Astrophysical Modeling: Stars

In this lecture and the next one, we will take a look at how to do astrophysical modeling in reality. There is both science and art in such modeling. For a given object or phenomenon, we need to decide which aspects are important and which are just happenstance. We also need to think of ways to simplify (so that we can make progress) that don't eliminate the important physics. Today we'll talk about a "solved" set of objects, stars, and in the next class we will talk about a most definitely unsolved phenomenon, gamma-ray bursts.

What do we know about stars?

Ask class: for suggestions on each of the following lists.

Masses, luminosities, colors, lifetimes and eventual fate, rotation. Single, binary, multiple, formation, pulsation, magnetic fields. Flaring, composition, winds, jets.

What do we really want to explain about stars?

Mass range, luminosities, temperatures, ages, composition. What makes them shine, what holds them up, what state is matter in.

How do we know these quantities from observations?

Major points: distances and ages. Must know distances to get luminosities. Age $\gg$ human lifetime, so need another way to see them evolve. Go through how this is done.

This is going to form the framework of the course. We will place ourselves in the position of talented physicists who want to work out the structure and evolution of the stars. That means that we will not only have to come up with answers to questions, but we need to come up with the questions themselves. Given the phenomenal inherent complexity of stars, we have to choose carefully what problems we are studying. These have to be problems that are simplified enough to be analyzed, but realistic enough to give us insight and to be compared with observations.

What simplifying approximations can we make?

First, what can we ignore? "Ignore" here means that what we are neglecting is small enough compared to what we are including that the accuracy we need is satisfied. Thus, need in every case to have some standard against which to compare.

Ignore rotation.—Start with the case of the Sun. The Sun has a rotation period of about a month. The Keplerian period at the surface is $2\pi\sqrt{R^3/GM}$, or about 3 hours. Thus, the rotation of the Sun is about 1/250 of Keplerian, and effects should be quadratic in that, so negligible at the 10^{-4} level.

Ignore magnetic fields.—Strongest fields seen are in sunspots, $B \sim 1000$ G. If that were the average in the entire Sun, we'd have a total energy of $(B^2/8\pi)\frac{4\pi}{3}R^3 = 6 \times 10^{37}$ erg. Compared with the $GM^2/R = 4 \times 10^{48}$ erg in gravitational potential energy, this is tiny and can be neglected.

Assume spherical symmetry.—This follows from the lack of rotation. To a high degree of accuracy the Sun is spherically symmetric.

Assume that the Sun has cosmic composition?—Actually, we can get information about this that doesn't require an assumption. That is, we can look at the composition of the solar photosphere, we can see what is in meteorites, and so on. For some approximations we can assume that the Sun is pure hydrogen, but actually for much of the development we don't need to worry about composition at all.

Ask class: how much has the Sun changed in their lifetimes?

Assume that the Sun is in equilibrium.—We can break this down into different types of equilibrium: hydrostatic equilibrium, thermal equilibrium (local), energy balance throughout the star, and so on.

Having decided on a few things that we can simplify, we can then go on to model various aspects of stars. In this lecture, we will focus on one in particular: what powers the Sun?

Of course nuclear fusion will be it, but in this lecture I want the students to come up with that realization and know why other things don't work. The overall perspective is that knowledge of the primary source of energy for a phenomenon is an important step towards modeling that phenomenon.

Ask class: without worrying about how realistic it is, what are some candidates for energy generation in the Sun? Stress to them that in this kind of process, you first jot down possibilities without regard to whether they are realistic, then examine them critically.

Next, **Ask class:** what do we need to explain for the Sun? Total energetics is a good start. The Sun has a luminosity of about 4×10^{33} erg s^{-1}, and from fossil records we know that it has been shining at this rate for something like 5 billion years, so that's around 5×10^{50} erg. Whatever energy source there is must be able to provide at least this amount. In addition, the Sun's energy source is stable.

For each candidate mechanism, **Ask class:** how can we decide whether this can be the energy source? After that's indicated qualitatively, **Ask class:** what information is necessary to evaluate it? Then, do calculation.

