Accretion and tidal forces

We've now gone far enough that we can start to consider specific astrophysical applications. In this class, therefore, we will apply our knowledge to two gravitationally related topics: accretion and tides.

We'll start with accretion. Although there are many aspects of accretion that depend critically on nongravitational processes (for example, emission of radiation!), we'll concentrate on gravitation and conservation laws. Continuing our discussion of Lagrange points, let's suppose that we have a star system consisting of two main sequence stars. Say that the stars are separated by enough that both of them are well within their Roche lobes; that is, their radii are much smaller than the distance to the L_1 point. **Ask class:** will there be any flow of matter from one star to the other? No, because all the mass in the system is gravitationally dominated by one star or another. Therefore, just like a particle that is in orbit around one star, ten times closer to it than the particle is to the other star, any gas in either star will simply stay with its parent. **Ask class:** in such a system, is there any way that there can ever be mass transfer? Yes! When a star evolves off the main sequence, it swells and becomes a red giant or supergiant. This can produce a radius of 1–10 AU, typically, so if the stars were within a few astronomical units of each other then they will transfer mass during their lifetimes.

Now suppose that one of the stars (Star 1) has expanded so that it is just slightly larger than its Roche lobe. Think of a molecule of gas that is closer to the other star (Star 2) than the L_1 point. To be clear, suppose that the other star is still *way* inside its Roche lobe. **Ask class:** what happens to it? The molecule gets inside the region of influence of the other star, but it does *not* have to fall onto that star immediately! The easiest way to understand this is to think of angular momentum. Relative to Star 2, the molecule has significant angular momentum per unit mass when it flows over past the L_1 point. In order to fall onto the star, the molecule must get rid of a lot of angular momentum. By itself, it can't do that. There might be some gravitational interactions with both stars, but these are likely to be chaotic and the molecule may well be ejected from the system. You can try this yourself with Doug Hamilton's orbit integration codes, linked to this class's web page.

Accretion, therefore, is linked up with the question of how angular momentum can be moved around. If the molecule can lose angular momentum relative to Star 2, it can eventually accrete onto that star. Nongravitational forces are one way to do this. Heuristically, imagine now that it isn't just one molecule, but a stream of gas that flows over the L_1 point from Star 1 to Star 2. The stream of gas will move around Star 2 and come back to nearly its starting point near L_1, but at that point the stream will intersect itself. This intersection causes shock heating, viscous interactions, and other stuff that allows transfer of angular momentum. Now let's think about the geometry of the situation.

Ask class: when considering all the gas together, will there be a net sense of the angular momentum? To answer a question like this, it is often helpful to determine whether there is a "special" direction in the problem, or if all directions are equivalent. In this case there is a special direction: the angular momentum axis of the binary itself. Therefore, yes, there will be a net sense of the angular momentum of the gas that flows onto Star 2. As a result, the gas will not flow in spherically, but with an axis; as it spreads out, the gas will form a flattened shape called an *accretion disk*.

Now let's consider some aspects of this problem from the point of view of conservation laws. We'll simplify by ignoring Star 1 and thinking about two particles, each of mass m, that are orbiting in a circle of radius r around a star of mass M. **Ask class:** what is the total angular momentum of the orbits? It's $2m\sqrt{GMr}$. Now suppose that somehow (e.g., through interactions between the two particles), one of them is moved to a radius $r_1 = r/2$. We can solve for the orbital radius of the other by conservation of angular momentum:

$$\begin{aligned} m\sqrt{GMr_1} + m\sqrt{GMr_2} &= 2m\sqrt{GMr} \\ \sqrt{r/2} + \sqrt{r_2} &= 2\sqrt{r} \\ r_2 &= (2 - 1/\sqrt{2})^2 r \approx 1.67r \ . \end{aligned} \tag{1}$$

Fine, so that's set. Let's check conservation of energy. In a circular orbit, the total energy is half the gravitational potential energy. **Ask class:** what is the total energy initially? It's $E = -2GMm/(2r) = -GMm/r$. **Ask class:** how about after the particles have moved? Then it's $E = -GMm/(2r/2) - GMm/(2 \times 1.67r) \approx -1.3GMm/r$. **Ask class:** how can this have happened? This means that substantial energy was released in this movement, likely in the form of radiation. Therefore, spreading of a disk of gas can release energy, but the angular momentum stays within the system unless there is escape of matter. This is one reason why conservation of angular and linear momentum is often much easier to track than conservation of energy.

