Intuition and Order of Magnitude Estimates

During this semester we're going to encounter all sorts of aspects of theoretical astrophysics. There will be a lot of topics covered, meaning that in many cases we'll only have a chance to skim the subject. However, in addition to looking at specific topics, I want to convey to you something about the process of problem solving in astrophysics.

Astrophysics is challenging and exciting in part because it involves so many different aspects of physics. It also is in a position unusual for sciences: in astronomy, we *observe* rather than *experiment*. That means that we can't just carefully vary one thing at a time to see how a system responds. We have to take what nature gives us, and we often have to make inferences without full information (e.g., we'll never live long enough to see a single star evolve!). That means that our tools are a little different than you may be used to.

For all these reasons, we're going to spend the first two classes talking about problem solving strategies in astrophysics. In a rough sense, this can often be broken up into:

1. Make a qualitative guess, based on any intuition you may or may not have. Even if you're wrong, you'll learn more this way than if you just blindly did the problem.
2. Make an order of magnitude estimate. These are quick and approximate calculations that give you an idea of the scale of the problem.
3. Do a specific calculation related to your question. The exact calculation may depend on how your order of magnitude problem turned out.
4. Check your calculation. Does it have the right units? Does it have the correct limits in easy simplified cases? Does it have the right symmetries?

For this class, we'll discuss qualitative guessing and order of magnitude estimates, then in the next class we'll move on to units, limits, and symmetries.

First, qualitative guessing. By this, generically, I mean "If I increase quantity X, will quantity Y increase or decrease?" and similar questions. This is always where you should start out. If you do a detailed calculation and find that the answer disagrees with your qualitative guess, you need to think hard about why that is. Was your intuition in error? Maybe, but many times you'll find a mistake in your calculation instead. One way or the other, this is the way to build intuition. Let's try a few examples.

1. You orient two refrigerator magnets so that they attract each other. As you pull them farther apart, does the attraction grow stronger or weaker?
2. You swing a ball on a string around your head. Keeping a fixed amount of tension in the string, does the angular velocity go up or down when the string is lengthened?

3. Which has a higher *linear* (not angular) orbital velocity: Mercury or Neptune?
4. Globular clusters are collections of hundreds of thousands of old stars that orbit galaxies. As they orbit a galaxy, they are sometimes pulled apart by the galaxy's tidal field. Which will survive longer (assuming comparable orbits): a globular with mass $10^5\,M_\odot$ and radius 10 pc, or one with mass $10^5\,M_\odot$ and radius 1 pc?
5. A star forms from a molecular cloud when gravity in the cloud overwhelms other forces. With all else being equal, is it easier to form a star from a hot cloud or a cold cloud?
6. Suppose electrons had ten times the mass that they actually do, but the same electric charge. If the laws of physics were otherwise the same, would the binding energy of hydrogen go up or down? This is not one you're expected to know, but it's in here to provide food for thought.

It's excellent practice to approach *every* problem like this. In some cases you may have no intuition, but make a guess anyway and see what happens!

Now let's move on to order of magnitude problems. The point of an order of magnitude problem is to get a quick qualitative guess as to an answer. In a real situation, the way it might work is that for a given astronomical phenomenon, you think of a possible explanation. Before diving into the details, you should find out whether the idea has any chance of working, by doing a quick check. I'll do a couple of examples, then it will be your turn.

First, a simple one. How many full-sized pages of paper could you carry, if they were bound up in boxes?

You may have no idea how much a page of paper weighs. But you know how much a book weighs; a nice 500 page novel might be a kilogram, most of which is the paper. If you adopt 1 kg=500 pages, then it's a matter of how much mass you can carry. For most adults it would be between 40 kg and 200 kg, or between roughly 20,000 and 100,000 pages.

Now a second straightforward example. How many liters of fluid have you drunk in your lifetime?

The amount you drink in a given day depends on the temperature, how much you're moving around, and so on. About two liters per day is probably typical. Let's say you are now 20 years old, which we'll round to 20x300=6,000 days old. Multiplying, you get about 10,000 liters, which is probably accurate to within a factor of three either way.

These two examples illustrate some of the keys to order of magnitude estimates. One point is that sometimes you won't know the exact thing you need (e.g., how much a sheet of paper weighs). You then need to draw on your knowledge to allow an estimate. You also should round your numbers to make it easier! Now we'll go through a more complicated astronomy-related example to see how these techniques can be applied.

Suppose someone suggests that the Sun produces energy by ordinary burning. Is there a quick way to see if this would suffice?

When presented with a problem like this, your first inclination may be to say "but I don't know how much energy is released in burning!". Fair enough. However, the great thing about an order of magnitude calculation is that you can make a reasonable guess, so that if the answer is way off what it needs to be you're safe, and if it's close then you know you need to do more work. In this case, you know that burning is a chemical process. You also know that digestion is a chemical process, so maybe that's similar. Great, so how much energy does digestion release? A typical person might have a diet of around 2000 kilocalories per day, to within a factor of 2. There are about 4 Joules in a calorie, so 2000 kilocalories is about 10^7 J, to an order of magnitude. We also need to know how much mass in food gives that energy. Let's guess that we eat 1 kg of food per day (in addition to some water). Then the efficiency is 10^7 J/kg. In cgs units, which are used in astronomy, 1 J=10^7 erg and 1 kg=10^3 g, so we have 10^{11} erg g^{-1}.

But what do we need? We can look up numbers in an astronomy book: the Sun has a mass of about 2×10^{33} g, and has been shining at a luminosity of about 4×10^{33} erg s^{-1} for about 5×10^9 yr$\approx 2 \times 10^{17}$ s. Therefore, it has generated $\approx 8 \times 10^{50} \approx 10^{51}$ erg. Dividing, this requires $10^{51}/2 \times 10^{33} = 5 \times 10^{17}$ erg g^{-1}. That's five million times what burning will give us. No dice.

Energy calculations like this (i.e., a phenomenon requires a certain amount of energy, can your process provide it?) are a great way to take a quick look at a model.

Now it's your turn. Here are a few order of magnitude problems.

1. Are there more stars in our galaxy than grains of sand in all the beaches in the world?
2. Which has a larger solid angle as seen by us: the Sun, or the rest of the stars in the universe combined?
3. Can Earth fling objects out of the Solar System? Can Jupiter?