## Failure of Hydrostatic Equilibrium

Most things in the universe that are supported by pressure are in approximate hydrostatic equilibrium. Why? **Ask class:** what would happen if an object were dramatically out of hydrostatic equilibrium? It would then evolve dynamically. **Ask class:** on what typical timescale would it evolve? If gravity is unopposed, then the object will collapse or explode, on roughly a free-fall time scale. To order of magnitude, this is the same as the orbital timescale, or $\sqrt{R^3/(GM)}$, where $M$ is the mass and $R$ the typical radius of the object (it doesn't have to be spherically symmetric). The average density is $\bar{\rho} \sim M/R^3$, ignoring inconvenient factors of $4\pi/3$ or whatever, so the dynamical time is $t_{\rm dyn} \sim 1/\sqrt{G\bar{\rho}}$. For $\rho = 1$ g cm$^{-3}$, the density of water (or the Sun, roughly), the dynamical time is about an hour.

**Ask class:** so, why is it that such a tiny minority of astronomical objects is seen to be dramatically out of hydrostatic equilibrium? It's because such an object evolves so fast that we have little chance to see it in that state. **Ask class:** what is an example of an object that is far out of hydrostatic equilibrium? A supernova! However, a star that goes supernova in another galaxy (i.e., almost all of those seen) might be visible to us for a few years, whereas the typical parent star lives for tens of millions of years, so the supernova (and its evolving remnant) is only visible for $\sim 10^{-7}$ of the lifetime. The core collapse itself lasts only a few seconds. **Ask class:** why are so many supernovae seen, if they are so rare? They are bright, so you can see them at incredible distances. This kind of tradeoff happens all the time in extragalactic astronomy. Rare things are less common (duh!), but bright things (that tend to be rare) can be seen much farther away than dim things. There is therefore a huge bias towards seeing bright things, unless you're really careful. Example: **Ask class:** if you went to a dark spot on the Earth and looked out, what would you guess to be the average spectral class of the stars you can see with your naked eye? It's about spectral class A, much brighter than the Sun. The overwhelming majority of stars are intrinsically dimmer than the Sun, but bright ones can be seen much farther away. In cosmology, the bias towards bright objects is called Malmquist bias.

Now back to our regularly scheduled program. Since objects far out of hydrostatic equilibrium don't last long in that state, one more typically finds objects that are slightly out of hydrostatic equilibrium, and hence evolve over long times. As we said in the last lecture, this typically means that the long-term evolution of many things reduces to a competition between gravity and everything else; since gravity is always attractive, other things must intervene to repel bits of matter and prevent everything from collapsing into a black hole! **Ask class:** what are examples of effects that can oppose gravity? Orbital motion, or centrifugal effects, are one example. Pressure or temperature or velocity shear are other examples. Magnetic fields also exert an opposing force, since field lines repel each other.

In order to consider all this in a specific context, let's think about the basics of star formation. The average density of the galaxy is around $10^{-24}$ g cm$^{-3}$, but the average density of a star is $\sim 1$ g cm$^{-3}$, so obviously a rather substantial density increase has happened! On large scales, gravity dominates, so we have to think about gravitational collapse. We need to find a condition for when something collapses and when it doesn't.

We can, as theorists, imagine that we have set up a nonrotating, nonmagnetic, nonturbulent cloud and ask about when gravity will beat thermal pressure. This leads to a minimum mass called the *Jeans mass*. **Ask class:** for a uniform-density spherical cloud of mass $M$ and radius $R$, what is the gravitational energy? $E_g = \frac{3}{5}\frac{GM^2}{R}$. **Ask class:** what is the thermal energy if there are $N$ particles at temperature $T$? $E_t = \frac{3}{2}NkT$. The condition for gravity to win is $E_g > E_t$, so

$$\frac{3}{5}\frac{GM^2}{R} > \frac{3}{2}\frac{M}{m}kT \; , \tag{1}$$

where $m$ is the average mass of a molecule, so $N = M/m$. Therefore, need

$$M > \frac{5}{2}\frac{kT}{Gm}R = \frac{5}{2}\frac{kT}{Gm}\left(\frac{M}{\frac{4}{3}\pi\rho}\right)^{1/3} \Rightarrow M > \left(\frac{5}{2}\frac{kT}{Gm}\right)^{3/2}\left(\frac{4\pi}{3}\rho\right)^{-1/2} \equiv M_J \; . \tag{2}$$