This, however, can give us a handle on how much total energy is released during accretion. Suppose that the star has mass M and radius R. If, in steady state, mass flows from very far away from the star to come to rest on its surface, and the rate of mass accretion is $\dot{M}$, then **Ask class:** what is the luminosity produced? It's $GM\dot{M}/R$. For compact objects, this can be a lot; coming to rest on a neutron star's surface releases 30–50 times more energy per mass than is released in hydrogen fusion!

Let's back up now and think about the angular momentum of the system as a whole. Consider a system in which the lower-mass star is transferring mass to the higher-mass star. **Ask class:** can they think of an example in which this might occur? A black hole in a binary with a solar-mass star is one possibility. To clarify the picture, suppose that the higher-mass star has mass M, and the lower-mass star has mass $m \ll M$. **Ask class:** what is the orbital angular momentum, roughly, for an orbital separation r? It's about $m\sqrt{GMr}$. Now suppose that mass transfer has occurred so that the smaller star has diminished to $m/2$. **Ask class:** what is the new orbital radius, assuming angular momentum conservation? It must be $4r$, to keep L constant. But that's a problem! The Roche lobe radius scales as $r(m/M)^{1/3}$, so the Roche lobe radius goes up by more than a factor of 3. If the small star is on the main sequence, its radius *decreases* with decreasing mass. Therefore, if it was just barely transferring mass before, after some mass has been donated it doesn't anymore.

Ask class: does this mean that once a light star starts to donate to a heavy star, it will shut itself off permanently? No, it can keep going. One way is if the light star evolves to a larger radius. Another is if there is a way to lose angular momentum. For example, many stars have winds, meaning that some matter can go to large radii (and, indeed, leave the system entirely). If the matter in the wind is connected magnetically to the star, then the wind will carry away angular momentum, slowing down the spin of the star. Spin-orbit coupling through tides (which we'll discuss later in this lecture) then takes away angular momentum from the orbit, bringing the stars closer together and continuing the mass transfer. For more compact binaries, gravitational radiation can drain the system of angular momentum.

Now consider the opposite case, of a massive star donating matter to a lighter star. **Ask class:** what might be an example of this? A high-mass star (say, $10\,M_\odot$) can be in a binary with, say, a $1.4\,M_\odot$ neutron star. It's paradoxical, because a high-mass star evolves faster than a low-mass star, but a high-mass star can lose enough mass (through winds, accretion, or even a supernova) that it ends up having less mass than its companion. **Ask class:** with angular momentum considerations, what happens to the orbit as the high-mass star loses mass to the lower-mass star? The orbit *tightens*. This, therefore, is an unstable situation: if the higher-mass star overfills its Roche lobe, then if mass transfer is conservative (i.e., all the lost mass ends up on the companion star), the stars get closer together. This may cause them to spiral in together, leading to so-called common envelope systems.

The example of accretion and accretion disks shows that one can often learn quite a lot about systems by tracking where the angular momentum goes. Even in cases where this doesn't provide an answer, it can at least clarify important questions. For example, consider the formation of a star from a molecular cloud. The cloud might have a density of 10^4 particles per cubic centimeter, so a $1\,M_\odot$ patch of the cloud would have a radius of roughly 1 pc. It will have some rotation, from motion through the galaxy if nothing else. **Ask class:** to order of magnitude, what is the angular momentum of a spherical blob of mass M and radius R, spinning at an angular frequency Ω? It's about

