a fractional radius $\Delta r/r < 0.01$ (see Figure 1), and that the orbital frequency at the transition radius increases steeply with increasing mass flux, as observed.

As described in [13], the sonic-point model accounts for the main features of the kilohertz QPOs, including the common occurrence of QPO pairs with a constant frequency separation, the high and variable frequency of the QPOs, the high amplitudes and coherences, and the similarity of the frequencies of the QPOs from



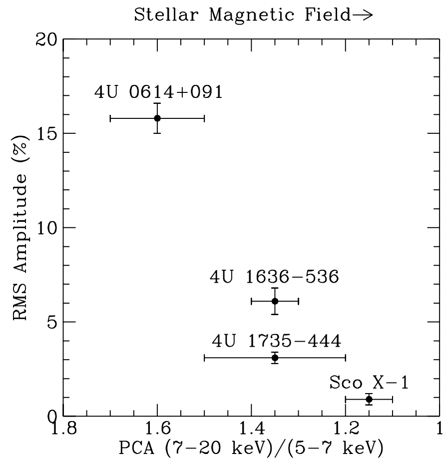


FIGURE 2. (Left panel) Measured amplitudes of the high-frequency QPOs seen in 4U 1608−52 (open circles) and 4U 1636−536 (filled circles) as a function of photon energy. Data for 4U 1636−536 were kindly provided by W. Zhang (personal communication) and reflects corrections made after the report by Zhang et al. [15] was published. The dotted curve shows the variation of the rms amplitude with photon energy caused by the optical depth variations expected in the sonic-point model [13]. (Right panel) Measured rms amplitudes of the high-frequency QPOs seen in Sco X-1, 4U 1735−444, 4U 1636−536, and 4U 0614+091 plotted against the PCA X-ray hard colors (defined as the ratio of the counts in the 7–20 keV bin to the counts in the 5–7 keV bin) of these sources. In neutron star LMXBs, the X-ray hard color is found to be greater for sources with weaker magnetic fields [11,12], so the correlation evident in this figure is striking confirmation that the amplitudes of these QPOs are lower for sources with stronger magnetic fields. The PCA colors were kindly provided by Rudy Wijnands.

sources with widely differing accretion rates and magnetic fields. It also provides a natural explanation for the observed increase in QPO amplitude with increasing photon energy and the decrease in QPO amplitude with increasing inferred magnetic field (see Figure 2). Another strength of the model is that it fits in well with the accretion rates, magnetic fields, and scattering optical depths inferred from pre-existing spectral and temporal modeling.

There are several unresolved issues related to the sonic-point model. Perhaps the most important is how much of the gas can continue spiraling in nearly circular orbits close to the stellar surface without coupling strongly to the stellar magnetic field, as required in this model. This is particularly important for the Z sources, some of which have inferred magnetic fields as high as 10^{10} G. Although it has been realized for more than two decades (see, e.g., [16–22]) that Rayleigh-Taylor and Kelvin-Helmholtz instabilities will allow some fraction of the gas to penetrate inside the magnetosphere without coupling, it has not been possible to calculate this fraction reliably, nor is it clear how much continues in nearly circular orbits.

There is, however, observational evidence that this does occur, because the orbital frequencies at the radii where gas is expected to begin coupling to the field are only a few hundred Hertz, much lower than the frequencies of the kilohertz QPOs.

MAGNETOSPHERIC BEAT-FREQUENCY INTERPRETATION

In the magnetospheric beat-frequency interpretation of the kilohertz QPOs, the frequency of the higher-frequency QPO in a pair is the orbital frequency at the main radius where the stellar magnetic field picks up and channels gas from the accretion disk onto the magnetic polar regions (we call this the "main gas pick-up radius"). The frequency of the lower-frequency QPO is then the beat frequency between this orbital frequency and the stellar spin frequency. The magnetospheric beat-frequency model [23,24] was developed to explain the ∼15–60 Hz "horizontal branch oscillations" (HBO) in Z sources [25], and was first suggested for the kilohertz QPOs by Strohmayer et al. [26].

