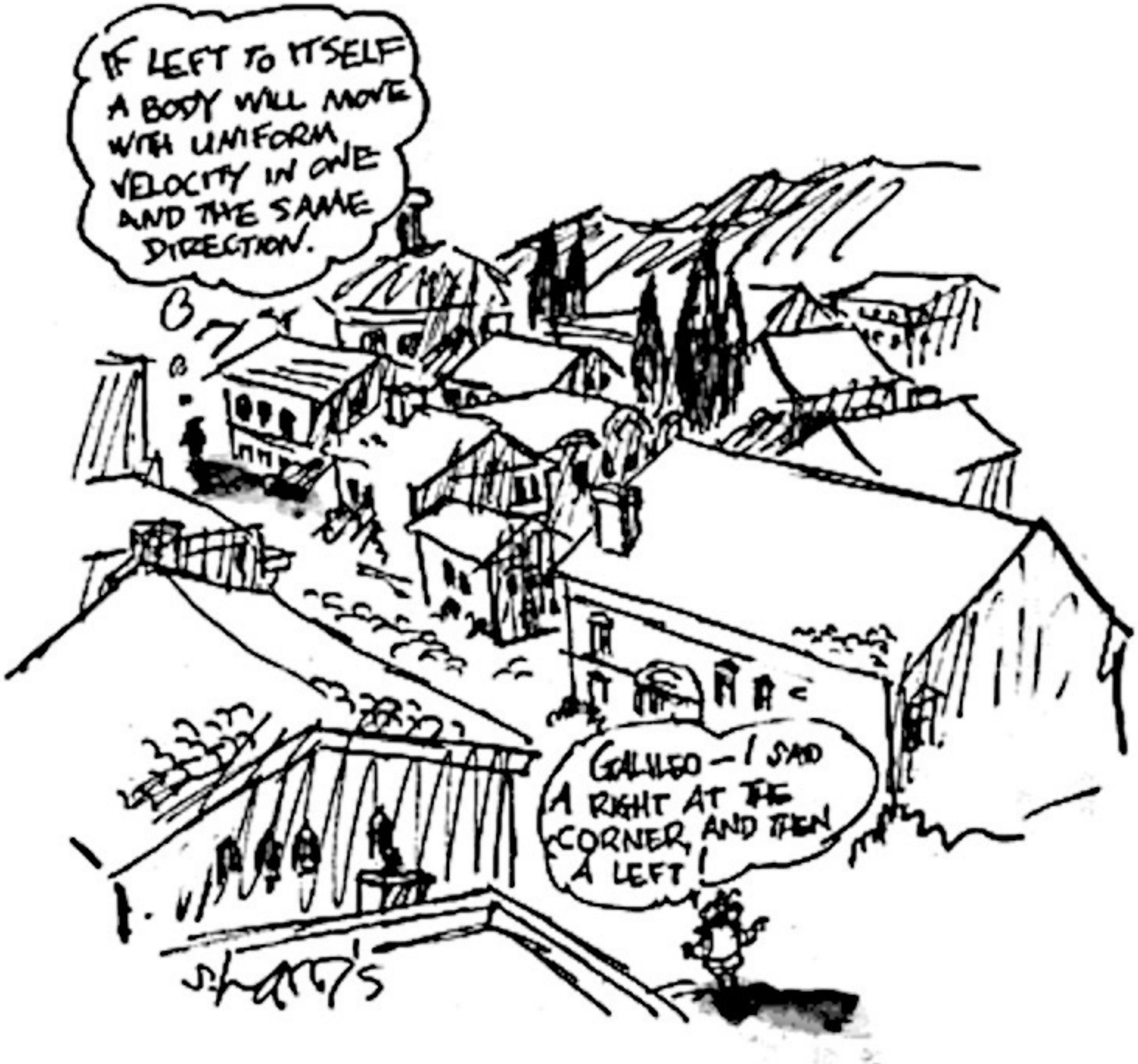
Class 2 :
Newton's Laws (in three acts)

ASTR350 Black Holes (Spring 2022)
Cole Miller

This class

- A bit about Newton

- Act I : Newton's Laws, the "Classic" version
- Act II : Newton's Laws in terms of momentum
- Act III : Newton's Laws from symmetry



Muddiest points

Any astro questions?

