

## EFFECTS OF RADIATION FORCES ON THE FREQUENCY OF GRAVITOMAGNETIC PRECESSION NEAR NEUTRON STARS

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### ABSTRACT

Gravitomagnetic precession near neutron stars and black holes has received much recent attention, particularly as a possible explanation of 15–60 Hz quasi-periodic brightness oscillations (QPOs) from accreting neutron stars in low-mass X-ray binaries, and of somewhat higher frequency QPOs from accreting stellar-mass black holes. Previous analyses of this phenomenon have either ignored radiation forces or assumed for simplicity that the radiation field is isotropic, and in particular that there is no variation of the radiation field with angular distance from the rotational equatorial plane of the compact object. However, in most realistic accretion geometries (e.g., those in which the accretion proceeds via a geometrically thin disk) the radiation field depends on latitude. Here we show that in this case radiation forces typically have an important, even dominant, effect on the precession frequency of test particles in orbits that are tilted with respect to the star's rotational equator. Indeed, we find that even for accretion luminosities only a few percent of the Eddington critical luminosity, the precession frequency near a neutron star can be changed by factors of up to  $\sim 10$ . Radiation forces must therefore be included in analyses of precession frequencies near compact objects in such varied contexts as low-frequency QPOs, warp modes of disks, and trapped oscillation modes. We discuss specifically the impact of radiation forces on models of low-frequency QPOs involving gravitomagnetic precession, and we show that such models are rendered much less plausible by the effects of radiation forces.

*Subject headings:* accretion, accretion disks — radiation mechanisms: thermal — relativity — stars: neutron

### 1. INTRODUCTION

The microsecond time resolution,  $\sim 6000$  cm<sup>2</sup> effective area, and  $\sim 256$  kbit s<sup>-1</sup> telemetry capability of the *Rossini X-Ray Timing Explorer (RXTE)* have made possible the discovery of brightness oscillations with frequencies  $\sim 300$ –1200 Hz in both accretion-powered and thermonuclear-powered emission (see, e.g., van der Klis 1998 for a review of the properties of these oscillations) from nearly 20 neutron star low-mass X-ray binaries (LMXBs). The remarkably coherent brightness oscillations observed during type I (thermonuclear) X-ray bursts are thought to occur at the stellar spin frequency or its first overtone (see, e.g., Strohmayer, Zhang, & Swank 1997). The commonly observed pairs of quasi-periodic brightness oscillations (QPOs) in the accretion-powered emission are generally thought to be generated by a beat-frequency mechanism, in which the frequency of the higher frequency QPO peak in a pair is the Keplerian orbital frequency at a special radius near the neutron star, and the frequency of the lower frequency peak is the difference between this Keplerian frequency and the stellar spin frequency (Miller, Lamb, & Psaltis 1998; Strohmayer et al. 1996). The observations and modeling of these brightness oscillations have produced a rapid advance in our understanding of these systems. For example, if these interpretations of the brightness oscillations are correct, then we know for the first time the spin frequencies of more than a dozen neutron stars in LMXBs, and as a result the evolutionary connection between LMXBs and millisecond pulsars has been strengthened greatly. Even more dramatic is the likelihood that the properties of the brightness oscillations provide robust and important constraints on the equation of state of neutron star matter (see, e.g., Miller et al. 1998).

