Overview of Neutron Stars

Neutron stars are extreme physics engines, and as such study of them can give us insight into many frontiers of physics. In the four lectures on neutron stars, we will give an overview of their properties (this lecture), X-ray binaries and magnetic accretion (next lecture), and gravitational waves as applied to neutron stars (tomorrow's two lectures). Here we will start with an overall description of neutron stars, then discuss their high densities and strong magnetic fields.

Summary of Neutron Stars

A typical neutron star has a mass of $\sim 1.2 - 2.2\,M_\odot$ and a radius of $\sim 10 - 13$ km. It can have a spin frequency up to $\sim$1 kHz and a magnetic field up to perhaps 10^{15} G or more. Its surface gravity is around $2 - 3 \times 10^{14}$ cm s^{-2}, so mountains of even perfect crystals can't be higher than < 1 mm, meaning that these are the smoothest surfaces in the universe. They have many types of behavior, including pulsing (in radio, IR, opt, UV, X-ray, and gamma-rays, but this is rarely all seen from a single object), glitching, accreting, and possibly gravitational wave emission. They are the best clocks in the universe; the most stable are more than a thousand times more stable in the short term than the best atomic clocks. Their cores are at several times nuclear density, and may be composed of exotic matter such as quark-gluon plasmas, strange matter, kaon condensates, or other weird stuff. In their interior they are superconducting and superfluid, with transition temperatures around a hundred million degrees Kelvin. All these extremes mean that neutron stars are attractive to study for people who want to push the envelope of fundamental theories about gravity, magnetic fields, and high-density matter.

High densities

Let's start, then, with high densities. An essential new concept that is introduced in high densities is *Fermi energy*. The easiest way to think about this is in terms of the uncertainty principle,

$$\Delta p \Delta x > \hbar\ . \tag{1}$$

For you sticklers for accuracy: yes, the actual uncertainty principle is $\Delta p \Delta x \geq \hbar/2$, but here we're not worried about exact factors. If something is localized to a region of size Δx, then its momentum must be at least $\hbar/\Delta x$ (I'm assuming you have taken basic quantum mechanics; think about a particle in an infinite square well of width Δx). That means that in a dense environment, there is a momentum, and hence an energy, associated with the confinement. Therefore, squeezing something increases its total energy, and this Fermi energy acts as a pressure (sometimes called degeneracy pressure). The existence of this energy has a profound role in the structure of white dwarfs and neutron stars. For example, unlike main sequence stars, which are larger if they are more massive, white dwarfs are *smaller* at higher masses.

We will begin with some basic numbers. If the energy and momentum are nonrelativistic, then the Fermi energy E_F is related to the Fermi momentum $p_F \sim \hbar/\Delta x$ by $E_F \approx p_F^2/2m$, where m is the rest mass of the particle. Since $\Delta x \sim n^{-1/3}$, where n is the number density of the particle, in this regime $E_F \sim n^{2/3}$. At a high enough density, however, $E_F > mc^2$, which means that the Fermi energy is relativistic. In the ultrarelativistic limit $E_F \sim p_F c$, so $E_F \sim n^{1/3}$. For electrons, the crossover to relativistic Fermi energy happens at a density $\rho \sim 10^6$ g cm^{-3}, assuming a fully ionized plasma with two nucleons per electron. For protons and neutrons the crossover is at about 6×10^{15} g cm^{-3} (it scales as the particle's mass cubed). The maximum density in neutron stars is no more than $\sim 10^{15}$ g cm^{-3}, so for most of the volume of neutron stars electrons are highly relativistic but neutrons and protons are at best mildly relativistic.

