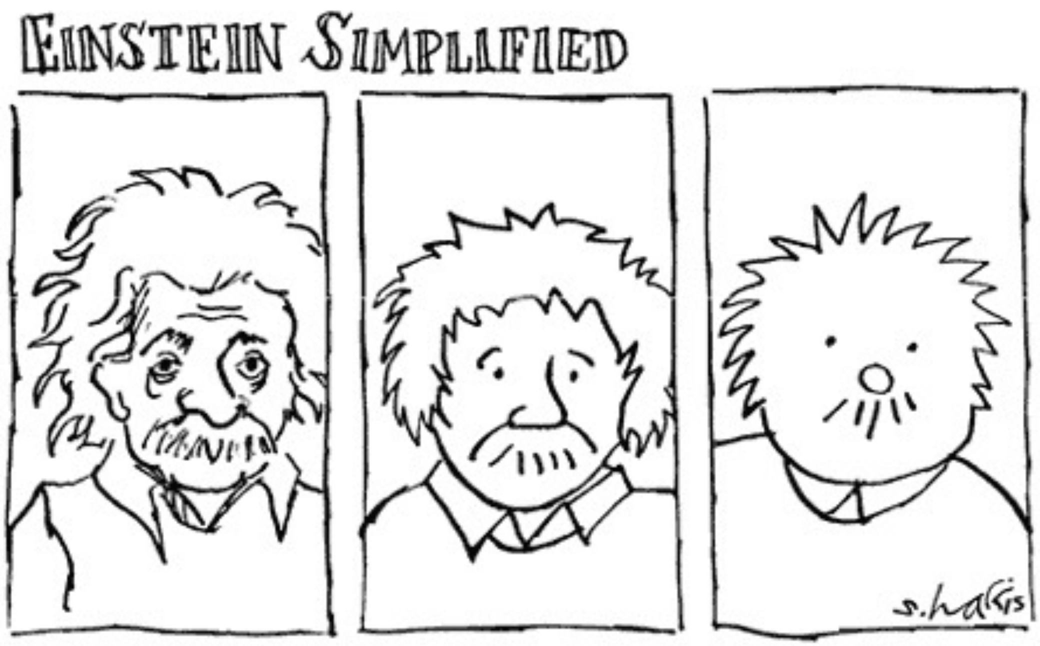
Class 4 : Special Relativity

ASTR350 Black Holes (Spring 2022)

Cole Miller

Today's class

- Speed of light problem
- Einstein's postulates
- Time – time is not absolute!!!



Muddiest points

Any astro questions?

RECAP

- Newton's Laws of Motion
  - Connection with notion of frame of reference
- Newtonian Gravity
  - Action at a distance?
  - Predicts notion and value of escape velocity, v_{esc}
- First ideas for black holes in late 1700s
  - Hypothetical objects with v_{esc}>speed of light
  - Rev John Michell and Laplace

Galilean Transformations

- In Galilean relativity **length intervals** and **time intervals are the same for all inertial observers of the same events**
  - if **meter sticks** are of the **same length** when measured and compared **at rest**, they are of the **same length** when compared in **relative motion to one another**;
  - if **clocks** are **calibrated** and **synchronized** when at rest, their **readings** and **rates will agree** even if they are in **relative motion** to one another.
    - seems perfectly obvious....but wait and see !
  - **What is an inertial observer??? – One who is not being accelerated or have a force on them- that is they have constant velocity**