The flood of new information from *RXTE* has also led to a reexamination of the physical picture of neutron star LMXBs that was developed earlier based on the 2–20 keV energy spectra and 1–100 Hz power spectra of these sources obtained using satellites such as *EXOSAT* and *Ginga*. A phenomenon that has played an especially useful role in the development of this picture is the so-called horizontal branch oscillations, or HBOs (van der Klis et al. 1985; see van der Klis 1989 for a review), which are a type of QPO that is observed in the horizontal branch spectral state of the persistently brightest neutron star LMXBs (the Z-sources). These oscillations have frequencies  $\nu_{\text{HBO}} \sim 15$ –60 Hz, fractional rms amplitudes of a few percent, and coherences  $\nu_{\text{HBO}}/\Delta\nu_{\text{HBO}} \sim 2$ –10, where  $\Delta\nu_{\text{HBO}}$  is the FWHM of the peak in the power spectrum. The most successful model of HBOs is the magnetospheric beat frequency model (Alpar & Shaham 1985; Lamb et al. 1985; Shibazaki & Lamb 1987), in which the observed frequency is the difference between the stellar spin frequency and the orbital frequency at the radius in the accretion disk at which the stellar magnetic field picks up and channels gas from the disk onto the magnetic polar regions. This model accounts for many of the main features of HBOs, including their range of frequencies, their amplitudes, and their dependence on inferred mass accretion rate. Moreover, the stellar magnetic moments that are required in this model are confirmed independently by model fits to the energy spectrum (Psaltis, Lamb, & Miller 1995; Psaltis & Lamb 1999), and the predicted stellar spin frequencies (Ghosh & Lamb 1992) are consistent with the  $\sim 300$  Hz spin frequencies inferred from observations.

An alternative to this model has been suggested recently by Stella & Vietri (1998; also Stella 1997a, 1997b). In their

interpretation, the frequency is the gravitomagnetic, or Lense-Thirring, precession frequency, which is the frequency at which an orbit tilted with respect to the stellar spin axis will precess about the spin axis. In this picture,  $\nu_{\text{HBO}} \propto \nu_{\text{kHz}}^2$  is predicted if the radius at which this frequency is generated is the same radius at which the kilohertz QPOs are generated. The slope of this relation is consistent with observations, although the proportionality constant derived from fits of this formula to observations is roughly 2–4 times (depending on the symmetry) the constant expected for the most realistic equations of state. The possible match of the observed frequency behavior is intriguing, but as yet no convincing physical mechanism has been suggested that would generate QPOs at the Lense-Thirring precession frequency.

To have precession at all requires that the gas generating the QPO be in an orbit that is tilted with respect to the spin equator of the star. One may then categorize possible mechanisms by whether they involve gas that is coupled over a range of radii (e.g., as a warp in a disk) or not coupled (e.g., as a thin annulus or orbiting clumps of gas decoupled from each other). In this paper we focus on the latter possibility.

Here we calculate the precession frequency of a test particle in an inclined orbit around a rotating and radiating neutron star. If the star does not radiate, then the precession frequency is just the Lense-Thirring frequency. However, these stars *are* radiating; indeed, the Z-sources have luminosities that are inferred to be at least tens of percent of the Eddington critical luminosity when the HBO is observed. Radiation forces can therefore be extremely important. Here we calculate their effect on the precession frequency. We show that radiation forces can change the precession frequency dramatically, by factors of several, even if the luminosity is only a few percent of the Eddington luminosity. This is especially true if, as expected, the radiation field is not independent of latitude (e.g., if the accretion occurs in an equatorial band instead of uniformly over the entire surface of the neutron star). Also, even relatively small azimuthal variations in the brightness at a given radius can reduce sharply the coherence of any resulting QPO. Hence, to explain the observed coherence of the HBOs in a model involving Lense-Thirring precession, one must explain why the radiation field is azimuthally symmetric to high accuracy. The axisymmetry requirements are not so severe for a Keplerian-frequency brightness oscillation, such as is thought to generate the higher frequency QPO observed in a pair, because the orbital frequency is not nearly so sensitive to radiation forces.

In § 2 we develop the methods used to estimate the effects of radiation forces on precession frequencies. We give our results and discuss their implications for HBOs in § 3.

