What can we learn about neutron stars from gravitational waves?

For our final lecture we will talk about gravitational waves from neutron stars, focusing on binaries.

Suppose that we have a well-separated binary, so that the components can be treated as points and we only need take the lowest order contributions to gravitational radiation. Temporarily restricting our attention to circular binaries, how will their frequency and amplitude evolve with time?

Let the masses be m_1 and m_2, and the orbital separation be R. We argued in the previous lecture that the amplitude a distance $r \gg R$ from this source is $h \sim (\mu/r)(M/R)$, where $M \equiv m_1 + m_2$ is the total mass and $\mu \equiv m_1 m_2/M$ is the reduced mass. We can rewrite the amplitude using the relation $f \sim (M/R^3)^{1/2}$ between the orbital frequency f and the mass and radius, to read

$$\begin{aligned} h &\sim \mu M^{2/3} f^{2/3}/r \\ &\sim M_{\rm ch}^{5/3} f^{2/3}/r \end{aligned} \tag{1}$$

where $M_{\rm ch}$ is the "chirp mass", defined by $M_{\rm ch}^{5/3} = \mu M^{2/3}$. The chirp mass is named that because it is this combination of μ and M that determines how fast the binary sweeps, or chirps, through a frequency band. When the constants are put in, the dimensionless gravitational wave strain amplitude (which, remember, is roughly the fractional amount by which a separation changes as a wave goes by) measured a distance r from a circular binary of masses M and m with a binary orbital frequency $f_{\rm bin}$ is (Schutz 1997)

$$h = 2(4\pi)^{1/3}\frac{G^{5/3}}{c^4} f_{\rm GW}^{2/3} M_{\rm ch}^{5/3}\frac{1}{r}\ , \tag{2}$$

where $f_{\rm GW}$ is the gravitational wave frequency. Redshifts have not been included in this formula.

The luminosity in gravitational radiation is then

$$\begin{aligned} L &\sim 4\pi r^2 f^2 h^2 \\ &\sim M_{\rm ch}^{10/3} f^{10/3} \\ &\sim \mu^2 M^3/R^5\ . \end{aligned} \tag{3}$$

The total energy of a circular binary of radius R is $E_{\rm tot} = -G\mu M/(2R)$, so we have

$$\begin{aligned} dE/dt &\sim \mu^2 M^3/R^5 \\ \mu M/(2R^2)(dR/dt) &\sim \mu^2 M^3/R^5 \\ dR/dt &\sim \mu M^2/R^3\ . \end{aligned} \tag{4}$$

Again using $f \sim (M/R^3)^{1/2}$, this implies

$$df/dt \sim M_{\rm ch}^{5/3} f^{11/3} . \tag{5}$$

This confirms what we said earlier: to lowest order, the rate of frequency evolution depends only on the chirp mass. An important implication is that to this order, we can determine only the chirp mass rather than both masses separately. Only at the next order in the post-Newtonian expansion does another quantity (the mass ratio) enter.

What if the binary orbit is eccentric? The formulae are then more complicated, because one must average properly over the orbit. This was done first to lowest order by Peters and Matthews (1963) and Peters (1964) by calculating the energy and angular momentum radiated at lowest (quadrupolar) order, and then determining the change in orbital elements that would occur if the binary completed a full Keplerian ellipse in its orbit. That is, they assumed that to lowest order, they could have the binary move as if it experienced only Newtonian gravity, and integrate the losses along that path.

We can understand one qualitative aspect of the radiation when the orbits are elliptical. From our derivation for circular orbits, we see that the radiation is emitted much more strongly when the separation is small, because $L \sim R^{-5}$. Consider what this would mean for a very eccentric orbit $(1-e) \ll 1$. Most of the radiation would be emitted at pericenter, so this would have the character of an impulsive force. With such a force, the orbit will return to where the impulse was imparted. That means that the pericenter distance will remain roughly constant, while the energy losses decrease the apocenter distance. As a consequence, the eccentricity decreases. In general, gravitational radiation will decrease the eccentricity of an orbit. More quantitatively, to the order that Peters and Matthews did their calculation,