Isaac Newton in 1689, by Sir Godfrey Kneller.

Isaac Newton (1643-1727)

Father of modern physics and cosmology

- Attended Cambridge University, originally to study law, but read Kepler, Galileo, Descartes
- Began to study mathematics in 1663
- While Cambridge was closed due to plague (1665-1667), Newton went home and
  - began to work out foundations of calculus
  - realized (contrary to Aristotle) that white light is not a single entity, but composed of many colors
  - *began to formulate laws of motion and law of gravity*
- Became professor of mathematics starting in 1669 (age 27!)
- Worked in optics, publishing “Opticks” (1704)
  - invented reflecting telescope
  - showed color spectrum from prism recombines into white light with a second prism
  - analyzed diffraction phenomenon

Newton's profound perspective

- Newton formulated a **universal** theory of motion and gravity
  - ***Same laws of physics operate anywhere and anytime in the Universe***
  - *Implies that experiments performed on earth are applicable to the rest of the universe*
- Helped spur the Age of Enlightenment

Newton and Gravity

Newton quantified this force and by considering the accelerations of objects on earth (the falling apple) with the acceleration of the earth's satellite (the falling moon) derived a "Universal" law of the attraction between objects
(see next lecture)

We use calculus to derive his results, but Newton used geometry because calculus wasn't yet familiar to his contemporaries!

ACT I – "CLASSIC" DISCUSSION

Why are we discussing Newton and Classical physics when black holes are a manifestation of General Relativity, a successor to classical physics?

Because without understanding (at least a little bit) of classical physics, understanding Relativity will be much more difficult and the radical changes will not be understandable

ACT I – “CLASSIC” DISCUSSION

Fundamental Assumptions in Newtonian Physics

Space is smooth and is infinite in all directions

Time is smooth and linear and flows into the infinite future

Space and time are distinct and independent

Seems pretty obvious....

I : Newton's laws of motion

- **Newton's first law** (The Law of Inertia*) **: If a body is not acted upon by any forces, then its velocity remains constant**
- Notes
  - Remember that velocity is a vector quantity (it has direction as well as magnitude)
  - This law sweeps away the idea that "being at rest" is a natural state… this was a major change of thinking, compared to the previous 2000 years- concept of what is 'natural'?