## 2. METHOD

Throughout this paper we use the procedures and notation given in Miller & Lamb (1996; see also Abramowicz, Ellis, & Lanza 1990). In particular, we assume that the only interaction between radiation and the test particle occurs via isotropic, frequency-independent scattering, which is a good approximation for the frequencies and ionization fractions expected near accreting neutron stars (see Lamb & Miller 1995 for further discussion). Hence the radiation

force is

$$f^\alpha = \sigma F^\alpha, \quad (1)$$

where  $\sigma$  is the scattering cross section,

$$F^\alpha = -T^{\mu\alpha}u_\mu - u^\alpha T^{\mu\beta}u_\mu u_\beta \quad (2)$$

is the radiative energy flux measured in the rest frame of the particle (see Miller & Lamb 1996), and  $T^{\mu\alpha}$  are the components of the radiation stress-energy tensor at a given event in the Boyer-Lindquist coordinate system. Here and below we set  $G = c \equiv 1$ , except where noted.

Let us assume that a particle moving in an almost circular orbit near the spin equatorial plane is perturbed by a small vertical velocity, which is therefore in the  $\theta$  direction. The 4-velocity is then

$$u^\mu = (u^t, 0, u^\theta, u^\phi), \quad (3)$$

where  $u^t$  and  $u^\phi$  are the same as for a circular equatorial orbit (note, however, that  $u^t$  and  $u^\phi$  are modified by radial radiation forces, and are therefore different from their values in the absence of radiation). The  $\theta$  component of the equation of motion is then

$$\frac{d^2\theta}{d\tau^2} + \frac{1}{2}g^{\theta\theta}(g_{\theta\mu,\nu} + g_{\theta\nu,\mu} - g_{\mu\nu,\theta})u^\mu u^\nu = \frac{f^\theta}{m}. \quad (4)$$

We expand only to first order in the dimensionless spin parameter  $j \equiv cJ/GM^2$ , because to higher order in  $j$  the spacetime external to the star must be calculated numerically (in particular, to order  $j^2$  and higher the spacetime deviates from the Kerr spacetime; see, e.g., Cook, Shapiro, & Teukolsky 1994). To this order, and assuming that the angular deviation  $\delta$  from the equatorial plane is small (i.e.,  $\delta \ll 1$ ), we find

$$\frac{d^2\theta}{d\tau^2} = \left[ \left( u^\phi \right)^2 - \frac{4jM^2}{r^3} u^t u^\phi \right] \delta - \frac{\sigma}{m} (T^{\mu\theta}u_\mu + u^\theta T^{\mu\beta}u_\mu u_\beta). \quad (5)$$

Dividing through by  $(u^t)^2$  and using the fact that in Boyer-Lindquist coordinates  $u^\phi/u^t = \Omega$ , the angular velocity as observed at infinity, we find after some further manipulation (see also Kato 1990)

$$\frac{d^2\theta}{dt^2} = -\Omega_\perp^2 \delta - \frac{\sigma}{m} (u^t)^{-2} (T^{\mu\theta}u_\mu + u^\theta T^{\mu\beta}u_\mu u_\beta), \quad (6)$$

where  $v^\theta \equiv u^\theta/u^t$ ,  $\Omega_K$  is the angular velocity of a circular orbit as measured at infinity, and

$$\Omega_\perp^2 = \Omega_K^2 - \frac{4jM^2}{r^3} \Omega_K \quad (7)$$

is the vertical epicyclic frequency in the absence of radiation forces to first order in  $j$ . The Lense-Thirring precession frequency is simply the difference between the vertical epicyclic frequency and the orbital frequency  $\Omega_K$ .

