Class 3 : Newtonian Gravity

ASTR350 Black Holes (Spring 2022)
Cole Miller

Today's class

- Newton's Law of Gravity
- Gravitational and Inertial Mass
- Free fall & the Weak Equivalence Principle
- Escape velocity (and Newtonian Black Holes)

Muddiest points

Any astro questions?

RECAP Newton's Laws of Dynamics*

- **Newton's 1st law** – $\underline{V}$ = *constant* if $\underline{F} = 0$
- **Newton's 2nd law** – $\underline{F} = M\underline{a}$
- **Newton's 3rd law** – for every action there is an equal and opposite reaction.

- Connection to momentum and momentum conservation
- Symmetry

* the study of forces and their effects on motion.

What Was Known About Gravity Before Newton

NOT MUCH (the concept did not really exist)

- But it was known how the planets move ! (Kepler's 3 Laws)
  - Planets move around the Sun in ellipses, with the Sun at one focus.
  - The line connecting the Sun and a given planet sweeps out equal areas in equal times.
    - Therefore, planets move faster when they are nearer the Sun
  - The square of the period *P* of the orbit is proportional to the cube of the semi-major axis *R*- $P^2 = \text{constant} \times R^3$; *constant determined by observations*

Kepler in perspective

- Based on accurate observations, Kepler calculated and thought his way to a major breakthrough

- Kepler's three laws of planetary motion
  - Represented a very simple (and correct!) model of the solar system
  - Swept away previous ideas thousands of years old
  - Were driven fundamentally by the **data**, including Tycho's data and error estimates

- Unlike previous models which quantified only what was observed already, Kepler's Laws had predictive power, consistent with modern idea of a meaningful scientific theory

What is a Scientific Theory??

Criteria for a scientific theory:

1. Relevant -- based on objective (reproducible) data
2. Falsifiable -- makes predictions that can be tested by expt./obs.
3. Consistent -- encompassing/extending existing established theories
4. Simple -- no more complications than necessary (Occam's razor)
5. Predictive -- does not just explain existing phenomena, but makes predictions for new discoveries

A theory is an attempt at explanation of physical phenomena- it is NOT speculation. However it is the best attempt at the time of its development and is subject to change- Science is always changing in response to new data and new ideas.

Galileo's Law of Falling Bodies

Galileo in a series of experiments measured the velocity and acceleration of falling bodies; he found that falling bodies accelerate at a constant rate.

Determining physical laws from experiment was completely new in the early 17th century !

- Galileo was the first clearly to understand that the forces acting upon objects could be broken into independent components- that a thrown stone had a force pulling it down as well as the force throwing it horizontally outward.
- These insights would be of great use to Isaac Newton, born the year Galileo died, in devising the calculus and his universal laws of gravity and motion