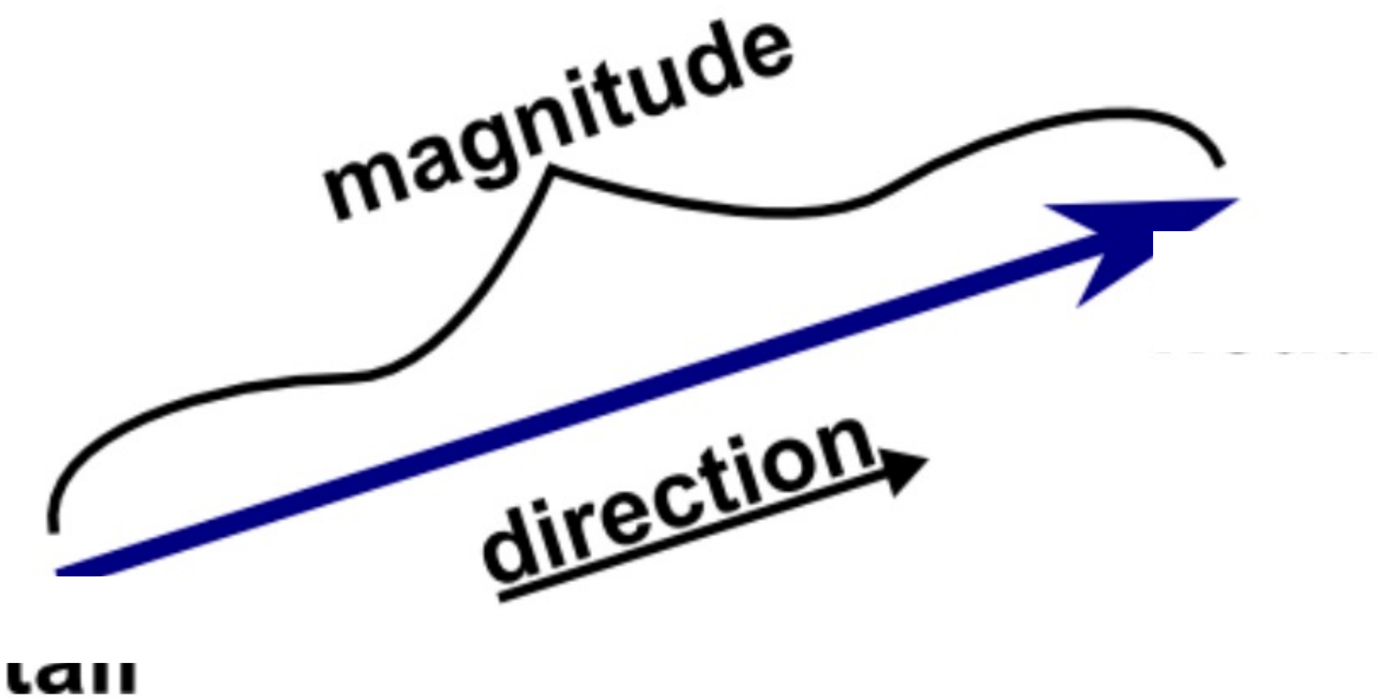
*Inertia is the resistance of any physical object to a change in its state of motion or rest, or the tendency of an object to resist any change in its motion

Vectors

A vector has a **magnitude** and **direction**

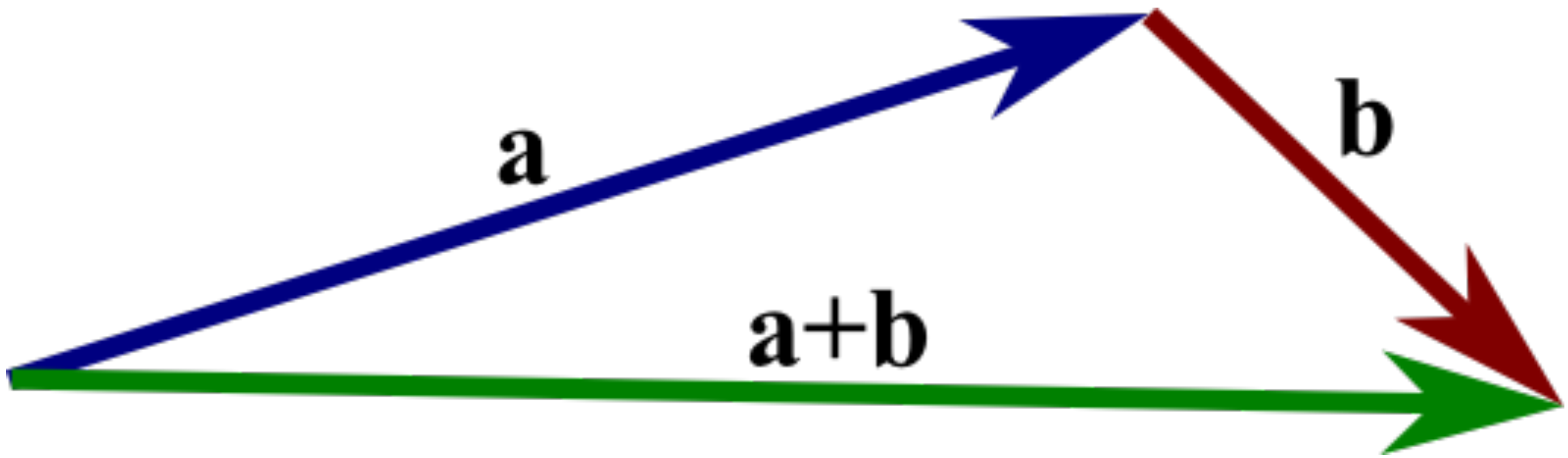
Two examples of vectors are those that represent force and velocity. Both force and velocity are in a particular direction.