astronomy.swin.edu.au

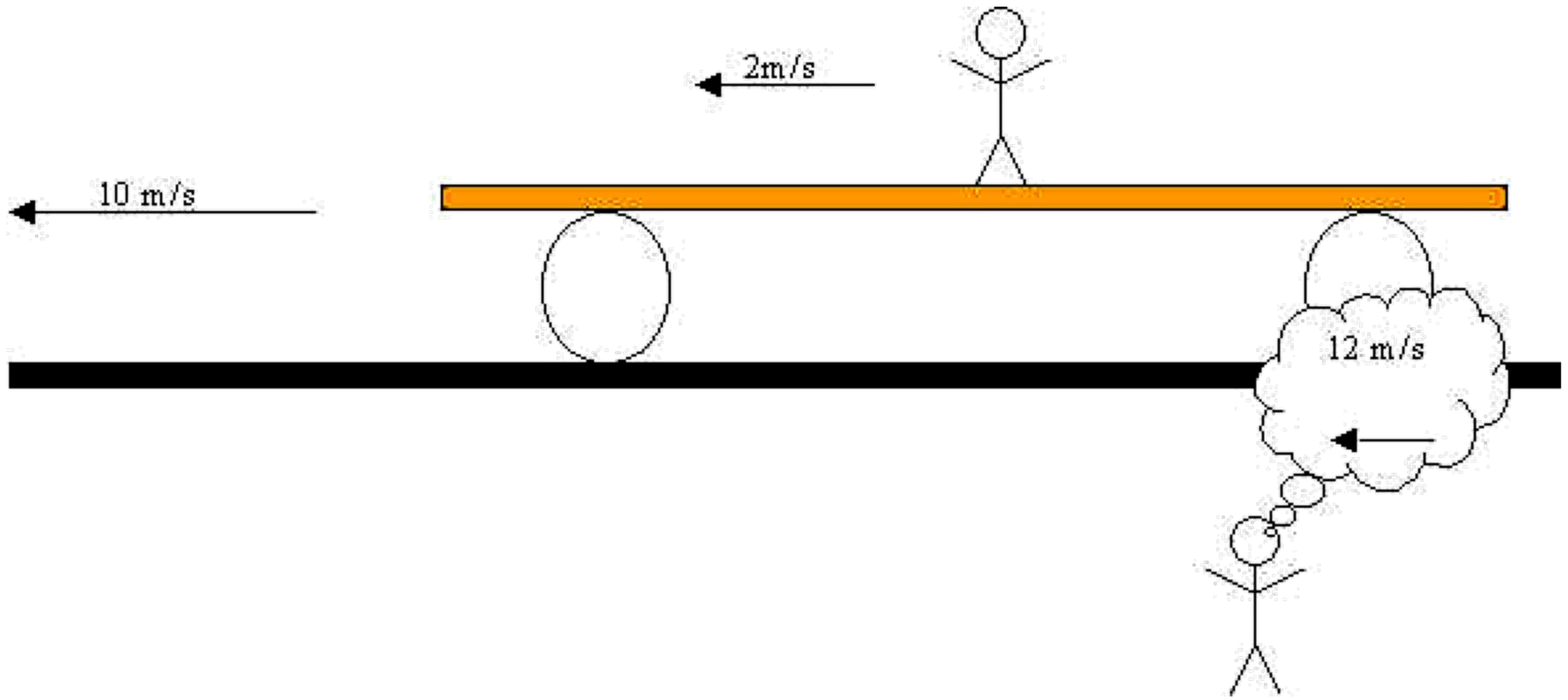
Physics at the End of the 19th Century

End of 19th century- physics almost 'solved'- (?,!)

- Newtonian mechanics and gravity
- Electricity and Magnetism – Maxwell

- *Small* outstanding issues
- nature of matter
- small problems with orbit of Mercury
- speed of light
- radioactivity (1896)

I: THE SPEED OF LIGHT PROBLEM



- Recap
  - “Relativity” tells us how to relate measurements in different frames of reference
  - Galilean relativity
    - *Simple velocity addition law :* $v_{total}=v_{run}+v_{train}$

Relativity

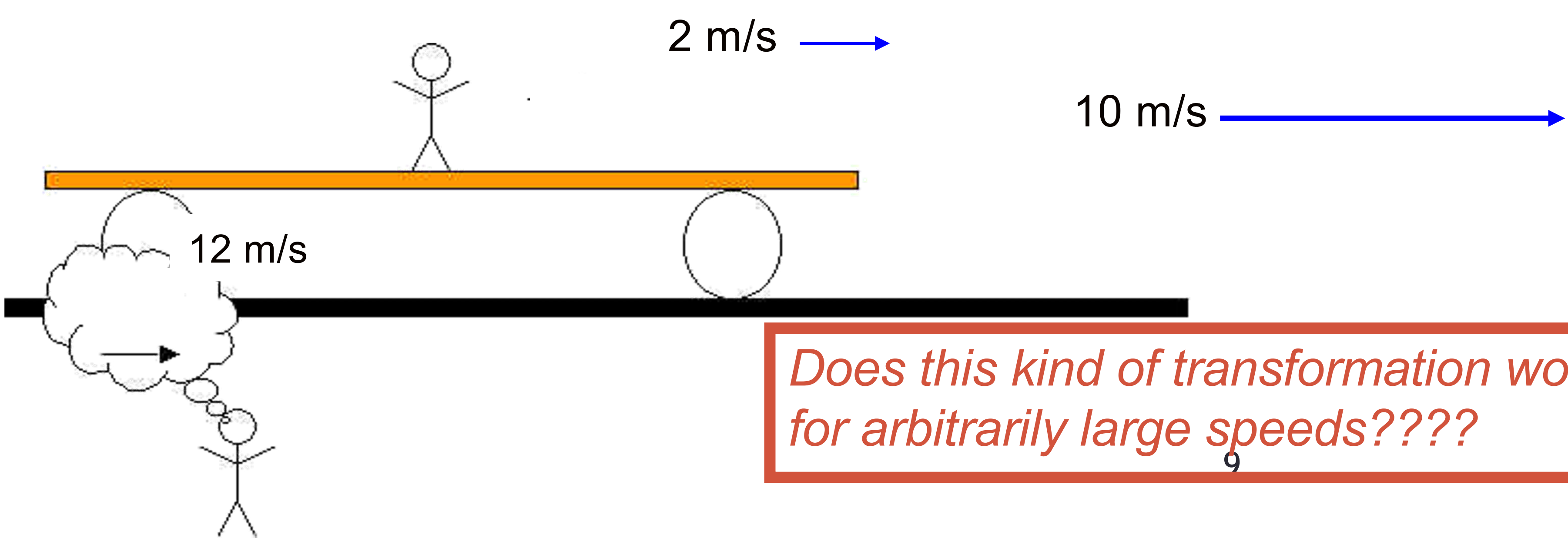
- "Relativity" refers in general to the way physical measurements made in a given inertial frame are related to measurements in another inertial frame.
  - An inertial observer is one whose rest frame is inertial (non-accelerating)
- A quantity is invariant if all inertial observers obtain the same value
- Under Galilean relativity, measurements are transformed simply by adding or subtracting the velocity difference between frames:

v_{ball}(measured on ground)=v_{train} (measured on ground)+v_{ball}(measured on train)

12 m/s = 10m/s + 2 m/s

v_{ball}(measured on train)=v_{ground}(measured on train)+ v_{ball}(measured on ground)

2 m/s = –10m/s + 12 m/s



Does this kind of transformation work for arbitrarily large speeds????

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

- Suppose you are in an *accelerating* car looking at a freely moving object (i.e., one with no forces acting on it). **You will measure** its velocity changing because you are accelerating (even though it has NO FORCE acting on it)!!!

- *In accelerating frames of reference (*a **non-inertial frame of reference)**, *N1 doesn't hold*

Real and fictitious forces

- In *non-inertial frames* you might be fooled into thinking that there were forces acting on free bodies.
- Such forces are call "fictitious forces". Examples –
  - G-forces in an accelerating vehicle.
  - Centrifugal forces in amusement park rides.
  - The Coriolis force on the Earth.
- Fictitious forces are always proportional to the inertial mass of the body.

...does this seem familiar?

Recall that from weak equivalence principle, inertial mass=gravitational mass $\Rightarrow$ *gravitational force is proportional to inertial mass.*

Maybe gravity is a fictitious force...

... and we live in an accelerating frame of reference???

Physics after Newton and Before Einstein: Electromagnetic waves

- The physics of electricity and magnetism:
  - In the late 1700's to mid 1800's scientists like Coulomb, Oerstead, Ampere, Faraday, Gauss developed ideas of how electricity and magnetism 'worked'-like Galileo, Kepler and Brahe described how bodies moved before Newton.
- These quantitative theories of electricity and magnetism were formulated in terms of *forces acting at a distance*, analogous to Newton's law of gravitation.
- The fields of electromagnetism and understanding of light (optics) were unrelated and treated as separate branches having no connection.

Electromagnetic **waves**

- James Clerk Maxwell (1831-1879)
  - Developed theory of electromagnetic fields in the 1860's (Maxwell's equations).

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} / c + (1/c)\partial \mathbf{E} / \partial t$$

Equations are just written out for fun… you do not need to know them!

Notice that a new constant "c" appears which is a velocity

B= magnetic field, **E**= electric field, **J** is the current, ρ is charge density, $\nabla\cdot$ is the divergenc
$\nabla\times$ is the curl and ∂ is the partial derivative

Solution to Maxwell's Equations

- Maxwell's Equations show that Electric and Magnetic Fields in "Free Space" - a region without charges or currents - travel at a single speed – c (which is the speed of light!)
- Known as electromagnetic radiation, these waves may occur at various wavelengths to produce a spectrum of light from radio waves to gamma rays.