This is $M_J \approx 2M_\odot T^{1.5} n^{-0.5}$, where $n$ is the number density. Therefore, if $M > M_J$ then the cloud will start to collapse, whereas if $M < M_J$ the cloud will not collapse. Note that this is the absolute minimum mass for a bound cloud. For $\rho = 10^{-23}$ g cm$^{-3}$ and $T = 100$ K, typical of the ISM, neutral hydrogen has $M_J \approx 10^4\, M_\odot$.

Suppose first that there is no support against collapse. In that case it would collapse on a free-fall timescale

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 3.4\times 10^7 n^{-1/2}\,\mathrm{yr} \; . \tag{3}$$

Typical densities in sterile (non-star-forming) regions are $\sim 50 - 100$ cm$^{-3}$, implying $t_{ff} \approx 5\times 10^6$ yr. This is $\sim 0.1\times$ the inferred lifetime of clouds, so something must hold up the collapse.

As the gas cloud collapses, its thermal energy goes up and can in principle halt (or at least slow) the collapse. We have

$$M_J \sim T^{3/2}\rho^{-1/2} \; . \tag{4}$$

If the equation of state is polytropic, so that $P \propto \rho^\gamma$ and $T \propto \rho^{\gamma-1}$ then

$$M_J \sim \rho^{(3\gamma-4)/2} \; . \tag{5}$$

**Ask class:** what does this mean for whether collapse will stop or run away? If the Jeans mass decreases as the cloud collapses, there will be a runaway; if it increases, the collapse will stop. Therefore, for $\gamma < 4/3$ there is a runaway.

**Halting collapse by rotation**

Here, it's a comparison of the rotational energy with the gravitational energy:

$$\beta = \frac{E_{\rm rot}}{|E_{\rm grav}|} \ll 1 \tag{6}$$

is the usual initial condition. Now, $E_{\rm rot} \sim (R\Omega)^2$ and $E_{\rm grav} \sim 1/R$, so $\beta \propto R^3\Omega^2$. If angular momentum is conserved during the collapse, then $L$=const=$R^2\Omega$, so $\Omega \sim 1/R^2$ and $\beta \propto 1/R$. Since the collapse is over many orders of magnitude (say, $10^{18}$ cm for a solar mass cloud to $10^{11}$ cm for a star), this means that rotation can halt the collapse even if it is unimportant initially.

Let's work this out. The Galaxy rotates with a period of about 200 million years, so let's say that the initial molecular cloud shares at least that rotation. If a 1 $M_\odot$ portion of the gas has a radius of $10^{18}$ cm initially, then to collapse to something the size of the Sun ($10^{11}$ cm) and conserve its angular momentum it needs to spin $10^{14}$ times faster, or in about 1 minute(!) compared with $\sim$3 hr for breakup and $\sim$30 days for the actual rotation period of the Sun. This is a serious problem! As a sidelight, let's think for a second about what other angular momentum exists in the solar system. **Ask class:** do they know what fraction of the mass of the solar system is in the Sun? About 99.8%. What fraction of the angular momentum? The Sun rotates at about 1/300 of the Keplerian orbital frequency at its radius. Jupiter orbits at Keplerian, of course. In addition, Jupiter is at a radius of 5 AU=$7 \times 10^{13}$ cm, or 1000 times the radius of the Sun, so it has a specific angular momentum about $1000^{1/2}$ times greater than a particle orbiting at the limb of the Sun. Therefore, the specific angular momentum of Jupiter is about $1000^{1/2} \times 300 = 10^4$ times that of the Sun. The Sun's mass is $10^3$ times Jupiter's so Jupiter's angular momentum is 10 times that of the Sun. In reality, the Sun's mass is centrally concentrated, and $J_J/J_\odot \approx 100$. So, that helps, but not enough.

Magnetic fields can also help halt collapse, but we will not consider them in this lecture (it's a bit far afield).