The magnitude of the vector is the strength of the force or the speed associated with the velocity.



Vector Addition

Given two vectors **a** and **b**, their sum, **a+b**, is the directed line segment from the tail of **a** to the head of **b** is the vector **a+b**.



From Math Insight. http://mathinsight.org/image/vector_a_plus_b

I : Newton's laws of motion-cont

- "Velocity" is a vector

  Thus Newton's "constant velocity" implies both constant speed and constant direction (and also includes the case of zero speed, or no motion).
- Galileo : based on his concept of inertia, it is impossible to tell the difference between a moving object and a stationary one without some outside reference to compare it against.
- This ultimately led to Einstein's development of the theory of Special Relativity.

Newton's second law

- **If a body of mass M is acted upon by a force F, then its acceleration a is given by F=Ma**
- Notes
  - Remember that both F and a are vectors but mass is not a vector
  - This law defines the "inertial mass" as the degree to which a body resists being accelerated by a force

Define acceleration =change of velocity/time e.g a=dv/dt

2^{nd} Law-Cont

- This is the most powerful of Newton's three Laws, - it allows quantitative calculations of dynamics: how do velocities change when forces are applied.
- According to Newton, force **causes** a change in velocity (an acceleration).

https://www.wisc-online.com/learn/general-education/technical-physics/tp1917/newtons-second-law-of-motion---video

Newton's second law- Again

- Another way of saying the 2nd law is that force = rate of change of momentum
- $\underline{p}$=momentum=m$\underline{v}$
- F= d$\underline{p}$/dt=m(d$\underline{v}$/dt)=m$\underline{a}$
- derivative d$\underline{p}$/dt ~ change in momentum/unit change in time or "$\Delta\underline{p}/\Delta t$"

time is **not** a vector even though we think of it having a "direction"

What is Mass?

1) **Mass** is a measure of the amount of matter in an object

2) **Mass** is a measure of the amount of energy in an object (Einstein $E=\mathbf{m}c^2$)

3) **Mass** is the degree to which a body resists being accelerated by a force (Definition of Inertial **mass**)

Weight is NOT the same as mass, weight depends on the gravitational field the mass is in.

A car on the Moon would weigh less than it does on Earth because of the lower gravity, but it would have the same mass.

This is because weight is a force, whereas mass is the property that (along with gravity) determines the strength of this force.