$$ae^{-12/19}(1-e^2)\left(1+\frac{121}{304}e^2\right)^{-870/2299} \tag{6}$$

is constant throughout the inspiral. If we ignore the final factor (which is always between 0.88 and 1), we can write this as $a(1-e)(1+e)e^{-12/19} \approx {\rm const}$. For high eccentricities such that $1-e \ll 1$, $1+e$ and $e^{-12/19}$ are roughly constant, so $a(1-e) = r_p \approx {\rm const}$, which means that the pericenter distance r_p is roughly constant as promised. For low eccentricities such that $1-e^2 \approx 1$, we get $ae^{-12/19} \approx {\rm const}$. The orbital frequency (which is half the dominant gravitational wave frequency when $e \ll 1$) is $f \propto a^{-3/2}$, which means that $f \propto e^{-18/19}$, or roughly $e \propto f^{-1}$. Thus for low eccentricities, the eccentricity roughly scales as the reciprocal of the frequency. This means that binary sources at the high frequencies detectable using LIGO can usually be considered to be effectively circular.

How compact binaries can merge

The basic ways that compact binaries can come together break down to two major categories:

1. Evolution of an isolated massive binary. That is, we start with a pair of massive stars that both evolve into black holes, and merge, without any other stars coming close enough to do anything.

2. Dynamical processes. Examples include single-binary interactions, the Kozai-Lidov resonance, and direct dynamical capture.

Isolated massive binaries.—The fine line that must be walked to result in a compact object merger is that the stars must begin far enough apart that they do not merge before both are compact objects, but close enough together that the final double compact object binary can then merge within a few billion years under the influence of gravitational radiation alone. The study of the evolution of massive binaries is particularly difficult because observational evidence is tough to obtain: massive stars are rare and short-lived, and the most critical evolutionary phases for compact object mergers occupy very small fractions of the short lives of these systems.

We are therefore largely dependent on theory to tell us what is likely to happen. Massive stars will under most circumstances expand out to be giants after they run out of hydrogen in their cores (an exception might be if they rotate rapidly enough to continue to cycle hydrogen into the core; this can lead to the so-called "chemically homogeneous" path of evolution). Thus it is possible that a pair of massive stars that are initially much too far separated to spiral in via gravitational wave emission can, in the "common envelope" phase (where the envelope of a giant encompasses its companion), be dragged much closer together. If the pair begins too close together, it might merge; if one of the stars was already a compact object, it could then reside in the center of the other star and thus form a hypothesized "Thorne-Żytkow object", but it will not produce a compact binary. Thus binaries need to start their lives far enough apart to avoid merger, but not so far apart that common envelope drag is insufficient to reduce the separation to a few tenths of an astronomical unit (which is needed for the inspiral to take a few billion years or less).

Unfortunately, the common envelope phase is *very* difficult to understand from a purely theoretical point of view, and given that no binary has ever been seen *in* a common envelope state, the uncertainties are huge. In fact, over the last decade plus there have been times when different treatments of common envelopes have given rate estimates (say, for double black hole binaries) that differ by more than two orders of magnitude! There are other problems in our understanding as well. For example, both neutron stars and black holes are produced by core-collapse supernovae. When we look at neutron stars it is clear that many of them have received kicks (i.e., net linear momentum) because of the core collapse. There is also evidence of supernova kicks for some black holes. However, the origin of these kicks is not known, and neither are the kick direction or the kick magnitudes as a function of the compact object mass.

Dynamical processes.—If a binary does have significant interactions with other stars and compact objects, then there are additional channels for mergers that open up. For example:

1. In globular clusters or nuclear star clusters, the stellar number density can be a million or more times higher than the ~ 0.15 stars per cubic parsec in our Solar vicinity. This still isn't enough to have stars collide directly with each other very often, but it does mean that binary systems, which act as if their collision cross sections are the sizes of the orbits, can have collisionless three-body (or four-body) encounters. The interactions are chaotic, but computer simulations show that when a binary and single interact, the binary that emerges from the interaction tends to contain the two most massive of the three original objects. Moreover, when the binary is hard (meaning that its binding energy is greater than the average kinetic energy of a star), interactions harden them further and drive them closer to merger.