As applied to the kilohertz QPOs, the magnetospheric model shares certain attractive features with the sonic-point model. For example, it is natural in both models that the frequency separation between pairs of QPO peaks remains approximately constant, as observed. However, the magnetospheric model suffers from many serious difficulties. For example, no mechanism has been found which will generate a QPO at the orbital frequency at the main gas pick-up radius. The high observed coherence of the kilohertz QPOs also poses grave difficulties for the magnetospheric interpretation. Moreover, HBOs and pairs of kilohertz QPOs have been observed simultaneously in all six Z sources (see [5]), so a magnetospheric beat-frequency model cannot explain both types of QPO. For further discussion of the difficulties with the magnetospheric beat frequency interpretation, see [13].

CONSTRAINTS ON NEUTRON STAR MASSES, RADII, AND EQUATIONS OF STATE

The requirement in beat-frequency models that the higher-frequency QPO have the frequency of a nearly circular orbit implies that (1) obviously, the orbit must be outside the neutron star, and (2) the orbit must also be outside the radius of the innermost stable circular orbit, because if the gas producing the QPOs were inside the innermost stable orbit then general relativistic effects would cause the gas to spiral rapidly towards the star, and any QPO thus produced would last for at most a few cycles and hence would produce a much broader QPO than is observed. As explained in [13], these two restrictions mean that, for a slowly rotating star and assuming that the orbit of the gas is prograde, the observation of a coherent QPO at a frequency $\nu_{\rm QPO}$ places upper limits to the mass and radius of the star of

$$M^0_{\rm max} = 2.2[1 + 0.75j(\nu_{\rm spin})](1000\,{\rm Hz}/\nu_{\rm QPO})\,M_\odot \qquad (1)$$

$$R^0_{\rm max} = 19.5[1 + 0.20j(\nu_{\rm spin})](1000\,{\rm Hz}/\nu_{\rm QPO})\,{\rm km} \tag{2}$$

where $j \equiv cJ/GM^2$ is the dimensionless angular momentum, which depends on the neutron star spin frequency $\nu_{\rm spin}$, the assumed equation of state, and the stellar mass. For a stellar spin frequency of 300 Hz, $j \sim 0.1$–0.3, and hence the maximum allowed mass is typically increased by ∼10–20% and the maximum allowed radius is typically increased by a few percent. For more details, see [13,27].

If compelling evidence is collected that an observed QPO frequency is the frequency of the innermost stable circular orbit, then the mass of that neutron star would be known to ∼10%, where the uncertainty is due to uncertainty in j. The detection of the innermost stable circular orbit would have profound implications. First, the mere detection of the presence of unstable orbits would be a confirmation of a key strong-gravity prediction of general relativity. Second, depending on the orbital frequency at the marginally stable orbit, many currently viable equations of state could be ruled out. For example, if the orbital frequency at the innermost stable orbit were 1200 Hz, and the spin frequency of the star were 300 Hz, then the estimated mass of 2–2.1 $M_\odot$ would eliminate the softer equations of state allowed by our current understanding of nuclear forces.

The most convincing evidence for the innermost stable circular orbit would be a QPO frequency that rises steeply with countrate but levels off at some frequency, and does so repeatedly at the same frequency (but not necessarily the same countrate) for a given source. This type of leveling off is shown in the right panel of Figure 1, which displays the dependence of frequency on countrate that emerges from the simplified model calculations described above (see also [13]). Other strong signatures of the innermost stable orbit would be a rapid drop in the amplitude or coherence of either of the QPOs, again always at the same frequency for the same source although not necessarily at the same countrate.

Currently, none of these compelling signatures have been observed from any source, and hence the evidence for effects due to the innermost stable orbit is only circumstantial. There is, nonetheless, reason for optimism, because the highest QPO frequencies currently observed are within only ∼100–200 Hz of where we expect the above signatures to be evident.

