ASTR 120 Problem Set 10: Due Tuesday, November 21, 2017

General reminders: You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the book you used a particular fact. This will be true in all homeworks.

In the first three problems you will use concepts you have learned throughout ASTR 120 to make calculations related to the asteroid impact hazard.

There has been a terrible discovery made: an asteroid with a 1 km diameter will hit Earth in 100 years! Because Bruce Willis won't be alive then, a desperate government has turned to you for help:

1. [5 points] Observations of the asteroid indicate that it is spherical and has a density of 2000 kg m^{-3}. To ensure that the asteroid misses Earth, it is necessary to push it sideways by 10,000 km. If the pushing force is constant over 100 years, calculate the required force. **Hint:** the orbital motion doesn't matter. Just calculate the force needed to move the asteroid 10,000 km in 100 years, assuming it starts at rest and has a constant force exerted on it.

2. [5 points] The average distance of the asteroid from the Sun is 1 AU. With that in mind, one group has suggested that the asteroid could be deflected by a system of mirrors. In particular, they can set up mirrors such that all of the light from the Sun that reaches the asteroid can be reflected in any desired direction and thus the full momentum of that light can be used to push the asteroid. Calculate this force; is it sufficient given your answer to problem 1?

3. [5 points] A different group proposes a "gravity tractor" as a way to move the asteroid. In this approach, they will park a spacecraft with mass m 600 meters from the center of the asteroid (and thus 100 meters from the asteroid's surface; some separation is needed for exhaust from the spacecraft to miss the asteroid) and the gravity of the spacecraft will pull on the asteroid. Calculate the mass m needed to exert the required force that you computed in problem 1.

4. [10 points] Before nuclear fusion was understood, when people tried to figure out the source of energy for the Sun the most efficient method they considered was *gravitational contraction.* The characteristic time scale for such energy is the *current* magnitude of the gravitational potential energy divided by the *current* luminosity of the Sun. We will do this in two steps, assuming in both cases that the Sun has a constant density and is spherically symmetric:

a. [5 points] We can calculate the magnitude of the gravitational potential energy by building it up in shells. That is, the magnitude of the gravitational potential energy of a spherical shell of mass

m a distance r from the center of a spherical mass M inside the shell is GMm/r. Because of the spherical symmetry, the mass *outside* the shell does not contribute to the gravitational potential energy of the shell. The mass of a shell of thickness dr and radius r is $dm = \rho dV = \rho 4\pi r^2 dr$, so we can integrate the total magnitude of the gravitational potential energy as follows:

$$E_{\rm grav} = \int_0^R \frac{GM(<r)\rho 4\pi r^2 dr}{r} \,, \quad (1)$$

where $M(< r)$ is the mass of the star *inside* radius r and R is the radius of the star. Perform this integral and express $E_{\rm grav}$ as a constant factor (which you must determine and specify) times GM^2/R, where M is the total mass of the star. Again, we assume that the density ρ is constant, which means that it is independent of r.

b. [5 points] Use your answer to 4a to compute the characteristic time scale $E_{{\rm grav},\odot}/L_\odot$ for the Sun, where L means the luminosity and $\odot$ indicates the Sun (the time scale should be in years). Finally, given that the Sun is actually centrally concentrated (the density at its center is greater than at the outside), make a case for whether the true time scale would be *more* or *less* than you got by assuming constant density.

Bonus Question [2 points]

We know now that the Sun is mainly made out of hydrogen and helium, but its spectrum is dominated by heavier elements (oxygen, iron, etc.). Summarize succinctly and accurately the argument made by Cecilia Payne (later Cecilia Payne-Gaposchkin) that the relative weakness of hydrogen and helium lines is consistent with those elements making up the majority of the atoms. **Note:** you'll need to go deeper than the Wikipedia article on Payne-Gaposchkin, which does not have a full enough description for credit on this bonus question.