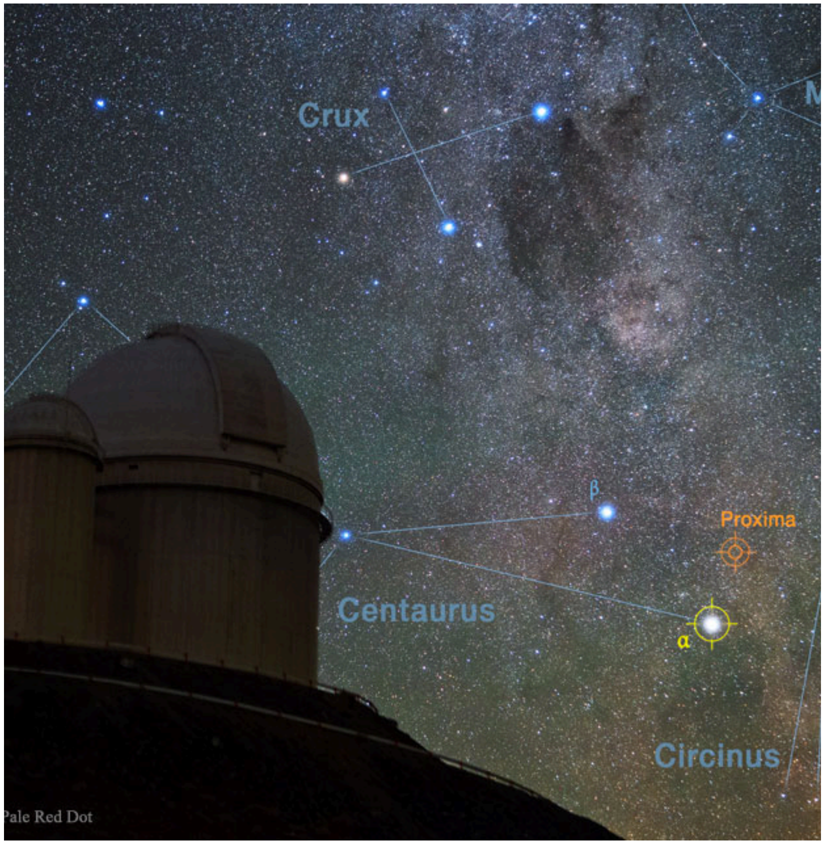
[02] Quantitative Reasoning in Astronomy (8/31/17)

Upcoming Items

1. Read Chapter 2.1 by next lecture. As always, I recommend that you do the self-study quizzes in *MasteringAstronomy*
2. Homework #1 due at start of next lecture.
3. Discussion tomorrow!
   - Friday 1–1:50 pm.
   - ATL 2400.

APOD



Any astro topics to discuss?

Any questions for today's lecture?

- Sorry, I'm still getting used to our campus webpage system...
- The Muddiest Points sections should be available now for your entries. In the meantime, any questions?

LEARNING GOALS

Appendix C

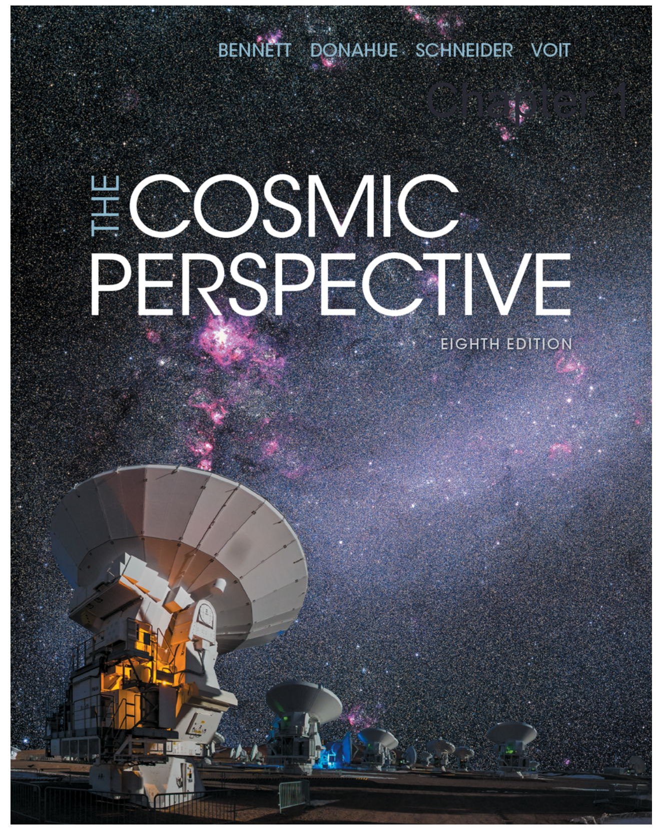
For this class, you should be able to…

… use proportionalities to express dependencies between variables;

… know when to express an answer in scientific notation and to what precision;

… use dimensional analysis and order-of-magnitude estimation as an aid to problem solving.