Burning

Quantity needed is energy released per gram of burning. This is about 10^4 cal g^{-1}, and since 1 cal≈4 J=4×10^7 ergs, that's 4×10^{11} erg g^{-1}. The total mass of the Sun is 2×10^{33} g, so the total energy available is 8×10^{44} ergs, which is much too small. Can get this approximate value from first principles by figuring that burning is a chemical reaction, so it will release something like 1–10 eV per reaction, or maybe about 1 eV per nucleon. That gets the rough order of magnitude of the energy release. A nucleon has a rest mass energy of about 1 GeV, so at best the efficiency is 10^{-9}. The total mass-energy of the Sun is about $Mc^2 = 2 \times 10^{54}$ erg, so this gives 2×10^{45} erg. **Ask class:** Is there an astronomical source in which burning (or more generally, chemical reactions) play the major role in generating energy? Not that I know of, but as Chris Reynolds suggested, molecular clouds may have a phase where the formation of molecules is energetically important.

Crystallization

Again, what is energy per gram? In this case one can again see that things are more or less electronic, in that it's electrostatic interactions that determine the energy. Since burning failed by such a large margin, crystallization will as well. If slightly more accuracy is needed, note that crystallization=freezing (more or less), and as a result the ions have fewer degrees of freedom than they did before. Typically you have something like $\frac{1}{2}kT$ per degree of freedom, so at some melting temperature T_m you'll release around $\sim kT_m$ per ion. A typical freezing temperature for metals is a few thousand degrees, so 0.1–1 eV per ion is released, even less than for burning. **Ask class:** can they think of a source for which crystallization is important? Here one needs to think of a situation in which other energy sources are unavailable; a very cool white dwarf is an example.

Cooling

From the virial theorem, the total internal energy of the Sun is of the order of GM^2/R, or about 4×10^{48} ergs. This falls short by a factor of 100. **Ask class:** is there a source for this one? More or less any small object that has reached hydrostatic equilibrium is likely to go through a phase where cooling releases energy. A small planet is an example. This phase would be bracketed by times when gravitational settling dominates, and when other processes such as crystallization may play a role.

Rotation

How fast could the Sun possibly rotate? At the break-up speed, at which each element in the Sun is moving at a Keplerian velocity. But, by the virial theorem, the total kinetic energy is again comparable to GM^2/R, which is way short of what is needed. In addition, the visible layers of the Sun rotate at frequencies 100 times less than the Keplerian frequency, and helioseismology results indicate that the central part of the Sun is also rotating at dramatically sub-Keplerian rates. **Ask class:** are there sources for which rotation is the dominant source of free energy? Pulsars are a good example.

Magnetic fields

What is the average magnetic field in the Sun required to have the necessary energy? Energy density in B fields is $B^2/8\pi$, so we need $E = (B^2/8\pi)\frac{4\pi}{3}R^3 = \frac{1}{6}B^2(7 \times 10^{10})^3 = 5 \times 10^{50}$. That means that the average field must be about 3×10^9 G. That is much larger than the field in even sunspots, which can get to $\sim$1000 G, but this by itself doesn't disprove things, because maybe the internal field is much greater than the external (although there are problems with this as well...). To be a little more convincing, we need another argument. If the pressure were dominated by the magnetic field, the star would be unstable. So, we need the matter pressure to be at least as great as the magnetic pressure. The magnetic pressure is $P_B = B^2/8\pi = 5 \times 10^{17}$, and if we approximate the matter by an ideal gas then it is $P_m = nkT$, where $T \sim 10^7$ K. Equating the two means that the average number density is $n = 5 \times 10^{26}$ cm^{-3}, implying a density of $\rho = 700$ g cm^{-3}. That's 500 times the density of the Sun, so no way. **Ask class:** can magnetic fields dominate the total energy of some type of source? Possibly, if other sources are weak. It's been suggested that "magnetars", neutron stars with magnetic fields up to 10^{15} G on the surface, may fit the bill. Tough to say for sure, though.

Gravitation

$E_G = -GM^2/R = 4 \times 10^{48}$ ergs. Short by a factor of 100, as was cooling. Appear to be satisfied by this, but then come back and be puzzled by the possibility that some part of the mass of the Sun is actually much denser, like a white dwarf or neutron star. If half of the Sun were a neutron star, energy release from accretion would be 10^{20} erg g^{-1}. That implies an accretion rate of 4×10^{13} g s^{-1}, or about 10^{31} g (=0.5% of the Sun's mass) in its lifetime. Can't rule this one out so easily. Bring in here the story of Landau and Oppenheimer (resolution is not so easy to see: such a star would look like a supergiant), and Thorne-Zÿtkow objects. Also bring up Shu's point that, due to the endothermic production of heavy elements in a supernova, the net result is that the energy has been primarily gravitational! **Ask class:** other sources for which gravitational energy dominates? A protostar that is settling releases gravitational energy, and any accreting compact object as well.