2. Careful observations of massive binaries in our Galaxy suggest that 10% or more of them could actually be triples, quads, or higher-order multiple systems. This opens up the possibility of Kozai (or Kozai-Lidov) resonances. Kozai and Lidov discovered independently in the early 1960s that if a binary is orbited by a third object in a hierarchical triple (such that the system has long term stability), and the binary orbital axis is strongly tilted with respect to the orbit of the tertiary, then over many orbits of both the binary and the tertiary, the relative inclination of the binary to the tertiary cycles between low and high values, while conserving the semimajor axis. Most importantly for merger possibilities, when the inclination goes down the eccentricity goes up and vice versa. Thus in the right range of orientations the binary could be driven to such a high eccentricity that gravitational radiation grinds it down to merger (although if the system is susceptible to such evolution it is likely that it would be driven to collisions on the main sequence or giant branch rather than when the objects become compact). In dense stellar systems such as globular clusters, hierarchical Kozai-susceptible triples can be created *after* evolution to compact objects, for example as an outcome of binary-binary interactions.

Dense stellar systems are expected to be far more efficient *per stellar mass* than isolated binaries in producing mergers. However, only $10^{-4}-10^{-3}$ of stars are in dense stellar systems, and that seems to tilt the balance in favor of binaries. Binaries in multiple systems, with their additional possibilities of mergers, might be the best compromise.

What we learned from GW170817

Most people expected double neutron star coalescences to dominate the overall LIGO event rate, but nature gifted us with a large rate of double black hole events (these can be seen over a much larger volume than the double neutron star mergers; it is probable that double neutron stars are more common *per volume* than double black holes). There was, however, one double neutron star coalescence that has been reported in detail, from 17 August 2017. Another double neutron star event was seen in the ongoing O3 run, but the report for that is likely to be in September 2019.

So what did we learn from GW170817? One thing we learned is that we are very lucky; just as the first direct detection of gravitational waves, GW150914, occurred even before Advanced LIGO was to make its first science data run, the first double neutron star event occurred soon after Virgo began its first science data run. Also, the event was much closer (just ~ 40 Mpc) than expected.

Astrophysically, the event yielded quite a bounty. Just ~ 2 seconds after the gravitational wave event, the *Fermi* satellite caught a gamma-ray burst that was consistent with being off-axis (i.e., we were some tens of degrees away from the jet direction). The subsequent ultraviolet, optical, and infrared development was consistent with ideas about "kilonovae" (emission from radioactive decay in ejecta) that had been suggested before the event. Indeed, the observations support the hypothesis that most of the elements much heavier than iron come from such mergers (although new supernova-related ideas have emerged since). Later radio emission helped us understand the jet. And from the standpoint of neutron stars, the lack of a clear signature in the gravitational waves of tides raised on the neutron stars meant that there is an upper limit on how big they can be (although that limit, $R \lesssim 13 - 13.5$ km, does not exclude the equations of state favored by nuclear physicists).

Other implications are intriguing but not completely robust. For example, it has been argued that if the remnant stayed as a stable "supramassive" neutron star (one held up against collapse by uniform rotation) then it would spin down and inject $\sim 100\times$ as much energy into the system as we saw. If that argument is correct, then it seems necessary that the merger produced a metastable remnant that collapsed after tens to hundreds of milliseconds. This, in turn, suggests an upper limit on the mass of neutron stars that, borrowing from work from several groups (including ours at Maryland!) on gamma-ray bursts prior to GW170817, might be as low as $\sim 2.15 - 2.2\ M_{\odot}$.

What can we expect in the future? Close events such as GW170817 will be rare, so we can't expect a high rate of such golden mergers. If double neutron star mergers with especially low (or high) chirp masses are found, that could be interesting. After enough events are seen, we will get better constraints on the equation of state of the dense matter in neutron star cores.

Mainly, though, as always we await the unexpected! Very low or very high neutron star masses? Mergers between black holes and neutron stars? Something we can't anticipate? The universe has been good to us so far; let's hope it keeps on giving!