… and maybe more, depending on how firm your current grasp of these topics is!



I'll be away next week

- Professor Alberto Bolatto will take over the class
  Radio observer, works on galaxies
- I'll be back for the following week and pretty much the rest of the semester
- Sorry to leave you temporarily, but have fun!
  Please still send in homeworks, Muddiest Points, etc.

Quantitative Reasoning in Astronomy

- You should use a consistent problem-solving strategy.
- Whenever possible, take advantage of proportionalities.
- Express very small or big numbers with scientific notation.
- Do not use more digits in an answer than are warranted.
  - Don't plug in numbers until the last step!
- Be systematic when converting between units.
- Use dimensional analysis, limits, and order-of-magnitude estimation to understand and check numerical problems.

General Problem-solving Strategy

- See Mathematical Insight 1.1 in the textbook.

1. Understand the problem.
   - What will the solution look like (big? small? units?).
   - What information is needed to solve the problem (data? formulas?).
   - Consider drawing a diagram or otherwise simplifying the problem.
2. Solve the problem.
   - Carry out the necessary mathematical manipulations.
   - Plug in numbers only at the very end! (Why is this a good idea?)
3. Explain your result.
   - Does the answer make sense?
   - What did you learn by solving the problem?

Scientific Notation

- Express 299,792,458 in scientific notation:
  A. 299.792458×10^5
  B. 3×10^8
  C. 2.99792458×10^3
  D. 3×10^{-5}
  E. 2.99792458×10^8

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  E. 2.99792458×10^{8}
- Does anyone know what this number is?

Scientific Notation

- Express 0.000001234 in scientific notation:
  A. 1.234×10^{-5}
  B. 1.234×10^{-6}
  C. 1.234×10^{-7}
  D. 1.234×10^{-8}
  E. 1.234×10^{-9}

Significant Figures

- Evaluate the following, using the correct number of significant figures: 12 x 394.265 x 173

  A. 818494.14

  B. 8.1849×10^5

  C. 8.184×10^5

  D. 8.185×10^5

  E. 8.2×10^5

Group Question

- The Juno spacecraft holds the record of fastest speed relative to Earth, 2.65×10^5 km/h. How long would it take Juno to travel to Proxima Centauri at this speed? Express your answer in years (and look up the distance to Proxima Centauri!)

- Solution:
  - Minimum travel time t is distance d divided by speed v, i.e., $t = d/v$.
  - Distance to Proxima Centauri $d = 4.25$ ly (Wikipedia) $= 4.02\times10^{16}$ m.
  - Speed of Juno $v = 2.65\times10^5$ km/h (given) $= 7.36\times10^4$ m/s.
  - We have consistent units, so $t = d/v = 5.46\times10^{11}$ s $= \underline{1.73\times10^4 \text{ yr}}$.

For other examples of worked problems, see Files → miscellaneous on ELMS.

Proportionalities

- Consider a sphere of mass M and radius R. Its *density* ρ is its mass divided by its volume:

$$\text{density} = \text{mass} / \text{volume, or } \rho = \frac{M}{\frac{4}{3}\pi R^3}.$$

- What is the density of a sphere of $M = 1\ \mathrm{kg}$, $R = 0.062\ \mathrm{m}$?
    - Plug in the numbers to find $\rho = 1.0{\times}10^3\ \mathrm{kg/m^3}$.
- What is the density of a sphere of $M = 2\ \mathrm{kg}$, $R = 0.062\ \mathrm{m}$?
    - We could use the formula again, but it's simpler (and less error prone!) to notice that if M is doubled (with R held constant), the density doubles. In other words, we're using the *proportionality*

$$\rho \propto M.$$

Proportionalities

- Suppose we changed both M and R. In this case notice

$$\rho \propto \frac{M}{R^3}.$$

- To solve, divide new quantities by old quantities:

$$\frac{\rho_{\text{new}}}{\rho_{\text{old}}} = \left(\frac{M_{\text{new}}}{R_{\text{new}}^3}\right) \Bigg/ \left(\frac{M_{\text{old}}}{R_{\text{old}}^3}\right).$$