In the  $\theta$  component of the force equation, the radiation terms may be divided into the velocity-independent flux term  $T^{\mu\theta}u_\mu$  and the remaining terms, which are velocity dependent. The velocity-dependent terms are “drag” terms and are dissipative. That is, for example, the amplitude of vertical oscillatory motion about the equatorial plane is damped by these terms. In contrast, the velocity-independent term is nondissipative and changes only the

frequency, not the amplitude, of the vertical oscillatory motion. If the radiation from the star is assumed to be isotropic, as is a standard assumption in many treatments of warped disk modes near compact objects (see, e.g., Pringle 1996; Markovic & Lamb 1998), then there is no net flux in the  $\theta$  direction, and  $T^{\hat{t}\theta} = 0$ . In general, however, the radiation field will not be isotropic and will have a  $\theta$  dependence, and hence a net flux, in the  $\theta$  direction. For example, if accretion occurs via a thin disk, then the radiation intensity near the rotation equator is greater than the radiation intensity far from the equator, and there is therefore a gradient of flux away from the equatorial plane.

In such a case, it is typical that for small  $\theta$  displacements, and hence small  $u^\theta$ , that the velocity-independent term

$$T^{\hat{t}\theta} u_t = (1 - 2M/r)^{-1/2} r^{-1} T^{\hat{t}\theta} u_t \quad (8)$$

is the largest of the radiation terms by 2 or more orders of magnitude. It is therefore likely that, for a realistic radiation pattern, the frequency (but not the amplitude) of vertical epicyclic motion will be changed significantly by radiation forces. As we show below, for displacements  $\delta \ll 1$  the radiation force is proportional to  $\delta$ , call it  $\kappa_{\text{rad}} \delta$ , and hence the vertical epicyclic frequency becomes, schematically,

$$\Omega_{\perp, \text{rad}}^2 = \Omega_K^2 - \frac{4jM^2}{r^3} \Omega_K - \kappa_{\text{rad}}. \quad (9)$$

We compute  $\kappa_{\text{rad}}$  in a simplified model, for one particular geometry, in § 3.1 (eq. [16]). Note that even if the epicyclic frequency is modified only slightly by the radiation forces, the precession frequency (which is  $\Omega_K - \Omega_{\perp, \text{rad}}$ ) can be changed dramatically. Indeed, as we now show, radiation forces can change the precession frequency by factors of several, even for luminosities that are only a few percent of the Eddington critical luminosity. In the next section we estimate the effects of radiation forces for one specific pattern of emission on the stellar surface to give an idea of the magnitude and dependences. We then discuss the implications that these effects have for models of HBOs involving Lense-Thirring precession.

### 3. RESULTS AND DISCUSSION

#### 3.1. Magnitude of Radiation Forces in Simplified Model

Consider a nonrotating star that radiates uniformly from a band of angular half-width  $\epsilon$  around the equator and does not radiate from any other part of the surface. Let the specific intensity  $I_s$  at the surface be isotropic in the outward direction. What is  $T^{\hat{t}\theta}$ , the  $\theta$  component of the flux as measured in a local tetrad?

Formally, this could be computed in a Schwarzschild spacetime from

$$T^{\hat{t}\theta} = I_s \frac{(1 - 2M/R)^2}{(1 - 2M/r)^2} \int_0^{2\pi} \int_0^{\alpha(\tilde{b})} \sin^2 \tilde{a} \cos \tilde{b} \, d\tilde{a} \, d\tilde{b} \quad (10)$$

(see, e.g., Miller & Lamb 1996), where  $\alpha(\tilde{b})$  is the angular extent of the band in direction  $\tilde{b}$  measured by an observer at Boyer-Lindquist radius  $r$ . For an observer in the equatorial plane ( $\theta = \pi/2$ ),  $T^{\hat{t}\theta} = 0$  because the contributions above and below the plane exactly cancel. For an observer out of the equatorial plane, the contributions do not exactly cancel, and the integral must be performed. Unfortunately, in a Schwarzschild spacetime the computation of  $\alpha(\tilde{b})$  requires numerical integration and does not yield much

insight. We therefore simplify further to straight-line photon propagation. A straightforward but tedious calculation then shows that  $\alpha(\tilde{b})$  is given implicitly by

$$\sin^2 \alpha(\tilde{b}) = \left(\frac{R}{r}\right)^2 \frac{\sin^2 \psi(\tilde{b})}{1 - 2(R/r) \cos \psi(\tilde{b}) + R^2/r^2}, \quad (11)$$

where  $\psi(\tilde{b})$ , the angle spanned by the emitting band in direction  $\tilde{b}$  as measured from the center of the star, is given by

$$\sin \psi(\tilde{b}) = \frac{\sin \epsilon}{(1 - \sin^2 \tilde{b} \cos^2 \epsilon)^{1/2}}. \quad (12)$$