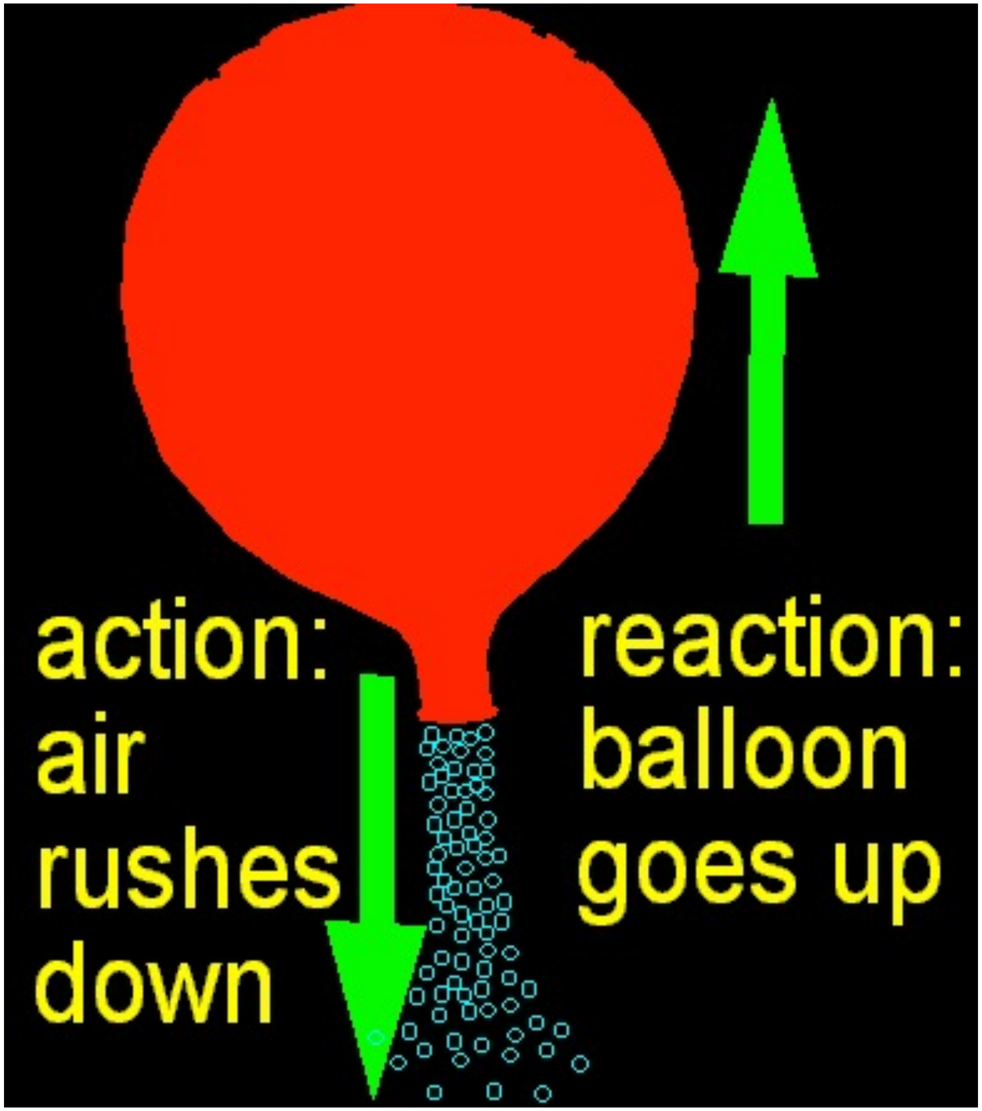
Third Law

If a body A exerts a force F on body B, then body B exerts a force -F on body A

Notes

This is the law of "equal and opposite reaction"

This law is closely tied to conservation of momentum



Rockets....

First liquid fuel rocket flight, March 16, 1926 (at his Aunt's farm in Massachusetts)

His work elicited relatively little interest in the United States

Robert Goddard 1882-1945

https://airandspace.si.edu/stories/editorial/robert-goddard-and-first-liquid-propellant-rocket

Review of Goddard's pioneering work on rockets

- "Professor Goddard... does not know the relation between action and reaction and the need to have something better than a vacuum against which to react -- to say that would be absurd. He only seems to lack the basic knowledge ladled out daily in high schools."

  \- January 13,*1920 New York Times editorial*

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"Further investigation and experimentation have confirmed the findings of Isaac Newton in the 17^{th} century...The Times regrets the error."