- So

$$\rho_{\text{new}} = \left(\frac{M_{\text{new}}}{M_{\text{old}}}\right)\left(\frac{R_{\text{old}}}{R_{\text{new}}}\right)^3 \rho_{\text{old}}.$$

No ¾ or π!

Proportionalities

- What happens to a sphere's density if…
  - …the mass is quartered?
  - …the radius is tripled?
  - …the mass is doubled and the radius is halved?
  - …the volume decreases by half?

IMPORTANT: When asked to compare two quantities, we usually mean to calculate the *ratio*, not the *difference*.

Proportionalities

- The “light-gathering power” of a lens is proportional to the square of the lens’ diameter ($P \propto D^2$).
  1. Estimating the diameter of a typical phone camera lens to be about 1 mm, how many times greater is the light-gathering power of a 1-meter telescope compared to a phone camera?
  2. What about a phone camera compared to Keck 1, which has a diameter of 10 meters?
  3. A 1-meter telescope to Keck?

Scientific Notation

- You should be comfortable doing operations with scientific notation:
  - $10^9 \times 10^{12} = 10^{(9+12)} = 10^{21}$.
  - $10^6 / 10^3 = 10^{(6-3)} = 10^3$.
  - $(8\times10^4) / (2\times10^3) = (8/2) \times10^{(4-3)} = 4\times10^1 = 40$.
  - $1 + 10^{-6} = 1.000001$ Any comment about this one in particular?
  - $(10^3)^2 = 10^{(3\times2)} = 10^6$.
  - $\sqrt{10^6} = (10^6)^{½} = 10^{(6\times½)} = 10^3$.

Precision

- The number of "significant figures" (sig figs) in your answer should be at most one more than the *least* number of sig figs in the data.
    - Mass of Earth $= 6\times10^{24}$ kg.
    - Radius of Earth $= 6{,}371$ km.
    - Density of Earth $= ?$
- Note: 400 (1 sig fig) is different than 400.0 (4 sig figs).
    - Sometimes see "400." for 2 sig figs—better as 4.00×10^2.

Precision

- Do the following calculations, keeping an appropriate number of sig figs.
  1. $(6.3825\times10^{-15})(7.3\times10^{2})$.
  2. (1.835×10^{4}) / (1.522×10^{15}).
  3. $(9.444\times10^{5})(9.32\times10^{11})$.
  4. $(6.21\times10^{3})(2.55\times10^{-6}) + 6.7\times10^{-2}$.
  5. (4.39×10^{-4}) / $(9.45\times10^{-8}) - 3.6\times10^{4}$.

Unit Conversion

- We will usually use metric ("SI") units in this course, but you should be comfortable switching between units. Note: in astronomy beyond the Solar System, "cgs" units (centimeters, grams, seconds) is usually used.
- The metric unit of density is $\mathrm{kg/m^3}$. What is $1{,}000\ \mathrm{kg/m^3}$ in $\mathrm{g/cm^3}$ (g/cc)?

$$\frac{1{,}000\ \mathrm{kg}}{\mathrm{m}^3} \times \frac{1{,}000\ \mathrm{g}}{\mathrm{kg}} \times \left(\frac{\mathrm{m}}{100\ \mathrm{cm}}\right)^3 = 1\ \mathrm{g/cm^3}.$$

Dimensional Analysis and Limits

- Checking units and limits can often tell you if an answer is wrong.

- Consider: distance = speed × time.
  - [meters] = [meters / second] [second]. ✓
  - [L] = ([L]/[T]) [T] = [L]. ✓

- How do we know "distance = speed / time" is wrong?
  - Bad units.
  - Bad limits (longer time ➔ shorter distance!).

Dimensional Analysis and Limits

- Sometimes you can derive physical equations (or at least proportionalities) by considering dimensions alone.

- For example, the angular frequency ω (units $[\mathrm{T}^{-1}]$) of a pendulum depends only its length l (units $[\mathrm{L}]$) and gravity g (units $[\mathrm{L/T}^2]$). Use this to estimate a formula for ω.