What we have found is that, although it is easy to find gas that will start to collapse (you just need $M > M_J$), there are three things that can prevent the gas from collapsing all the way and forming stars: (1) if the polytropic index exceeds 4/3, the cloud can heat up fast enough to stop collapse with thermal pressure, (2) rotation and a centrifugal barrier will generally set in if the cloud conserves its angular momentum, (3) in some cases the magnetic field may halt collapse. We will now consider ways out of these problems.

**Heating and Cooling**

First, the thermal problem. A molecular cloud is heated by external radiation (X-rays, gamma-rays, UV) when it is low-density, and by cosmic rays more generally. At low densities and high temperatures, cooling is relatively inefficient. It tends to proceed via molecular radiation, such as from $H_2$ and CO. At low temperatures and high densities, cooling from dust grains dominates. This radiation occurs in the IR because that's where it is able to escape. This is why IR mapping tends to track dust.

The net result is that when a cloud becomes optically thick (say, $\tau > 1/2$), then it is self-shielded from external radiation and the interior portions of the gas can cool in peace. This happens by formation of molecules, for example, and the equilibrium temperature drops to about 10 K. Therefore, the inner part of the gas can radiate away the energy it gets from gravitational collapse ("settling" might be more appropriate), and continue to contract.

**Angular momentum**

As we indicated before, the specific angular momentum ("specific" means "per mass", so it's $L/M$) of giant molecular clouds is vastly greater than that of stars, so you have to get rid of most of it. For a $\sim$1 pc giant molecular cloud, $L/M > 10^{23}$. For a dense cloud core, $\sim$0.1 pc, $L/M \sim 10^{21}$. For the Sun, $L/M \sim 10^{15}$. Lots of orders of magnitude. Where does the excess go?

You might think it could go to binaries, or planets, or that young stars might have a lot of angular momentum, but it isn't so. A 3-day binary has $L/M \sim 10^{19}$. **Ask class:** how would the angular momentum go with orbital period? Like $P^{1/3}$, so even at a $10^4$ yr binary, $L/M \sim 10^{21}$. We already found that Jupiter doesn't have enough either, $\sim 10^{20}$. Young stars like T-Tauris have $L/M \sim 10^{17}$, so more than the Sun but nothing close to that of the initial cloud.

Therefore, specific angular momentum must be transported away from the system entirely. **Ask class:** what are some ways that this can happen? Winds (magnetic, especially), jets, disks. Still a lot of discussion about how this happens. In somewhat more detail, angular momentum can be removed by:

(1) Magnetic braking. If a magnetic field threads a cloud, it will try to enforce uniform rotation. This moves angular momentum outward. This can happen before the collapse of the cloud. It is also effective in slowing down the rotation of stars.

(2) Collapse to a disk. Stresses within the disk transport angular momentum outward and mass inward. An especially important source of such stress is the "magnetorotational instability", or MRI. If the disk has enough ionization, even a weak magnetic field is amplified if fluid at smaller radii has a higher angular momentum than fluid at larger radii.

If the ionization fraction is really low (or more properly if the magnetic Reynolds number is high enough), this mechanism is ineffective. This may lead to "dead zones" in some protoplanetary disks.

(3) Star-disk coupling. If the star has a significant magnetic field, it can get slowed down by interaction with the disk (or spun up, for that matter).

The net result of all of this is that once the gas starts to contract, it is likely that at least parts of it will eventually form stars (the efficiency of star formation, i.e., what fraction of the gas becomes stars, is debated). However, we are presented with an interesting issue: the initial Jeans mass is often about $10^4\,M_\odot$ or even larger. **Ask class:** why, then, don't we have lots of $10^4\,M_\odot$ stars? A key is that as the gas settles and cools, the Jeans mass decreases. Therefore, smaller subclumps in the matter become unstable, so that instead of one huge star we end up with lots of normal stars. In the early universe, at redshifts of $z \sim 10 - 20$ where the first stars formed, there are essentially no "metals" (in astronomical lingo, elements heavier than helium). This means that cooling was much less efficient then, so objects are hotter and the Jeans mass is larger. This leads current researchers to believe that the first generation of stars ("Population III" stars) could have been much more massive than current-day stars, perhaps up to hundreds of solar masses.