- July 17, 1969 New York Times correction

ACT II – INTRODUCING MOMENTUM

Momentum

- **Definition :** If an object of mass m is moving with velocity V, its momentum p is given by p=mV

- **Definition :** total momentum p_{tot} of a number of objects with masses m_1, m_2, … and velocities V_1, V_2, … is the (*vector*) sum of the objects' separate momenta

- **Conservation of momentum :** The total momentum of a system of particles is constant *if no external forces act on the system*

Momentum

- Newton's laws can be rephrased entirely in terms of momentum…
  - **<u>Second law</u>**… the rate of change (derivative) of momentum of a body is equal to the force applied to that body*
  - **<u>First law</u>** is special case of the Second law… the momentum of a body is unchanged if there are no forces acting on body
  - **<u>Third law</u>**… the momentum of an isolated system of objects is <u>conserved</u>

$\underline{F}=m\underline{a},= m d\underline{V}/dt=d\,\underline{p}/dt$

Conservation? In physics, the term *conservation* refers to something which doesn't change.

- The law of *momentum conservation*: (more fundamental than Newton; true for relativity and quantum mechanics, too!)
- For a collision occurring between object 1 and object 2 in an isolated system (one that is not acted on by force external to the system), the total *momentum* of the two objects before the collision is equal to the total *momentum* of the two objects after the collision.
- *Momentum* is neither created nor destroyed, but only changed through the action of forces as described by Newton's laws of motion.
- Optional challenge: can you prove, using Newton's laws, that momentum is conserved in any interaction within an isolated system?

Frames of Reference?

- A frame of reference is a set of coordinates that can be used to determine positions and velocities of objects in that frame; different frames of reference can move relative to one another.
- Objects changing positions with time with respect to the **frame of reference** are in motion while those which do not change position are at rest.

Galilean Relativity

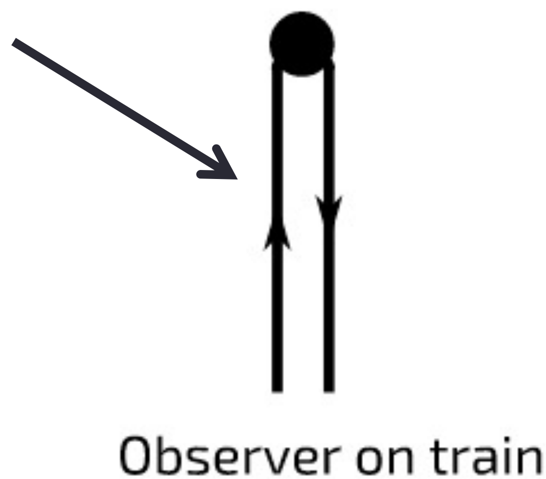
- Consider two **frames of reference** that differ by some uniform velocity difference (so we are not considering accelerated frames of reference)
  - How do we relate velocity in one frame to the other??

- We use a "velocity addition rule" $\mathbf{v_{total}=v_1+v_2}$– this is an example of a **Galilean transformation**