I: Newton's Law of **Universal** Gravitation

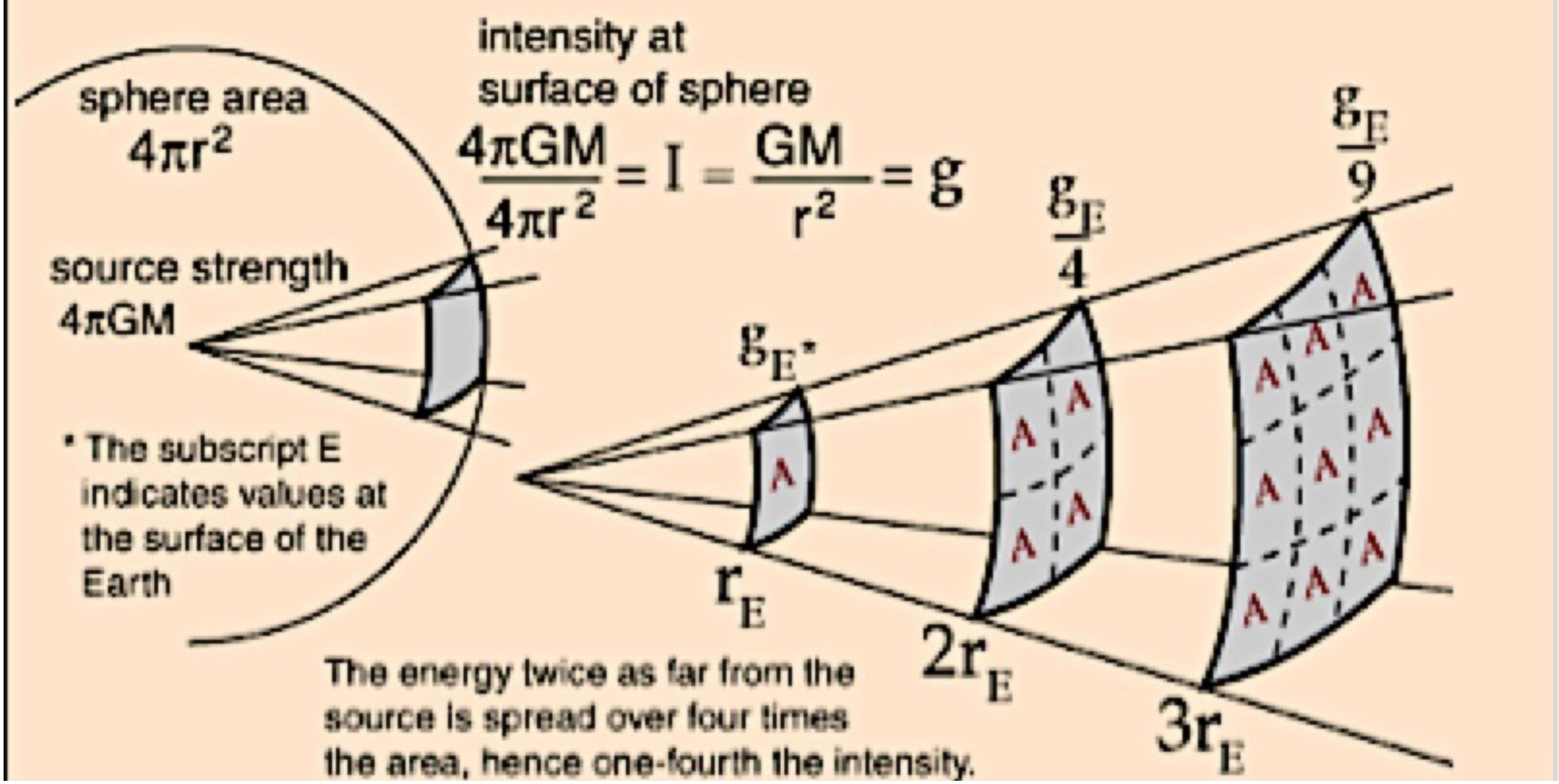
<u>Newton's law of Gravitation:</u> A particle with mass m_1 will attract another particle with mass m_2 at distance r with a force F given by

$$F = \frac{Gm_1 m_2}{r^2}$$

- "G" is called the gravitational constant ($G = 6.67\times10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$ in SI units) G is obtained by measurement
- This is a **universal** attraction. <u>Every particle in the universe attracts every other particle!</u> Gravity often dominates in astronomical settings- but weak compared to other forces (electricity and magnetism, nuclear forces)

Inverse Square Law , Gravity

As one of the fields which obey the general inverse square law, the gravity field can be put in the form shown below, showing that the acceleration of gravity, g, is an expression of the intensity of the gravity field.



http://hyperphysics.phy-astr.gsu.edu/hbase/forces/isq.html

Newton "Discovered" Gravity

- Newton was the **first** to create a theory that applied to all objects, large and small, using mathematics (calculus- which he invented) which described the motion of planets and earthly bodies (how apples fall and artillery shells move).