- Notice $(g/l)^{1/2}$ has units of $[\mathrm{T}^{-1}]$. We surmise $\omega \propto (g/l)^{1/2}$.
  - In fact, $\omega = (g/l)^{1/2}$, for small oscillations.
  - The oscillation period $P = 2\pi/\omega = 2\pi\,(l/g)^{1/2}$.

Dimensional Analysis and Limits

- You're taking an exam when you get to a problem that requires use of the small-angle approximation. Trying to remember the equation to use, you write down:

$$\theta = dD.$$

- Here D is the object size, d is the distance to the object, and θ is the angular size.
- Check the units: can this possibly be the correct formula?

Dimensional Analysis and Limits

- Realizing that the units of the last equation didn't make sense, you try to remember again. Recalling that d is the distance to the object and D is the object's size, you come up with:

$$\theta = d / D.$$

- You think this might be the right equation, since the units now make sense. But then you try considering limits of the equation; can this equation possibly be correct? (Hint: consider the case where the object is very large, or the case where the object is very far away.)

The Carrot and the Stick

- I think that "sanity checks" of answers for units and limits are extraordinarily important; even more than cutting down on errors, you gain important insight.
- In order to encourage your development of these approaches, I will therefore be my own good cop/bad cop:
- The carrot: if you get a clearly wrong answer in a HW or exam, and (1) give the reason it is clearly wrong, ***and*** (2) say roughly what the answer should be, you will get substantial partial credit.
- The stick: if you get a clearly wrong answer and do **not** say anything, more points will be taken off.
- Note: you have to commit. Saying "I might be wrong" gets you no extra credit, and could take off points if you're right!

Order-of-magnitude Estimation

- As you're watching the news one night, you hear that the Earth will be struck by a space pebble (much smaller than a person) the next day. What is the probability that it will hit a person? That it will hit you?

Statistics and Coincidence

- “Mr. Bond, they have a saying in Chicago: ‘Once is happenstance. Twice is coincidence. The third time it’s enemy action’.” Goldfinger (the book, not the movie)
- Unfortunately we don’t have time to do statistics in this class, but it’s really important in life generally! Bayesian statistics in particular!
- There are a lot of things in the universe; thus there are many opportunities for coincidences to seem significant. For example, should you be stunned and amazed that the angular sizes of the Moon and Sun are so similar?
- Order of magnitude estimates can help you determine whether the blip in your data needs explanation! And be sure to take “trials” into account; there are, for example, many moons of many planets.

Order-of-magnitude Estimation

- Estimate the total mass of leaves that fall from trees on Earth in a single autumn.
  - First, define what variables you will need to know (e.g., number of deciduous trees on Earth).
  - Second, determine how the variables are related (do you multiply them? Add them?).
  - Finally, make a reasonable estimation for each variable and do the calculation.

More examples: http://www.physics.umd.edu/perg/fermi/fermi.htm

Astronomy in the News

- Exoplanet around Proxima Centauri.
- Milky Way-size dark matter galaxy in Coma cluster.
- Making a black hole in the lab.
- Extraterrestrial signal from HD164595?
- Juno at Jupiter.

Scientific Notation (Powers of 10)

- Use scientific notation for values < 0.001 or $> 9{,}999$.
  - $0.000001 = 10^{-6}$.
  - Radius of the Earth $= 6{,}371$ km $= 6.371 \times 10^{6}$ m.
  - Distance to Andromeda galaxy $= 2.5 \times 10^{6}$ ly $= 2.5$ Mly (shorthand).
  - Planck length = 1.6×10^{-35} m.

Scientific Notation

- Write each of the following numbers in scientific notation, where appropriate:
  - 57
  - 0.0000009
  - 600
  - 402000
  - 3.14159

Unit Conversion

- You've come into contact with an alien civilization that is prepared to give you instructions on interstellar travel. However, because their last observation of human beings was in the 1700s, the speed given in the instructions is in units of furlongs per fortnight. In order to make use of their instructions, find the conversion from furlongs per fortnight to meters per second.

ClassAction: Introductory Concepts

- **G3: Scientific Notation**
- G4: Calculation in Scientific Notation
- **G5: Special Units**
- G6: Dimensional Analysis
- **G11: Precision/Accuracy**