Suppose now that  $\epsilon$  is small enough that we can ignore the small angular extent (near  $\tilde{b} = \pm \pi/2$ ) where the band, in the direction  $\tilde{b}$ , extends beyond the visible horizon. This is a good approximation for  $\epsilon \lesssim 0.2$ . The integral (10) then simplifies to

$$T^{\hat{t}\theta} \approx I_s \frac{(1 - 2M/R)^2}{(1 - 2M/r)^2} \int_{-\pi/2}^{\pi/2} \cos \tilde{b} \, d\tilde{b} \, 2\delta \frac{d\alpha(\tilde{b})}{d\theta} \sin^2 \alpha(\tilde{b}). \quad (13)$$

In equation (13) we have subtracted the contribution of the radiation field at latitudes greater than that of the observer ( $\tilde{b} = -\pi/2$  to  $\pi/2$ ) from the contribution of the radiation field at latitudes less than that of the observer ( $\tilde{b} = \pi/2$  to  $3\pi/2$ ). Combining these equations, we find

$$T^{\hat{t}\theta} \approx \delta I_s \frac{4(1 - 2M/R)^2}{(1 - 2M/r)^2} \frac{R^3}{r^3} \cos \epsilon \sin^2 \epsilon \times \int_0^{\pi/2} \frac{\cos^3 \tilde{b} \, d\tilde{b}}{[1 - 2(R/r) \cos \psi(\tilde{b}) + (R^2/r^2)]^2 (1 - \sin^2 \tilde{b} \cos^2 \epsilon)^{5/2}}. \quad (14)$$

This estimate is in excellent agreement with the numerical results described in § 3.2.

At large radii, we can obtain a simple analytic expression, which, however, underestimates the radiation force at small radii (that is, where  $r \lesssim \text{few} \times R$ ). In the limit  $r \gg R$ ,

$$T^{\hat{t}\theta} \approx \delta I_s \frac{8}{3} \left(1 - \frac{2M}{R}\right)^2 \frac{R^3}{r^3} \cos \epsilon \sin \epsilon, \quad (15)$$

and

$$\kappa_{\text{rad}} \approx \left(\frac{I_s}{I_E}\right) \frac{M}{r^3} \left(1 - \frac{2M}{R}\right) \frac{R}{r} \cos \epsilon \sin \epsilon, \quad (16)$$

where  $I_E$  is the critical specific intensity (if the star were emitting from its entire surface with a specific intensity  $I_E$ , then the luminosity measured at infinity would be the Eddington luminosity; see also Miller & Lamb 1996). The ratio of the radiation term to the Lense-Thirring term is then

$$\frac{\kappa_{\text{rad}}}{4jM^2\Omega_k/r^3} \approx \frac{(I/I_E)(1 - 2M/R)(R/M) \cos \epsilon \sin \epsilon}{4j(M/r)^{1/2}}. \quad (17)$$

At  $r = 12M$  for a star with  $j = 0.2$ , radius  $5M$ , and an emitting band of half-width  $\sin \epsilon = 0.2$ , equation (17) predicts that the two terms are equal when  $I_s = 0.4I_E$ . In fact, the effects of radiation increase very rapidly with decreasing radius, and the two terms are instead approximately equal

when  $I_s = 0.1I_E$ . Note that because we have assumed an emitting band with an angular half-width  $\sin \epsilon = 0.2$ , this intensity corresponds to a luminosity of only  $L = 0.02L_E$ .