Matter-antimatter reactions

There is plenty of energy available, in principle at least. To produce the $\sim 10^{51}$ erg necessary, 10^{30} g of matter+antimatter needs to be annihilated. That's 5×10^{-4} of the Sun's mass, so it's a small fraction. However, if this were spread diffusely throughout the star then it would all react very quickly and would blow the star apart. If it were concentrated in a central core, then the luminosity would be limited to the Eddington luminosity of about $10^{38}(M/M_\odot)$ erg s^{-1} at which radiation forces would prevent additional matter from falling in. However, as with gravitation onto a dense central core, the Sun would end up like a supergiant. Therefore, this too is not trivial to rule out, but nonetheless it can't power the Sun. **Ask class:** does antimatter power exist somewhere in the universe? Annihilation radiation from near our Galactic center has been seen, which must come from antimatter. Here, however, one can quibble, because there had to be some primary source of energy that produced the antimatter in the first place.

Hamsters

Suppose we were to power the Sun with hamsters on treadmills, or with their body heat. The average hamster apparently puts out about 1/3 Watt, or 3×10^6 erg s^{-1}. Suppose for argument's sake that a hamster has a mass of 10^3 g. Then in order to power the Sun's 4×10^{33} erg s^{-1} you'd need about 10^{27} hamsters, or about 10^{30} g total. That means that, mass for mass, hamsters are about 2000 times more efficient in generating energy than the nuclear fusion in the Sun! This is because, as we'll see later, fusion in the Sun via the pp chain proceeds very slowly. However, it can continue over $\sim 10^{10}$ yr, unlike the average hamster. Also unlike the average hamster, the Sun doesn't need to be fed.

Now suppose that we consider hamsters of different sizes. A typical mammal spends most of its energy keeping a constant body temperature, and hence the energy output scales like its surface area, or $M^{2/3}$ if we make the standard approximation of a spherical (or at least non-fractal) hamster. Thus, the energy output per mass goes like $M^{-1/3}$, so the larger the hamster the less efficient its output per mass. If we wanted to power the Sun with a single uberhamster, we would need a single hamster about $(10^{27})^{3/2}$ times more massive than a standard Terran hamster, or about 10^{43} g give or take a factor of a few. That's about the mass of the Large Magellanic Cloud, and at typical hamster densities would stretch out from the Sun to about the orbit of Saturn. Going in the other direction, suppose we enlisted femtohamsters of mass about 10^{-15} of the typical hamster. The total mass of femtohamsters required to power the Sun would be about 10^{25} g, or about the mass of Pluto. Unfortunately, however, none of the hamster family would be able to go on very long, and there'd be an astronomical mess to clean up.

Fission

Fission releases approximately 1 MeV per nucleon, or about 10^{-3} of the rest mass energy. For the Sun the rest mass energy is about 2×10^{33} g c^2, or 2×10^{54} erg, so 0.1% of that is 2×10^{51} erg, which is enough. So why couldn't fission be a viable mechanism? Two reasons. First, it would require that a large fraction of the Sun, at least 25%, would be made up of heavy elements such as uranium(!). Not so. Even more conclusively, fission is a process that has a critical mass, and you'd better believe that a solar mass exceeds that critical mass! Therefore, if you did set up that much uranium, it would blow itself to bits within a ridiculously short time. So no dice on this mechanism. **Ask class:** are there astronomical objects for which fission is an important energy source? The Earth, for one, and probably other terrestrials as well.

Fusion

Fusion of hydrogen to helium releases about 7 MeV per nucleon. That's plenty of energy. The main fuel, hydrogen, is the most abundant element in the universe, so that isn't a problem. The mechanism is self-regulating, because if too much energy is released there is a slight expansion of the star, cooling it and decreasing the energy generation rate (and vice versa). The only question is how this actually works, because there are many important details about how hydrogen burns to helium that are essential for being really convinced that this works.