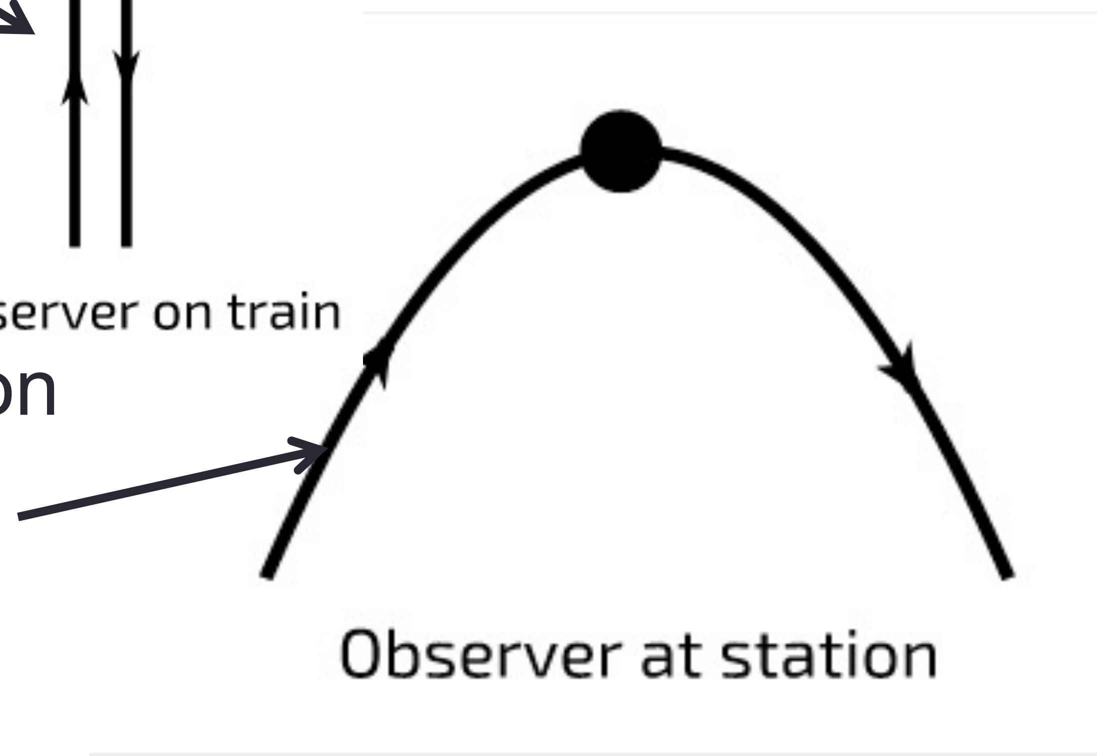
- The idea that the laws of nature are the same for a moving observer as for a stationary observer (two different frames of reference connected by a Galilean transformation) is called the **Principle of Galilean Relativity**

Imagine...

- 2 Observers: one on a train, the other at the station
- I am on the train and throw a ball up-what do I see

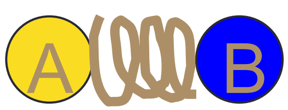


What does the person on the station see?
Frame of reference matters!



An illustration of Newton's laws and Momentum

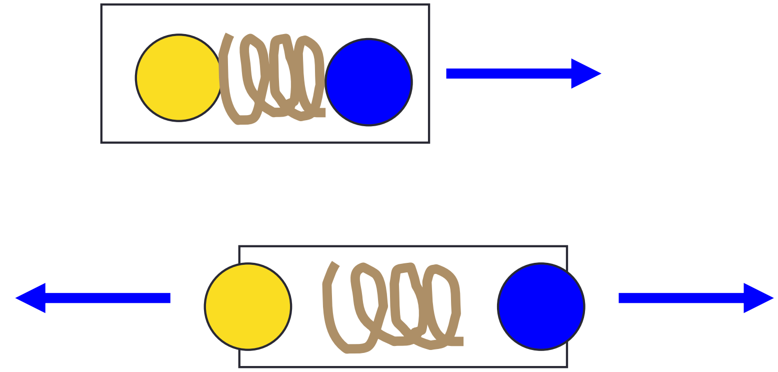
- Consider two equal masses M at rest. Initial momentum is $p = 0$.
- Masses are suddenly pushed apart by a spring… and will move apart with the same speed V *in opposite directions* (*by symmetry of space!*). Total momentum is $p = mv - mv = 0$. **<u>Total</u> momentum is unchanged.**