The Apple Story

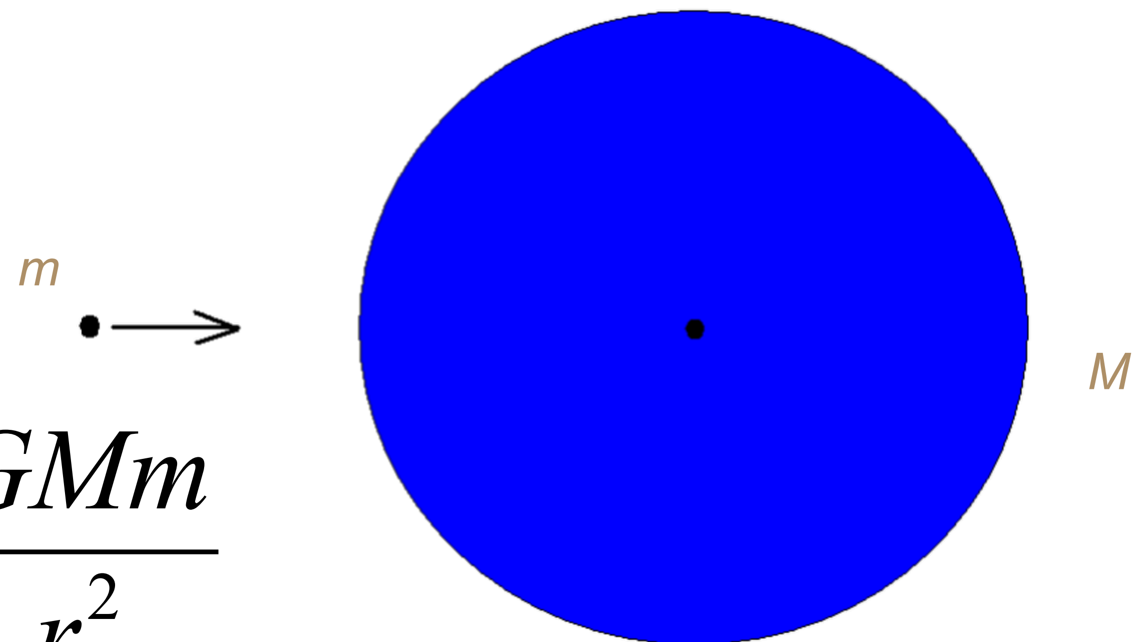
- *After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees…he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself…"*

https://thonyc.files.wordpress.com/2016/05/newton.jpg w=500

https://www.newscientist.com/article/2170052-newtons-apple-the-real-story/#ixzz6Ck4e8hFN

Gravitational Mass

- Newton's Law of Gravitation **defines** the "gravitational mass" of a body
- Using calculus, it can be shown that a spherical object with mass *M* (e.g. Sun, Earth) creates the same gravitational field as a particle of the same mass *M* at the sphere's center.



$$F = \frac{GMm}{r^2}$$

II : Inertial and gravitational mass: the weak equivalence principle

Newton's 2nd law says:

$$F = m_I a$$

m_I=inertial mass

Newton's law of gravitation says:

$$F = \frac{GMm_G}{r^2}$$

m_G=gravitational mass

So, acceleration due to gravity is:

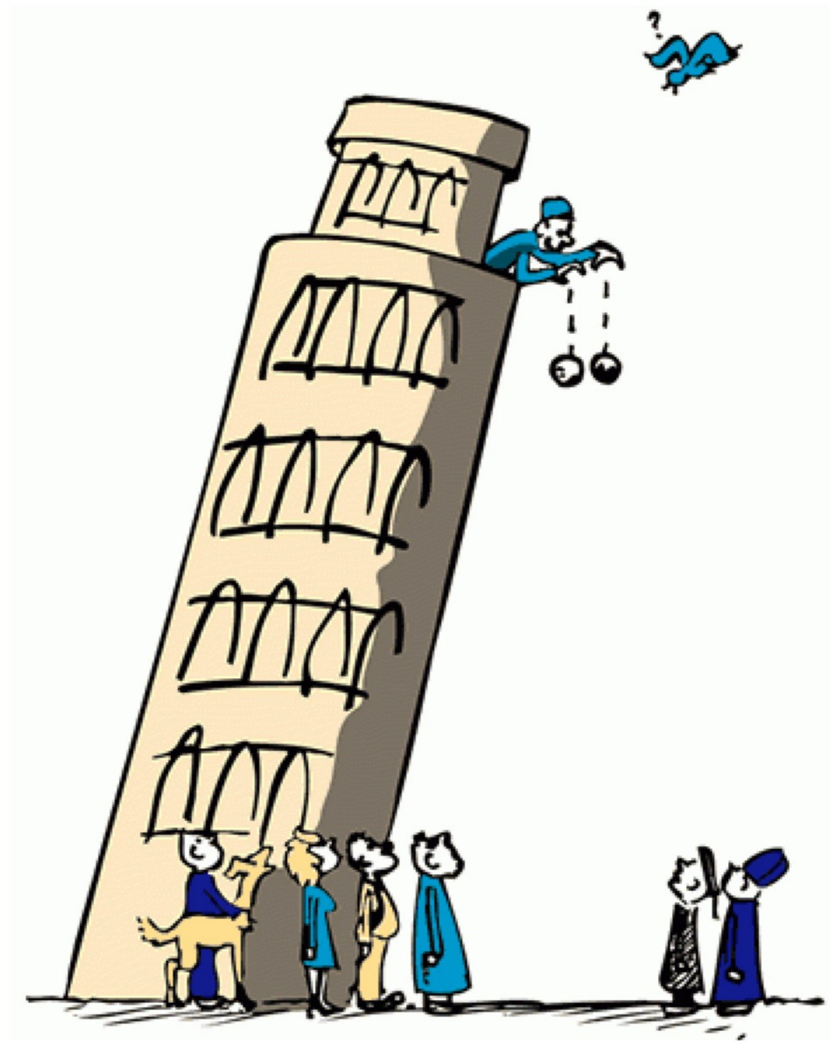
$$a = \left(\frac{m_G}{m_I}\right)\frac{GM}{r^2}$$

