Neutron Stars: Exercise Set 2

1. I claim in the notes that the frequency of a sound wave that involves most of a gravitationally bound object can't exceed $\sim (G\rho)^{1/2}$. But is that correct? Consider a spherical star of mass M and radius R. Recalling that the sound speed is $c_s = (dP/d\rho)^{1/2}$, where P is the pressure and ρ is the density, do a *simple* order of magnitude calculation to determine the sound speed and thus the crossing time of a sound wave across the star. The inverse of that time is of order the frequency. What are some objects that could potentially rotate or pulsate at a frequency much greater than $\sim (G\rho)^{1/2}$, and why?

2. Dr. I. M. N. Sane, noted astrophysical gadfly, has had a startling revelation: laser interferometers cannot possibly detect gravitational waves, and therefore the LIGO and Virgo collaborations have been fabricating data! He has realized that a gravitational wave will stretch or shrink *all* "measuring sticks" in the same way. One consequence of this is that if an arm of a gravitational wave detector is changed by some fractional amount $\epsilon \ll 1$, then the wavelength of the laser light in the cavity is also changed by precisely that same factor. Therefore, the length of the cavity as measured in laser light wavelengths is unchanged by the gravitational wave, leading Dr. Sane to conclude that the interference patterns will be unchanged. Dr. Sane is considering this in the LIGO-like limit that the frequency of the gravitational wave is much less than c/L, where L is the unstretched length of an arm.

You have been approached by the US National Science Foundation to evaluate this argument. The future of gravitational wave detection depends on your response!

3. Dr. Sane doesn't understand all this focus on binary compact object mergers. He thinks that direct collisions of single neutron stars in clusters with each other will make wonderful burst sources. He has requested that you work out the numbers for him. Suppose that you consider a dense globular cluster, such that in the center the number density of neutron stars is 10^6 pc^{-3} and there are 1000 total neutron stars per cluster. Suppose that each neutron star has a radius of 10 km and mass of $1.5\,M_\odot = 3\times 10^{33}$ g, and that the typical random speed in the cluster is 10 km s^{-1}. To within an order of magnitude, calculate how often two neutron stars in a given cluster will hit each other. If there are 10^{10} such clusters in the universe, how often will this happen in the universe? **Note:** be careful when you calculate the cross section for collisions, because gravitational focusing is important. Thus your first step will be to determine the *impact parameter* b that results in a collision: if an object's velocity vector would have a closest approach b if it continued in a straight line, then b would be the impact parameter. Then the cross section is πb^2. If gravitational focusing were unimportant then $b \approx R$, where R is the radius of the neutron star, but you should find that $b \gg R$ in this case.