This analysis shows that radiation forces can have a major impact on the precession frequency near neutron stars. We now give numerical results for this precession frequency as a function of radius and radiation intensity.

### 3.2. Numerical Results

A detailed description of the codes used to calculate the radiation stress-energy tensor and to follow the motion of test particles around rotating, radiating neutron stars is given in Miller & Lamb (1996). In essence, the codes simply calculate the stress-energy tensor by ray tracing, then calculate the motion of test particles using the relativistic force equation. In the calculations reported here, we use a radiating band, centered on the rotational equator, of half-width 0.2 times the radius of the star. The stellar radius is set to  $R = 5M$ , and the rotation parameter is  $j = 0.2$ . This is a high value of  $j$ ; for example, a  $1.8 M_\odot$  star with a spin frequency 300 Hz has  $j \approx 0.1$  for realistic equations of state. Therefore, if radiation forces alter the precession frequency significantly for the extreme value of  $j = 0.2$ , the effects will be even more important for smaller rotation parameters. For the external spacetime we use the Kerr spacetime, which for  $j = 0.2$  is insignificantly different from the true spacetime around a rotating neutron star. These calculations therefore neglect the corrections caused by classical precession, which are small for the radii and spin rates of interest (see Stella & Vietri 1998).

Figure 1 shows the results of these calculations. In this figure, we plot the precession frequency as seen at infinity as a function of the luminosity at infinity,  $L/L_E = 0.2I_s/I_E$ . For

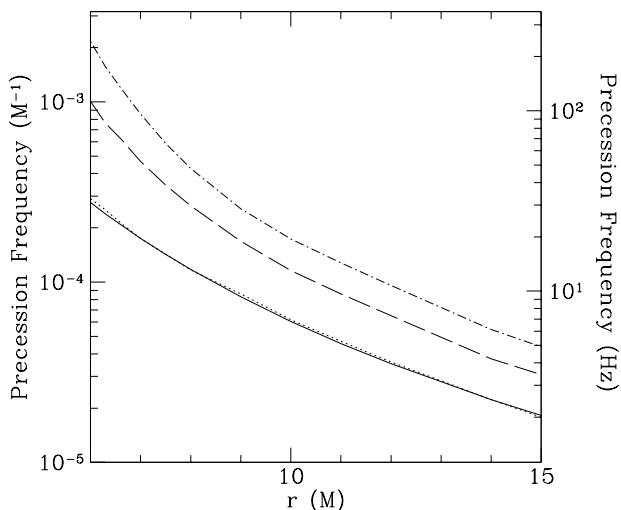


FIG. 1.—Precession frequencies as a function of radius for various luminosities. The left axis gives the frequency in units of  $M^{-1}$ , and the right axis gives the frequency in hertz, assuming a stellar gravitational mass  $M = 1.8 M_\odot$ . As discussed in the text, these calculations were carried out assuming a rotation parameter  $j = 0.2$ , a stellar radius  $R = 5M$ , and a uniformly radiating band of half-width 0.2 times the stellar radius. The solid line gives the analytical Lense-Thirring precession frequency, calculated exactly in a Kerr spacetime with  $j = 0.2$ . The dotted line gives the numerically computed precession frequency for no radiation for comparison with the analytical frequency. The long dashed line gives the precession frequency for a luminosity 0.02 times the Eddington critical luminosity, and the dash-dotted line gives the frequency for a luminosity 0.04 times the Eddington luminosity. This figure shows that the precession frequency can be changed dramatically by radiation forces, particularly for radii close to the star.

comparison, we also plot the analytic value of the Lense-Thirring frequency for zero radiation (*solid line*). In Figure 2 we focus on the radial dependence of the frequency for two different luminosities, with the  $r^{-3}$  dependence of the Lense-Thirring frequency divided out. Clearly, close to the star the radiation component of the precession frequency increases much faster than does the gravitomagnetic component. In Figure 3 we focus on the effect of increasing the luminosity at a fixed radius  $r = 10M$ , and with  $R = 5M$ ,