Let's now think about what that means. Suppose we have matter in which electrons, protons, and neutrons all have the same number density. For a low density, which of the three has the highest Fermi energy? The electrons, since at low densities the Fermi energy goes like the inverse of the particle mass. Given what we said before, what is the approximate value of the electron Fermi energy when $\rho = 10^6$ g cm^{-3}? That's the relativistic transition, so $E_F \approx m_e c^2 \approx 0.5$ MeV. Then at 10^7 g cm^{-3} the Fermi energy is about 1 MeV, and each factor of 10 doubles the Fermi energy because $E_F \sim n^{1/3}$ in the relativistic regime. What that means is that the energetic "cost" of adding another electron to the system is not just $m_e c^2$, as it would be normally, but is $m_e c^2 + E_F$. It therefore becomes less and less favorable to have electrons around as the density increases.

Now, in free space neutrons are unstable. This is because the sum of the masses of an electron and a proton is about 1.5 MeV less than the mass of a neutron, so it is energetically favorable for a free neutron to decay. What happens, though, at high density? If $m_p + m_e + E_F > m_n$, then it is energetically favorable to combine a proton and an electron into a neutron. Therefore, at higher densities matter becomes more and more neutron-rich. First, many of the protons in atomic nuclei combine with electrons to make neutrons, so you get nuclei such as ^{120}Rb, with 40 protons and 80 neutrons. Then, at about 4×10^{11} g cm^{-3} it becomes favorable to have free neutrons floating around, along with some nuclei and a sea of electrons (this is called "neutron drip" because the effect is that neutrons drip out of the nuclei). At even higher densities, the matter is essentially a smooth distribution of neutrons plus a $\sim 5 - 10\%$ smattering of protons and electrons. At higher densities yet (here we're talking about nearly 10^{15} g cm^{-3}), the neutron Fermi energy could become high enough that it is favorable to have other particles appear.

It is currently unknown whether such particles will appear, and this is a focus of much present-day research. If they do, it means that the energetic "cost" of going to higher density is less than it would be otherwise, because energy is released by the appearance of other, exotic particles instead of more neutrons. In turn, this means that it is easier to compress the star: squashing it a bit doesn't raise the pressure and energy as much as you would have thought. Another way of saying this is that when a density-induced phase transition occurs (here, a transition to other types of particles), the equation of state is "soft".

Well, that means that it can't support as much mass. That's because as more mass is added, the star compresses more and more, so gravity becomes more important. If pressure doesn't increase to compensate, the star collapses and forms a black hole (or, according to some ideas, it might undergo a phase transition to a different type of compact object). What all this means is that by measuring the mass and radius of a neutron star, or by establishing the maximum mass of a neutron star, one gets valuable information about the pressure-density relation (which is called the equation of state in this context), and hence about nuclear physics at very high density. This is just one of many ways in which study of neutron stars has direct implications for microphysics.

Comment: the extra "squishiness" of matter when it is near a density-induced phase transition may also have importance in the early universe. It's been pointed out that when the universe goes from being a quark-gluon plasma to being made of nucleons (at about 10^{-5} s after the Big Bang), this is a density-induced phase transition, so it may be comparatively easy to form black holes then. Perhaps this led to the formation of so many black holes that they form dark matter; incidentally, because this event happened before big bang nucleosynthesis, no baryon number constraints are violated. This is not the leading model for dark matter, but it is thought-provoking and has received extra attention since the discovery of many black hole binaries with ground-based gravitational wave detectors such as LIGO and Virgo.

Huge Magnetic Fields

In addition to ultrahigh densities, another unique aspect of neutron stars is their magnetic fields. The most magnetic neutron stars have fields that are $\gtrsim 10^7$ times stronger than any other fields in the known universe. The fields can therefore have extremely important effects on matter in ways not approached anywhere else. The range of magnetic field strengths in neutron stars is enormous: from the $B \sim 10^{8-9}$ G of millisecond pulsars (already stronger than almost anything else in the universe!) to the $B \sim 10^{12}$ G that is typical of young pulsars to the $B \sim 10^{15}$ G of many magnetars.

It is always useful, in astrophysics, to get a sense for the importance of an effect by comparing it with something else. In the case of magnetic fields we can begin by computing a characteristic energy: the *cyclotron energy*. The cyclotron energy is the energy of a photon at the cyclotron frequency

$$\omega_c = \frac{qB}{mc} \,, \tag{2}$$

where q is the electric charge of a particle spiraling around the field, B is the strength of the field, m is the mass of the particle, and c is the speed of light. This equation is in cgs units.