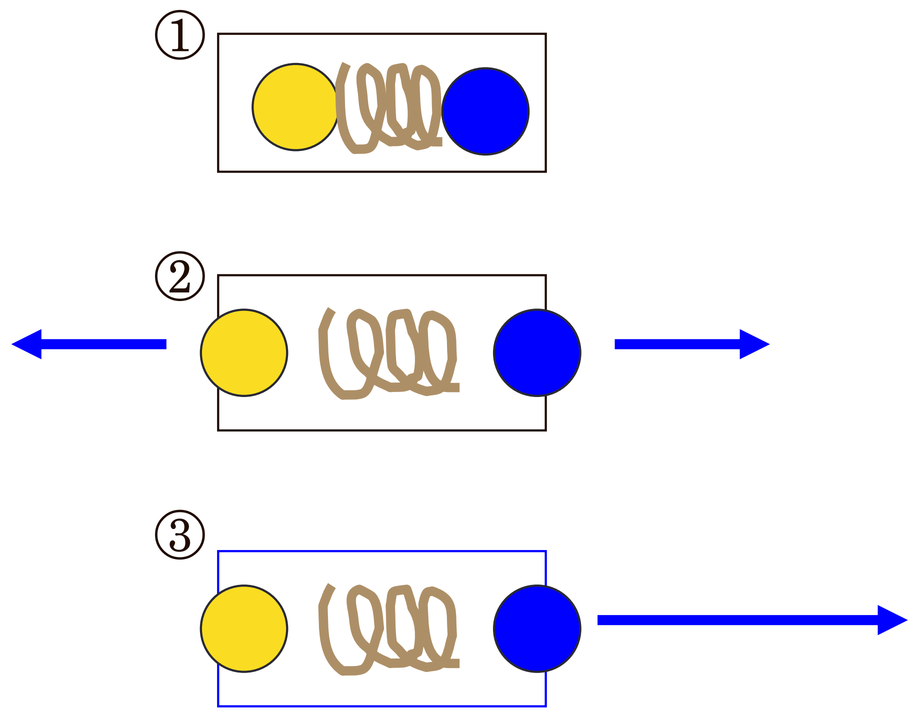
Before: $v_A = v_B = 0 \Rightarrow p_{tot} = 0$

After: $v_A = -V,\ v_B = V \Rightarrow$
$p_{tot} = Mv_A + M v_B = -MV + MV = 0$

- Same situation, but masses are now both initially moving at velocity V. Initial momentum is p_{tot}=2MV.
- Can turn into the previous situation by "moving along with them at velocity V".



1. Change of perspective [subtract V from all velocities] brings masses to rest…
2. Do same problem as before…
3. Change back to original perspective [add V to all velocities] …
4. Final velocity of one ball is 2V; final velocity of other ball is 0. Final total momentum is p_{tot}= 2MV. **No change in total momentum.**



ACT III – SYMMETRIES AND FRAMES OF REFERENCE

Symmetries and frames of reference

- Can we derive Newton's laws / conservation of momentum from more fundamental considerations? Yes! Use the idea of symmetry…

- The idea of symmetry is very important in modern advanced physics! A symmetry means that you can change the system in a way that does not make a difference.
- For example: there are no preferred locations in empty space; all locations are equivalent.
- Brilliant insight by the great Emmy Noether: for every symmetry there is a conservation law! Noether's Theorem

Symmetry and Conservation Laws

- Empty space possesses the symmetries that it is the same at every location (homogeneity) and in every direction (isotropy)
- These symmetries in turn lead to the invariance principles that the laws of physics should be the same regardless of changes of position or of orientation in space.
  - The first invariance principle implies the law of conservation of linear momentum,
  - the second implies conservation of angular momentum.
- The homogeneity of time leads to the laws of physics remain the same at all times, which in turn implies the law of conservation of energy.
- Much of modern physics is based on Noether’s Theorem

Summary

- **Newton's 1st law :** $\underline{V}$ = *constant* if $\underline{F} = 0$
- **Newton's 2nd law :** $\underline{F} = M\underline{a}$
- **Newton's 3rd law:** for every action there is an equal and opposite reaction.
- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference ($\mathbf{v}_{tot}=\mathbf{v}_1+\mathbf{v}_2$)
- **Galilean Relativity** – the idea that the laws of nature are the same for a moving observer as for a stationary observer.
- **Symmetry leads to conservation law**

- $\underline{\mathbf{v}}$=velocity, $\underline{\mathbf{a}}$=acceleration, $\underline{\mathbf{F}}$=force, M=mass

Next time…

- Newtonian gravity!