OTHER MODELS

Klein et al. [28] have suggested that when the local accretion rate per unit area is super-Eddington, the radiation produced by accretion gets partially trapped and tends to escape in "photon bubbles", which release their energy in a quasi-periodic fashion. Their extensive numerical simulations have shown that the frequencies can be in the kilohertz range, that several peaks in the power spectrum can be produced, and that the rms amplitude of the QPOs can be as high as 3%. The complexities of the three-dimensional photohydrodynamics are such that it is not easy to search parameter space numerically, and analytic insights are difficult to reach (although see [29] for an analytic overview of the phenomenon of photon bubbles). For this oscillation to exist there must be super-Eddington accretion per unit area, and because this is not expected to be the case for the atoll sources (in which the overall accretion rate can be less than 1% of Eddington and the magnetic fields are thought to be weak), photon bubbles are not promising as an explanation of all the kilohertz QPOs. However, the required conditions may exist in the Z sources, and it is thus worth considering photon bubble oscillations in that context. Much further numerical work has yet to be done, and it remains to be seen if photon bubble oscillations can explain other observational trends, such as the increase in relative amplitude of the QPOs with increasing photon energy.

Titarchuk & Muslimov [30] have proposed that the frequencies of kilohertz QPOs are the frequencies of several specific oscillation modes of the accretion disk. This model, like all models in which the luminosity producing the QPOs is released in the accretion disk, has difficulty explaining the high observed amplitudes, because only a small fraction of the total luminosity (typically $<$20%) is released from the disk. In addition, the frequencies of disk modes depend on the distance from the star, so in such a model one must explain why only a small range of radii is selected. One must also explain why only two of a typically large number of oscillation modes produce high-amplitude brightness oscillations.

Titarchuk & Muslimov [30] have made no proposal for how the modes are excited, why they are only excited in a small range of radii, or why their amplitudes can be so high. Moreover, the frequency relations discussed in [30] generally predict that the *ratio* of the frequencies of different modes will remain constant as the frequencies change, in contradiction to the observation that the *difference* of the frequencies is approximately constant in most sources. For the difference to be constant in this model, one must appeal either to a coincidence or to some physical principle that has not yet been elucidated. More seriously, the formalism used in [30] was developed for a spherical star supported by gas pressure in which rotation is a small perturbation, rather than for an accretion disk which is by its nature strongly sheared, and in which the gas is centrifugally supported.

DISCUSSION

The current observations of kilohertz QPOs strongly favor a beat-frequency model. It is, therefore, important to consider further observations that will help to discriminate between the two beat-frequency mechanisms that have been suggested, i.e., the sonic-point mechanism and the magnetospheric mechanism. For example, the sonic-point model predicts that the stronger the magnetic field is, the weaker will be the kilohertz QPOs; in the magnetospheric interpretation, one expects just the opposite, as is true for the horizontal branch oscillations in Z sources. Figure 2 shows that the current data on the amplitude versus the inferred magnetic field clearly prefer the sonic-point model, but more data, and in particular more quantitative data, will help to clarify the situation further.

Serendipity is also likely to continue to play a major role. For example, suppose that QPOs that share all the major properties of the kilohertz QPOs (high and variable frequencies, high amplitude and coherence, plus the common occurrence of pairs of QPO peaks with approximately constant separation) are discovered in sources known to contain black holes because their mass function exceeds three solar masses. This would rule out the beat-frequency models, because they require a surface or a strong and coherent magnetic field. In this case, disk models would likely be favored. If similar high-amplitude kilohertz QPOs were detected from a strongly magnetized accretion-powered pulsar, then again both beat-frequency models would be in great difficulty: the sonic-point model because it would predict an extremely low amplitude, and the magnetospheric interpretation because the frequencies at the characteristic coupling radius would be Hertz, not kilohertz. In this case, one would examine carefully models such as the photon bubble oscillation model, for which the characteristic frequencies are generated at the surface, and a strong magnetic field is beneficial; indeed, the study of photon bubbles was initiated with accretion-powered pulsars in mind.

The detection of QPOs at other frequencies predicted by specific models could help to choose among current models. For example, the detection of a signal at the spin frequency would greatly strengthen the evidence for a beat frequency model, and would tie in the persistent QPOs with the burst oscillations. We now have hundreds of thousands of seconds of data for some sources, and because the frequency separations suggest where the spin frequency should be, searches for the spin frequency can be performed with much greater sensitivity than was possible before. The beat-frequency models also predict that weak signals will be present at other special frequencies, such as at the first overtone of the beat frequency or at the sum of the spin frequency and the orbital frequency. It is important to continue searching for such QPOs, because either detections of them or strong limits on their amplitudes will constrain the models and, in the context of the models, could provide valuable information on the optical depth of the hot central corona or even on the compactness of the neutron star.