When we use $q = e$ (the magnitude of the charge of an electron) and $m = m_e$ (the mass of an electron), and multiply by $\hbar$ to get an energy, we find that the electron cyclotron energy is

$$\hbar\omega_c = \frac{\hbar eB}{m_e c} = 11.6 \text{ keV} \left(B/10^{12} \text{ G}\right) \,. \tag{3}$$

This is a large energy! We can compare it with two other characteristic energies: the ground-state binding energy of hydrogen, 13.6 eV, and the rest-mass energy of an electron, 511 keV. These comparisons tell us that above $\sim 10^9$ G magnetic fields have a nonperturbative effect on atomic binding energies, and at fields comparable to or larger than 4.4×10^{13} G (where $\hbar\omega_c = m_e c^2$) other effects can come in. Indeed, at such large magnetic fields (and a bit lower) there are multiple new quantum electrodynamic processes, including single-photon pair production ($\gamma B \rightarrow e^- e^+ B$), photon splitting ($\gamma B \rightarrow \gamma\gamma B$), and even weirder effects such as the "vacuum resonance" in a plasma.

Yet another effect comes in when we simply compare photon energies to the electron cyclotron energy. When we think about the scattering of a photon off of an electron, we can think of it classically as having a wave oscillate an electron which, being thus accelerated, radiates. But if the magnetic field is strong enough, then it is tougher to oscillate the electron across the field than along the field. This extra resistance means that it is more difficult to accelerate the electron across than along the field, and thus there is less radiation in that polarization. This, in turn, tells us that the cross section for scattering is suppressed when the field is strong enough and the polarization of the photon is perpendicular to the field direction. The suppression factor is $\sim (\omega/\omega_c)^2$ for a photon with frequency $\omega < \omega_c$, and that polarization is called "extraordinary" in contrast with the "ordinary" polarization that is aligned with the magnetic field (and for which the cross section is not suppressed because the electron can move freely along the direction of the field). Because ordinary and extraordinary photons can convert into each other, energy leaks out primarily in the low cross section mode, and thus there is hope for seeing polarized X-rays from some neutron stars.

Superconductivity and Superfluidity

Nature is lazy! This is why there is progressive neutronization of matter at higher and higher densities: it's a lower energy state than it would have with more protons and electrons. In that same general spirit, we also can have superconductivity and superfluidity in neutron stars.

The general idea is that if there is an attractive pairing interaction between fermions, they can couple to form a state with integral spin, and therefore can act like bosons. At a low enough temperature, these "bosons" can form a condensate-like state in which all of the bosons occupy the same quantum state and form a superfluid. If the component fermions are charged, this forms a superconductor. In normal laboratory experience, the pairing is electronic and happens only at very low temperatures (other than the ceramic high T_c superconductors, which do their thing at liquid nitrogen temperatures or a bit above, almost all laboratory superfluid or superconducting phenomena are observed at temperatures less than 20 K). However, in the dense core of neutron stars, nucleonic pairing can happen.

As with all highly degenerate systems, pairing occurs between states near the Fermi surface (recall that in the cores of NS, both protons and neutrons are degenerate, just not relativistically so). Since there are many more neutrons than protons, neutrons and protons can't pair up easily because their momenta are substantially different. We can therefore consider only $n - n$ and $p - p$ pairings. The first gives a superfluid, the second a superconductor. The transition temperatures are extremely difficult to compute from first principles, but the guess is that they are in the range of a hundred million to a billion K.

What are the effects of superconductivity and superfluidity? The evidence is pretty indirect, but superfluidity has been invoked to explain glitches (sudden but small changes in the spin frequency of pulsars) and the evolution of the temperature and magnetic fields of neutron stars. It is also possible that superfluidity in magnetars has a significant impact on quasi-periodic oscillations seen in especially large bursts.