$$L \approx M\Omega R^2 \ , \tag{2}$$

which one can get from units or other considerations. If a cloud shrinks and keeps its angular momentum and mass, then this means its spin frequency must scale as $\Omega \sim R^{-2}$. However, note that the orbital frequency scales only as $\Omega_K \sim R^{-3/2}$; so as the cloud contracts the ratio of angular frequency to Keplerian orbital frequency increases. If something is spinning faster than its orbital frequency, it will break apart. Consider the Sun. If the initial frequency of spin of the molecular cloud is no less than $1/2 \times 10^8$ yr$^{-1} \approx 10^{-16}$ s^{-1} (this is the rotation period of the Galaxy), then by the time it has contracted from 1 pc to 7×10^{10} cm (the radius of the Sun) its spin frequency would have to be about 0.4 s^{-1}, so the Sun would have a spin period of less than three seconds! Obviously absurd. The conclusion is that contracting stars must lose almost all their angular momentum during the protostellar phase. There has been a lot of work focused on finding out how this happens, but I'll leave it to you to look up the ideas!

We will now switch our focus to another phenomenon that may be considered as part of a "2+1" body problem: tides. The external gravitational field of a spherical star or planet is exactly the same as that of a point at the object's center. However, this does *not* mean that the object *itself* responds to gravitational fields in the same way that a point mass does! **Ask class:** what is the difference? For an extended object in a gravitational field, different parts of the object are at different distances from the source of the field, so they experience different accelerations. Indeed, from the general relativistic standpoint, this is the only way gravity can exert a force that can be felt; if a point mass falls freely in a gravitational field it feels no "push" or "pull". However, if an extended object (like a person!) is in a gravitational field, then even when freely falling there are different accelerations at different points. This would be felt as a push or pull, which could grow to fatal proportions if there is a plunge into a black hole!

Now consider the Earth-Moon system. Both objects exert tides on each other. This raises tidal bulges. In the case of the Moon, it is in synchronous rotation, meaning that as seen from a distant location it spins at the same angular velocity that it orbits. It got that way because if it were to spin faster than the orbit, the tidal bulge would lag the spin and therefore slow it down, and vice versa. The Moon is small enough (and the Earth's gravity large enough) that there has been enough time for synchronization. On the other hand, the Earth has not come into synchronous rotation. The Pluto-Charon system is mutually synchronized.

Let's investigate this a bit more by following angular momentum. **Ask class:** if angular momentum is conserved in the Earth-Moon system, what has to happen as the Earth's spin is slowed down? The angular momentum must be placed somewhere else, in this case the orbit. That means that the orbit expands (since the angular momentum goes like $\sqrt{GMr}$, larger L means larger r). Suppose that the Sun were eternal (i.e., no nasty evolution to worry about). **Ask class:** by conserving angular momentum, what would be the long-term evolution of the Earth-Moon system? As the Earth slows down, the orbit expands. This will continue to take place until the Earth is in synchronous rotation with the orbit. That can happen, because the Earth has a smaller moment of inertia than the Earth-Moon system, meaning that its spin angular velocity will drop faster than the orbital angular velocity. Now consider the effect of the Sun. The Earth-Moon system orbits the Sun, and the orbital rate of one per year is much less than the time for the Moon to go around the Earth. Therefore, angular momentum is taken out of the Earth-Moon orbit.

What happens now? **Ask class:** how does the Earth-Moon separation change as a result? It shrinks. That means that the orbital angular velocity is greater than the spin angular velocity of the Earth or Moon. **Ask class:** now how is angular momentum transferred? Orbital angular momentum is then given to the spins. The decrease in orbital angular momentum means that the Earth and Moon get closer together, so they orbit even faster, and again give angular momentum to the Sun/Earth-Moon orbit. Net result: the Earth and Moon move slightly farther from the Sun, but they themselves get ever closer until they collide! This will, however, take much longer than the evolution time of the Sun.

As a final cool example of tides, consider Mars and its moon Phobos. Phobos is actually orbiting *inside* the synchronous radius, i.e., its orbit is faster than the spin period of Mars. The spin angular momentum of Mars is much greater than the orbital angular momentum of Phobos, due to an enormous mass ratio. **Ask class:** how will this system evolve? Phobos will raise a tidal bulge on Mars that will speed up Mars' rotation slightly. This will therefore take angular momentum away from Phobos. This causes the orbital radius to drop, so Phobos speeds up further, and the process is a runaway. In fact, the best guess is that Phobos will crash into Mars in about 30 million years. It's a bit of a mystery why it should be in this orbit, but maybe it's just happenstance.