ASTR 120 Problem Set 6: Due Tuesday, October 17, 2017

General reminders: You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the textbook you used a particular fact. This will be true in all homeworks.

1. [10 points] In this problem we will use energy considerations to derive the behavior of a narrow annulus of gas, or a narrow ring of particles. Many phenomena in astronomy only happen if they are *energetically favorable*; part of your task in this problem is to determine what that means. Please read carefully through the problem so that you can make optimal use of the hints.

Our setup is that we imagine that we have two particles, both of mass m, that are initially in the same circular orbit, of orbital radius r, around a much larger mass $M \gg m$.

a. Something happens to move one of the particles to a new circular orbit, where the orbital radius is now $r(1+\epsilon)$. Here we assume that $\epsilon \ll 1$. If the other particle moves to a circular orbit so that the *total* angular momentum of the two particles combined is the same as before, what is its new orbital radius? To start on this problem, recall that for $m \ll M$ in a circular orbit of radius r, the angular momentum of the orbit is $L = m\sqrt{GMr}$. You will also use a result that you can derive from calculus: if $\epsilon \ll 1$, then $\sqrt{1+\epsilon} \approx 1 + \epsilon/2 - \epsilon^2/8$ (this is the expression "to second order" in ϵ, meaning that we truncate what would actually be an infinite series after the ϵ^2 term). **Hint:** you can approach this problem by assuming that the other particle's orbital radius in the new configuration is $r(1+\delta)$, with $|\delta| \ll 1$ (δ might be negative), and solve for δ given ϵ, to second order in ϵ.

b. Compare the total orbital energy in the new configuration (where the particles have different orbital radii) with the total orbital energy in the original configuration (where the particles had the same orbital radius, r). Recall that the total energy of a circular orbit of a single particle of mass m around a body of mass $M \gg m$ at an orbital radius r is $E = -GMm/(2r)$. Here, also, we can use a result derivable using calculus: that for $\epsilon \ll 1$, $1/(1+\epsilon) \approx 1 - \epsilon + \epsilon^2$. When you have your expressions for the original energy and the final energy (to second order in ϵ), state whether the new energy is larger or smaller than the old energy; recall that because the energy is negative, "larger" means "less negative" in this case.

c. As we have emphasized in class, we need to use equations as paths to greater understanding of the systems we study. With that in mind, please use your answer to part b to answer the following: left to itself, will a narrow annulus of gas or ring particles (i.e., one whose radial thickness is much less than r) spread, or narrow further, if the gas or particles can interact with each other and radiate energy? Put another way, is spreading "energetically favorable"?

Overall hint: note that you can simplify your algebra considerably by realizing that there are factors that can be taken out. For example, in the angular momentum, there is a factor of $\sqrt{G}$ that will appear in all the expressions. You could thus factor this out and concentrate on the parts that will change from before to after the move of the particles.

2. [10 points] In class we discovered, surprisingly, that the Sun's force on the Moon is more than twice as great as the Earth's force on the Moon. As the Moon hasn't left us in the last month, it must be that this isn't the right comparison. Let's instead compare the *tidal* force exerted by the Sun on the Moon's orbit, with the force exerted by the Earth on the Moon.

a. In anticipation of part c, let's do a general problem. Suppose that a planet of mass M_p is orbited by a moon of mass M_m in a circle of radius $r_{\rm moon}$. The two of them together orbit a star of mass M_s, in a circle of radius $r_{\rm orbit}$. We will assume that $M_m \ll M_p \ll M_s$, and that $r_{\rm moon} \ll r_{\rm orbit}$. Calculate the gravitational force of the planet on the moon; call this $F_{\rm planet}$. Also calculate the *difference* between the gravitational force of the star on the moon when the moon's distance from the star is $r_{\rm orbit} - r_{\rm moon}$ (i.e., the moon is on the close side to the star), and the gravitational force of the star on the moon when the moon's distance from the star is $r_{\rm orbit}$; call this difference in forces $F_{\rm tide}$. Note that for $r_{\rm moon} \ll r_{\rm orbit}$,

$$\frac{1}{(r_{\rm orbit} - r_{\rm moon})^2} \approx \frac{1}{r_{\rm orbit}^2}\left[1 + 2\frac{r_{\rm moon}}{r_{\rm orbit}}\right] . \tag{1}$$

Please use this approximation; it will simplify your expressions!

b. Calculate the *ratio* $F_{\rm tide}/F_{\rm planet}$ for the Earth-Moon-Sun system. For this purpose, assume that the Moon orbits the Earth in a perfect circle, and that the Earth-Moon system orbits the Sun in a perfect circle. For this ratio, you should note that certain quantities cancel; it will make your calculation easier.

c. Using all the approximations above, set $F_{\rm tide} = F_{\rm planet}$ and determine $r_{\rm moon}$ as a function of $r_{\rm orbit}$, M_p, and M_s; this would be the absolute maximum distance that a satellite could orbit. Use http://janus.astro.umd.edu to look up the most distant *regular* satellites (those that orbit the planet in the same direction that the planet orbits the Sun) and *irregular* satellites (those that orbit in the opposite direction) for Jupiter and Saturn. How close do the satellites come to your maximum distance? Do the regular or irregular satellites extend farther out?

3. [5 points] We have emphasized that many of the trends in our Solar System (nearly circular orbits, giant planets far from star, planets in the same plane as the rotation plane of the Sun) don't hold for all exoplanets. For this problem, do a Web search (and as always give the URLs) to find:

a. The exoplanet with the highest eccentricity.

b. The orbital radius of the first exoplanet found around a Sunlike star; compare this to the orbital radius of Mercury, which is the closest planet to the Sun.

c. The name of at least one *retrograde* exoplanet, i.e., one with an orbital direction opposite to the direction that its host star rotates.

Bonus Question [2 points]

Most moons in our Solar System, including Earth's Moon, are outside *synchronous orbit*, which is where the time for them to go around their planet would be equal to the planet's rotation period. You can show that for such moons, the effect of the tidal force of the moon on the planet is to move the moon farther away from the planet. Do a Web search to find an example of a moon in our Solar System that is *inside* synchronous orbit (and indicate the URL you found). This is not a required aspect of this problem, but what do you think will happen to that moon?