

## ASTR 120 Problem Set 12: Due Thursday, December 7, 2017

**General reminders:** You must show all your work to get full credit. Also, if any website was useful, you need to give the URL in your answer. Note that any website is fair game; you just have to cite it. If any book including our textbook was useful, you need to indicate where in the book you used a particular fact. This will be true in all homeworks.

1. [15 points] In this problem we will explore quantitatively one of the proposed scenarios that might produce “hot Jupiters”. In the hypothetical system we will consider, a copy of Jupiter is initially in a circular orbit with a radius of 5.2 AU around a copy of the Sun. Something happens (maybe a gravitational encounter) and the planet is thrown into an orbit with an apocenter distance of 5.2 AU but a pericenter distance of 0.01 AU (note that this is measured from the *center* of the star). From that point on, tidal forces and energy dissipation circularize the orbit. Note that in this class we have encountered all the formulae needed; for this problem it is your responsibility to use the correct formulae and to look up specific numbers as you need them.
  - a. [5 points] Compute the ratio of the tidal force from the star to the self-gravitational force from the planet, at the pericenter distance of 0.01 AU. Assume that the “Jupiter” is spherical. Can the planet survive?
  - b. [5 points] As we have discussed, circularization will occur while very nearly keeping the angular momentum of the orbit constant. With this in mind, compute the final radius of the (initially highly eccentric) orbit after it has circularized.
  - c. [5 points] Compute the orbital energy released by the circularization from the initially highly eccentric orbit. Compare this energy with the self-gravitational energy of the planet (assuming that the planet is spherical and has constant density throughout its interior). Discuss whether the planet will survive and if so, how (**Hint:** think about the total energy emitted by the Sun in its lifetime, compared with its gravitational self-energy).
2. [5 points] Dr. I. M. N. Sane has demonstrated that not just living organisms, but even simple chemicals, could never have arisen naturally. His example is table sugar, which has the chemical formula  $C_{12}H_{22}O_{11}$ . To form this would require the simultaneous collision of all 45 atoms, a process so ridiculously improbable that it would never happen in the history of the universe. A reporter from Time magazine, sensing a major story, has contacted you for a comment. Give a few-sentence response about Dr. Sane’s idea.
3. [5 points] A single *E. coli* bacterium has a mass of (rounded) about  $10^{-15}$  kilograms, and can divide itself in about 20 minutes. Therefore, if you started with one, after 20 minutes there would be two, after 40 minutes there would be four, after 60 minutes there would be eight, and so on.

(a) [2 points] Starting with one *E. coli* bacterium, and assuming an unlimited supply of nutrients, calculate how long it would take for the total mass of the bacteria to reach (i) the mass of a human (about 50 kilograms), (ii) the total mass of all humans on Earth (about 300 billion kilograms), (iii) the total mass of living things on Earth (estimated at  $10^{15}$  kilograms), and (iv) the total mass of the Earth itself (about  $6 \times 10^{24}$  kilograms).

(b) [3 points] Write a few sentences indicating what this thought experiment has to do with evolution and natural selection.

Bonus Question [2 points]

Do Web research to find an animal which has a larger ratio of brain weight to body weight than humans do. Write at most 3-4 sentences indicating whether you therefore consider this animal more intelligent than humans, discussing why or why not.