CONCLUSIONS

The detection of kilohertz QPOs is a testament to the outstanding capabilities of RXTE and an interesting phenomenon in its own right. It may also provide the strongest astrophysical constraints to date on the mass and radius of the neutron stars in LMXBs, and on the equation of state of neutron star matter. Current observations strongly favor a beat-frequency interpretation of the QPOs detected during persistent emission, and in particular the sonic-point mechanism. New observations of these sources continues to yield important new insights, and we can expect a high pace of discovery in the coming years.

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REFERENCES

1. Lamb, F. K. 1981, in X-Ray Astronomy in the 1980's, ed. S. S. Holt (NASA Technical Memorandum 83848), 77
2. Lamb, F. K., & Pines, D. 1979, Compact Galactic X-Ray Sources: Current Status and Future Prospects (Urbana: UIUC Physics Department)
3. Bradt, H. V., & Swank, J. H. 1989, in Timing Neutron Stars, ed., H. Øgelman & E. P. J. van den Heuvel (Dordrecht: Kluwer, NATO ASI Vol. C262), 393
4. Swank, J., et al. 1995, in The Lives of Neutron Stars, ed. M. A. Alpar, Ü. Kızıloğlu, & J. van Paradijs (Dordrecht: Kluwer, NATO ASI Vol. C 344), 525
5. van der Klis, M. 1997, this volume
6. Ford, E., et al. 1997, ApJ, 486, L47
7. Méndez, M., et al. 1997, ApJ, submitted (preprint astro-ph/9712085)
8. Wijnands, R. A. D., & van der Klis, M. 1997, ApJ, 482, L65
9. Hasinger, G., & van der Klis, M. 1989, A&A 225, 79
10. Lamb, F. K. 1989, in Proc. 23rd ESLAB Symp. on X-ray Astronomy, ed. N. E. White (ESA SP-296), 215
11. Psaltis, D., & Lamb, F. K. 1997, in preparation
12. Psaltis, D., Lamb, F. K., & Miller, G. S. 1995, ApJ, 454, L137
13. Miller, M. C., Lamb, F. K., & Psaltis, D. 1997, ApJ, in press
14. Lamb, F. K., & Miller, M. C. 1995, ApJ, 439, 828
15. Zhang, W., Lapidus, I., White, N. E., & Titarchuk, L. 1996, ApJ, 469, L17
16. Arons, J., & Lea, S. M. 1976a, ApJ, 207, 914
17. Arons, J. 1976b, ApJ, 210, 792
18. Elsner, R. F., & Lamb, F. K. 1976, Nature, 318, 345
19. Lamb, F. K. 1975a, in X-Rays in Space, ed. D. Venkatesan (Calgary: University of Calgary), p. 613
20. Lamb, F. K. 1975b, in Proc. 7th Texas Symposium on Relativistic Astrophysics (Ann. NY Acad. Sci., 262, 331)
21. Wang, Y.-M., & Nepveu, M. 1983, A&A, 118, 267
22. Wang, Y.-M., Nepveu, M., & Robertson, J. A. 1984, A&A, 136, 66
23. Alpar, A., & Shaham, J. 1985, Nature, 316, 239
24. Lamb, F. K., Shibazaki, N., Alpar, A., & Shaham, J. 1985, Nature, 317, 681
25. van der Klis, M. et al. 1985, Nature, 316, 225
26. Strohmayer, T., Zhang, W., Swank, J. H., Smale, A., Titarchuk, L., & Day, C. 1996, ApJ, 469, L9
27. Lamb, F. K., Miller, M. C., & Psaltis, D. 1997, these proceedings
28. Klein, R. I., Jernigan, J. G., Arons, J., Morgan, E. H., & Zhang, W. 1996, ApJ, 469, L119
29. Arons, J. 1992, ApJ, 388, 561
30. Titarchuk, L., & Muslimov, A. 1997, A&A, 323, L5
31. Unno, W., Osaki, Y., Ando, H., & Shibahashi, H. 1979, Nonradial Oscillations of Stars (Tokyo: University of Tokyo Press)