ASTR 120 Problem Set 9: Due Tuesday, November 14, 2017

General reminders: You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the book you used a particular fact. This will be true in all homeworks.

1. [5 points] Explain why Saturn is more oblate than Jupiter, even though Saturn rotates more slowly. As part of your answer, you need to write the ratio of two quantities, such that *in general* for a self-gravitating fluid body (the Jovian planets qualify) you expect that if the ratio is larger then the body will be more oblate. Also, give the value of that ratio for Jupiter and for Saturn.

2. [10 points] For this problem we will develop some intuition about orbits, and to do that we need a bit of a mathematical setup.

Suppose that we have a mass m orbiting in a circle around a mass $M \gg m$, so that the center of mass is basically at the center of M. If we go into a *rotating reference frame*, such that in that frame neither M nor m move, then in that frame the total net acceleration at a location $\mathbf{r}$ relative to the center of mass is

$$\mathbf{a}_{\rm net} = -\frac{GM}{|\mathbf{r}|^3}\mathbf{r} - \frac{Gm}{|\mathbf{r}-\mathbf{r}_m|^3}(\mathbf{r}-\mathbf{r}_m) + \Omega^2\mathbf{r}\ . \tag{1}$$

Here the vertical bars mean the magnitude of the vector, $\mathbf{r}_m$ is the location of the smaller mass m, and $\Omega = \sqrt{GM/|\mathbf{r}_m|^3}$. The $\Omega^2\mathbf{r}$ term is a centrifugal acceleration term.

Whew! Now here are your questions:

a. [5 points] Points where $\mathbf{a}_{\rm net} = 0$ are especially interesting for planets, moons, and artificial satellites. Suppose that we consider the line joining M and m (still in the rotating frame). By "line" we mean a line that extends indefinitely in both directions; thus there is an infinite part beyond M, a finite part between M and m, and an infinite part beyond m. Make a convincing *intuitive* argument about the number of different points along that line for which $\mathbf{a}_{\rm net} = 0$.

b. [5 points] One of those points is between M and m, and in fact is very close to m for $m \ll M$. Suppose that the distance between the center of mass and this point is $r_m(1-\epsilon)$, where $0 < \epsilon \ll 1$. Then the acceleration equation, in magnitude, becomes

$$a_{\rm net} = 0 = -\frac{GM}{[r_m(1-\epsilon)]^2} + \frac{Gm}{(\epsilon r_m)^2} + \left(\frac{GM}{r_m^3}\right) r_m(1-\epsilon)\ . \tag{2}$$

Solve for ϵ, using the approximation $1/(1-\epsilon)^2 \approx 1 + 2\epsilon$. Compare your answer with the answer we got when orbital motion was *not* taken into account; using our current notation, we obtained

$\epsilon = [m/(2M)]^{1/3}$. Comment on the robustness of the answer (i.e., how much does the answer depend on specific assumptions, such as a circular orbit?).

3. [10 points] In the class slides we mention some non-gravitational forces that can influence the orbits of boulders and dust grains. In one of those forces, light from the Sun bounces off an asteroid. Because the asteroid is orbiting, the scattered light has a net momentum at a given instant (as a whole; individual photons can bounce in any direction), and because this momentum comes from the asteroid, the asteroid is slowed down and spirals into the Sun.

a. [5 points] The force exerted by this effect on a spherical particle of radius r, moving in a circular orbit around the Sun at speed v, can be written as

$$f = \frac{F_s}{c^2}\pi r^2 v \tag{3}$$

where F_s is the flux of radiation from the Sun. The particle has a momentum in its orbit $p = mv$. The *characteristic time* during which the particle's momentum changes significantly is defined as $T_{\rm char} = p/(dp/dt)$, where dp/dt is the rate of change of the momentum. Assuming that the particles have uniform density ρ, derive an expression for $T_{\rm char}$ as a function of ρ, r, and F_s.

b. [5 points] Calculate the radius r of a particle that has $T_{\rm char} = 1$ billion years, assuming a density of $\rho = 2000$ kg m^{-3} and that the particle's circular orbit is 3 AU from the Sun. Particles much larger than this have little effect from this radiation drag, whereas particles much smaller than this are swept rapidly toward the Sun.

Bonus Question [2 points] The first clear depiction of a well-known comet is in a well-known tapestry from centuries ago. Do a Web search to determine the comet, the tapestry, and the tapestry's historical context.