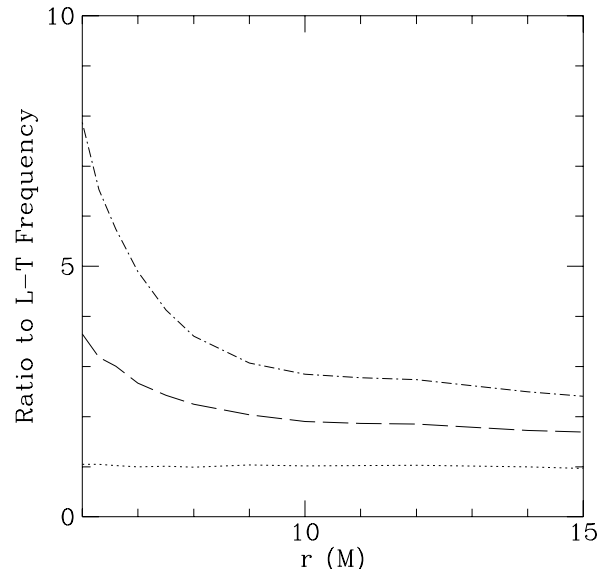


FIG. 2.—Ratio of precession frequencies to the Lense-Thirring frequency for the same luminosities and parameters as in Fig. 1. This figure highlights the deviation from the  $r^{-3}$  scaling of the precession frequency at small radii.

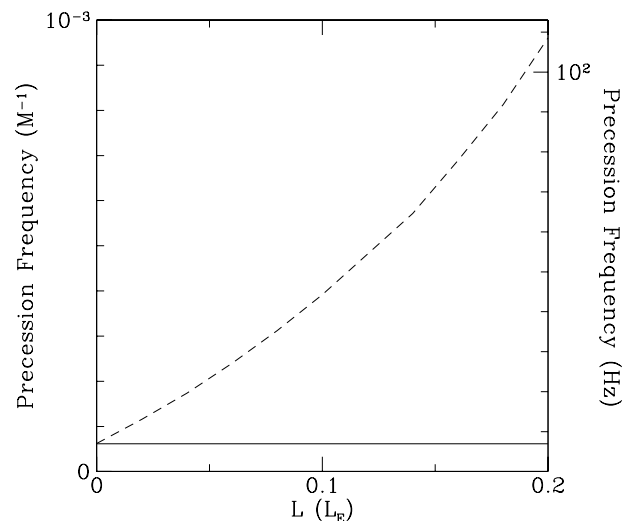


FIG. 3.—Precession frequencies as a function of luminosity (*dashed line*) at the fixed radius  $r = 10M$ . As in Fig. 1, we assume  $j = 0.2$ , a stellar radius  $R = 5M$ , and a uniformly radiating band of half-width 0.2 times the stellar radius. The solid horizontal line gives the precession frequency in the absence of radiation forces. The left- and right-hand axes are as in Fig. 1, except that here the frequencies are plotted linearly instead of logarithmically. As expected from the analytical treatment in § 3.1, for low luminosities the deviation from the Lense-Thirring precession frequency is linear, but at high luminosities the precession frequency is higher than a simple linear extrapolation would predict.

$j = 0.2$ , and a radiating band with fractional half-width 0.2, as before.