So, if the ratio (m_G/m_I) *varies*, the rate at which objects fall in a gravitational field will vary…

Galileo Tower Experiment

- Galileo said 1638 in *Dialogues Concerning Two New Sciences*: "[T]he variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that … I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed."

https://science.sciencemag.org/content/347/6226/1096.full?intcmp=collection-general relativity



At the end of the last Apollo 15 moon walk (July 1971), Commander David Scott performed a live test of m_I/m_G for the television cameras.

Equivalence Principle in other words

- Inertial mass determines how much an object resists moving when pushed by a force—as when you shove a car.
- Gravitational mass determines how strongly gravity pulls on the object.

- **The equivalence principle says: the two masses are one and the same, regardless of how heavy a thing is or what it's made of.**

- That explains Galileo's experiment: If the two types of mass are identical, then for all objects the pull of gravity varies in strict proportion to the resistance to motion, ensuring that all things fall at the same rate

https://science.sciencemag.org/content/347/6226/1096.full?intcmp=collection-generalrelativity

Equivalence Principle

- **The Weak Principle of Equivalence** states all the laws of motion for freely falling particles are the same as in an unaccelerated reference frame.
  - the Weak Equivalence Principle is a restatement of the equality of gravitational and inertial mass.
  - **There is no way of distinguishing between the effects on an observer of a uniform gravitational field and of constant acceleration-** the equality of gravitational and inertial mass. This is the foundation of the **General** Theory of Relativity.

  Einstein said that this was 'the happiest thought of his life' (http://physicsbuzz.physicscentral.com/2015/03/the-happiest-thought-of-my-life-100.html)

“Weightlessness”

Apollo 10, in orbit (May 18-26, 1969)

Orbital speed and period

- An object of mass m going in a circle of radius r with speed v requires a force F given by

$$F = \frac{mv^2}{r}$$

to keep it in orbit

- **For a**n object mass m (satellite, space station, Moon) in a circular orbit about a large body of mass M (such as the Earth), this force is provided by gravity. **So equate the two to get**...

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Orbital speed and period

$$\frac{GM\cancel{m}}{r^2} = \frac{\cancel{m}v^2}{r}$$

The mass of the smaller object cancels (the weak equivalence principle that we've just seen)

Take square roots and get

$$v = \sqrt{\frac{GM}{r}}$$

- Consequences of this…
  - The mass of the smaller object cancels (the weak equivalence principle that we've just seen)
  - The speed is given by

$$v = \sqrt{\frac{GM}{r}}$$

  - The period T (time taken for one orbit) is given by

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Predictions from Newton's theory match observations !

