Neutron Stars: Exercise Set 1

1. Suppose that you have a star that is supported by nonrelativistic degeneracy (of electrons or neutrons, it doesn't matter). Derive the dependence of the radius R of the star in equilibrium on its mass M (for example, if radius depended on mass to the seventh power, which it doesn't, you would write $R \propto M^7$).

To do this, first make the simplifying assumption that the density is constant, and that the mass is carried by protons and neutrons and the degenerate particles form a fixed fraction of the total particles. Determine the total energy per particle (including both Fermi energy and gravitational potential energy). Then, at a fixed stellar mass M, minimize with respect to the radius R of the star.

2. Do the same thing as in problem 1 but assuming that the star is supported by ultrarelativistic degeneracy (again, of electrons or neutrons). You should find that there is a critical mass $M_{\rm crit}$ above which the star is unstable. What would happen below that critical mass?

3. If a neutron star has a purely quadrupolar magnetic field, make a qualitative argument for the relative sharpness of the transition between the material-dominated and magnetically-dominated regimes in the accretion flow, compared with the case in which the star has a purely dipolar magnetic field.

4. Consider a binary with component stellar masses m_1 and m_2 in a circular orbit with semimajor axis a. The total mass is $M = m_1 + m_2$, and the reduced mass is $\mu = m_1 m_2/M$. Then the orbital angular momentum of the system is

$$L = \mu\sqrt{GMa}\,. \tag{1}$$

Suppose that accretion happens from one star to the other, in such a way that the total mass M and the orbital angular momentum L are constant. Show that if the accretion goes from the lower-mass to the higher-mass star, the system widens (thus a increases), but that if the accretion goes from the higher-mass to the lower-mass star, the system tightens (thus a decreases). With this in mind, go back to the discussion in the notes about HMXBs and LMXBs to understand better the dichotomy between the types of accretion in such systems.