### 3.3. Discussion

These figures demonstrate that radiation forces can produce a precession frequency that is many times the gravitomagnetic precession frequency. Here we have presented results for one particular emission pattern and for test particles. However, the main result of this paper, that precession frequencies can be changed dramatically by radiation forces, is much more generally applicable. For example, similar qualitative effects (although differing in detail) are to be expected when radiation is absorbed by the accretion disk and then reradiated outward from the disk. There are, therefore, circumstances in which the vertical component of the radiation force could alter the stability or properties of radiation-driven warps around stars. For instance, if the angle subtended by the radiating layer on the star, as seen at some radius  $r$ , is comparable to the warp angle of the disk, then the radiation absorbed by the disk is increased by a factor of a few. The torque on the disk would therefore be changed by similar factors. Hence close to radiating stars the radiation-driven warping of disks can be altered by vertical radiation forces (for discussion of the importance of warping in a variety of astrophysical situations see, e.g., Pringle 1996, 1997; Maloney, Begelman, & Pringle 1996; Livio & Pringle 1997; Armitage & Pringle 1997; Maloney & Begelman 1997; Markovic & Lamb 1998).

The test particle approximation is strictly valid only if the optical depth to the stellar surface is much less than unity. If instead there is significant shadowing in some directions, the qualitative effects of radiation forces can be different than they are in the simple calculations presented here. For example, if there is substantial emission above and below the disk but shadowing decreases emission in the midplane, the epicyclic frequency is increased by radiation forces, and hence the precession frequency is decreased. What can be said generally is that because the magnitude of the radiation term can exceed the magnitude of the gravitomagnetic term even for low luminosities, the precession frequency is highly sensitive to details of the radiation field.

This has important consequences for any model of HBOs that invokes precession. For instance, it means that for many realistic emission patterns the precession frequency is much greater than the Lense-Thirring frequency. As is clear from Figure 2, the precession frequency will therefore depend strongly on both luminosity and radius (indeed, even at a fixed luminosity the precession frequency is much steeper than  $r^{-3}$  near the star). Hence it would require an improbable coincidence for the precession frequency to have an  $r^{-3}$  dependence on radius, as has been claimed for GX 17+2 and GX 5-1 (Stella & Vietri 1998).

The production of narrow HBOs by Lense-Thirring precession also requires that the radiation field be very axisymmetric. Consider, for example, a  $Z$ -source radiating at a few tens of percent of the Eddington luminosity. From the results in §§ 3.1 and 3.2 it is clear that a fractional azimuthal variation in the intensity of only  $\Delta I/I \sim 0.05-0.1$  would produce a much larger FWHM to the QPO than is observed, because the change in precession frequency would exceed the Lense-Thirring precession frequency. Such a fractional azimuthal variation arises in many plausible scenarios; for example, this could happen if the optical depth from that radius to the stellar surface varies slightly with azimuth. In contrast, the orbital frequency, which in this picture gives the higher frequency QPO peak in a pair, is much less sensitive (see § 5 of Miller et al. 1998), and in such a situation would vary by less than 1%.

Coherence of the HBO is even more difficult to maintain in a Lense-Thirring precession model if the HBO is generated by the precession of the footprint of impact on the stellar surface. This is because, in that case, one needs to map the precession frequency at some orbital radius onto the stellar surface. As discussed in Miller et al. (1998), this demands near axisymmetry from the entire spiral of gas from the orbital radius to the surface, and not just from the movement of gas near the orbital radius. In turn, this means that the radiation field must be nearly axisymmetric from the orbital radius inward, because otherwise the gas from even a single clump would become dephased. This appears difficult, particularly given that radiation stresses have a rapidly increasing effect on the precession frequency of gas as the gas moves closer to the star. Note that this problem does not exist for the magnetospheric beat frequency model of HBOs, because in that model the flow of gas onto the star is controlled by the magnetic field, and hence radiation forces have only a minor effect.

In conclusion, we have shown that radiation forces have an extremely strong effect on orbital precession near accreting neutron stars. Combined with recent results on warped disk modes near neutron stars (Markovic & Lamb 1998), this makes Lense-Thirring explanations of neutron star QPOs much less promising than had been thought previously. More generally, vertical radiation forces may modify torques or precession frequencies in other contexts as well, and they must therefore be considered in treatments of warps or disk modes around accreting neutron stars and black holes.

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