The period (time taken for one orbit) is given by

$2\pi r$= circumference of circle/velocity of body

substitute above formula for v and do a little algebra

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

square both sides

$T^2=4\pi^2r^3/GM$

Kepler's third law

Period^2 α r^3

This is one of Kepler's laws

$T^2 \sim r^3$ Newton determined the constant from his theory of gravity

The constant=**$4\pi^2/G$** is calculated from first principles and not from measurements (they are equal)

Applications and impact of Newtonian physics

- With Newton's laws, it was possible to make predictions
  - these laws can be applied to stars in galaxies, galaxies in clusters, planets around other stars etc., to understand orbits and "weigh" the system, since the mass is proportional to the inverse-square of the orbital period and cube of the orbital distance.
  - Its the *deviation* of the observations from our understanding of how much mass objects have that has led to the idea of dark matter.
- As Newton's physics came to be widely known, there was a huge cultural impact.
- With the Universe describable by precise mathematical laws, it supported the idea of "rationality" in other arenas -- including architecture, government, history, etc.
- *Key to shift in thought known as the Enlightenment. The universe is a giant machine! (?)*

III : Escape Velocity

- What goes up must come down. Ah, actually…

- Suppose we are on the surface of a planet with mass M and radius R. Then there is a critical speed above which we can leave the surface and *never fall back*. This is the **escape velocity, v_{esc}.**
- Using Newton's Laws and a little calculus, we find that

$$V_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

Bigger the mass the higher the escape velocity

- Some numbers…
  - $G=6.67 \times 10^{-11}\ Nm^2kg^{-2}$
- The Earth
  - $M_{Earth}=6.0 \times 10^{24}kg$
  - $R_{Earth}=6.4 \times 10^6m$
- The Sun
  - $M_{Sun}=2.0 \times 10^{30}kg$
  - $R_{Sun}=7.0 \times 10^5km$

$V_{esc}=11.2km/s$ (25,020 mph)

7 miles per second

$V_{esc}=617km/s$

On a test you will be given the formulae and constants

Applications and impact of Newtonian physics

- Herschel, in 1781, discovered Uranus; its orbit showed enough variations to predict there must be another as-yet-unknown planet.
  - In 1845, John Adams and Urbain Leverrier, predicted the existence of an unseen planet, to account for the fact that Uranus was being pulled slightly out of position in its orbit by the gravitational effect of an unknown body, and calculated its position and motion in the sky. Observations confirmed this leading to the discovery of **Neptune** in 1846

Early indications of black holes

- ``The gates of hell are open night and day;
  Smooth the descent, and easy is the way:
  But to return, and view the cheerful skies,
  In this the task and mighty labor lies."
  - Virgil, The Aeneid, Book VI [John Dryden translation]

John Michell

- Founded field of seismology, first to apply statistics to astronomy, invented apparatus to measure strength of gravity
- Understood concept of escape velocity and how it depended on **mass and size** of object
- If following Newton, light was a particle, it should be slowed down by gravity and he knew that light traveled at a certain speed- so if the escape velocity of a star was greater than the speed of light, it could not escape- **dark stars**

https://www.famousscientists.org/john-michell/

Big Problem

- However: in 1801 Thomas Young showed that light behaved like a wave- and the whole calculation assumed that light was a particle.
- Unlikely (in Newtonian physics) that gravity would act on a wave.

Next Lecture- Special Relativity

- Speed of light problem
- Einstein’s postulates
- Time dilation – time is not absolute!!!

RECAP

- **Galilean Transformation** – the "usual" velocity addition/subtraction rule for changing frames of reference $\underline{v}=\underline{v}_1+\underline{v}_2$ (vector addition)

- **Galilean Relativity** – the idea that the 'laws of nature' are the same for a moving **non-accelerated** observer as for a stationary observer.
  - If everything is moving together at constant velocity, there can be no apparent difference from the case when everything is at rest.
    - Ball dropped from top of moving ship's mast hits near bottom of mast, not behind on deck.

(reminder of terms, $\underline{\mathbf{V}}$= velocity, $\underline{\mathbf{F}}$= force, $\underline{\mathbf{a}}$= acceleration, all are vectors, **m**=mass NOT a vector)

RECAP

- **Galilean Relativity**
  - Galilean invariance or Galilean relativity:**the laws of motion are the same in all inertial frames.**
    - Galileo described this in 1632 in his Dialogue Concerning the Two Chief World Systems using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer below the deck would not be able to tell whether the ship was moving or stationary.
  - Newton's laws hold in all frames related to one another by a Galilean transformation. In other words, all frames related to one another by such a transformation are inertial (meaning, Newton's equation of motion is valid in these frames).
    - An inertial frame is a reference frame in relative uniform motion to absolute space and there exists an absolute space.