(there is a nice derivation for the mathematically inclined at http://maxwells-equations.com/equations/wave.php)

- **BUT, what frame of reference is this speed measured relative to?**
  - **If light is a wave, what is waving? Thought that waves need a medium (air, water, rock, ...). How could light go through the vacuum of space? Maybe there is an “aether” that fills space?**

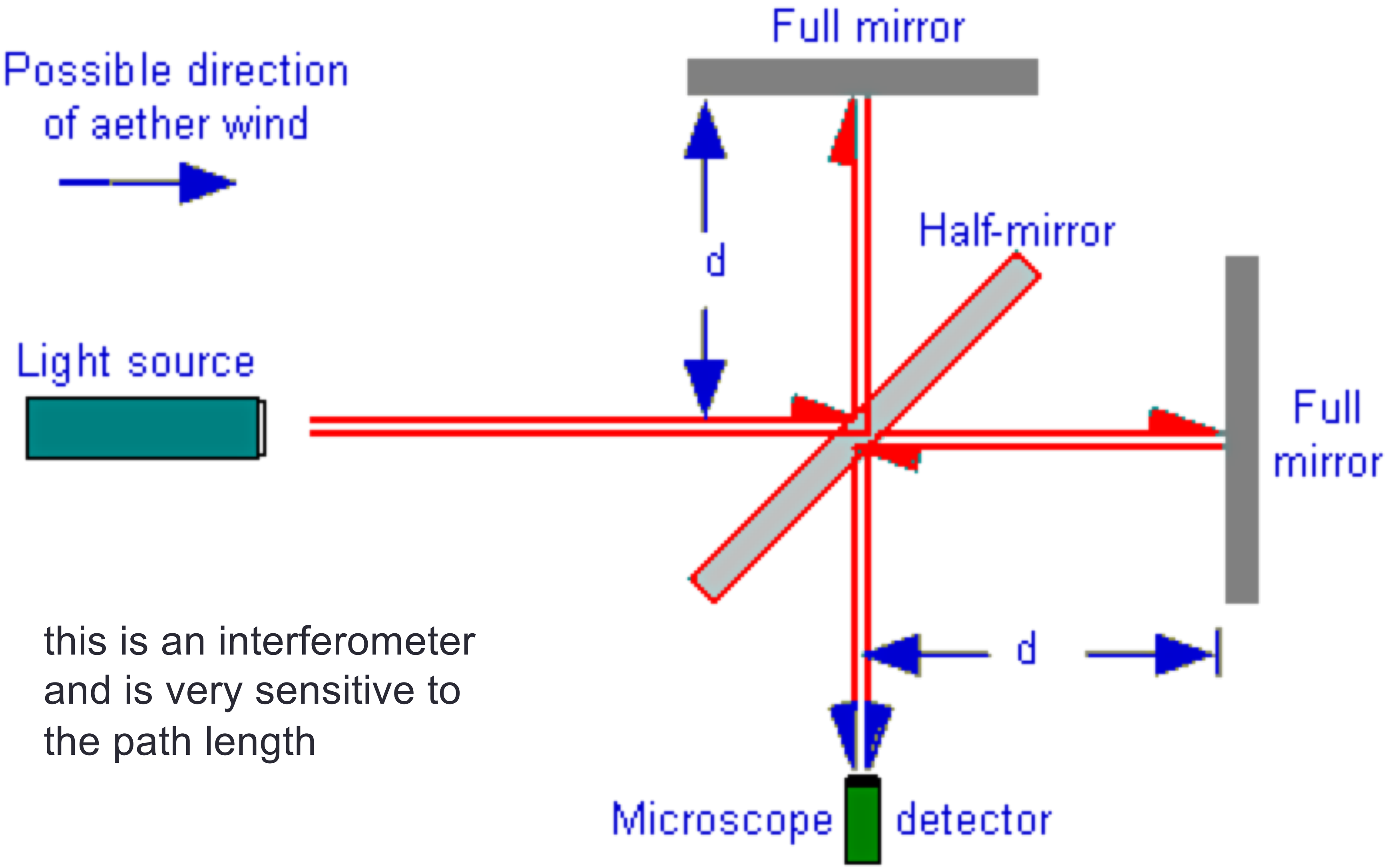
How to Measure Motion with Respect to the Aether

- As the Earth moved in its orbit around the sun, the flow of the ether should produce a detectable "ether wind".
- The speed of a beam of light emitted from a source on Earth would depend on the magnitude of the ether wind and on the direction of the beam with respect to it.

The experiment was designed to measure the speed of light in different directions in order to measure the speed of the ether relative to Earth, thus establishing its existence.

https://www.aps.org/programs/outreach/history/historicsites/michelson-morley.cfm

Michelson-Morley Expt (1887)



this is an interferometer
and is very sensitive to
the path length

Michelson-Morley results

- Repeated at different times of year, when Earth's motion differs; Earth's speed around the Sun is ~30 km/s
- Experiment performed in 1887
- Results
  - **M-M showed that speed of light was same in any direction to within 5 km/s**
  - Modern versions of the experiment show constancy to better than 1 micron/s (1 part in a 1000 trillion)
- So, what's going on???

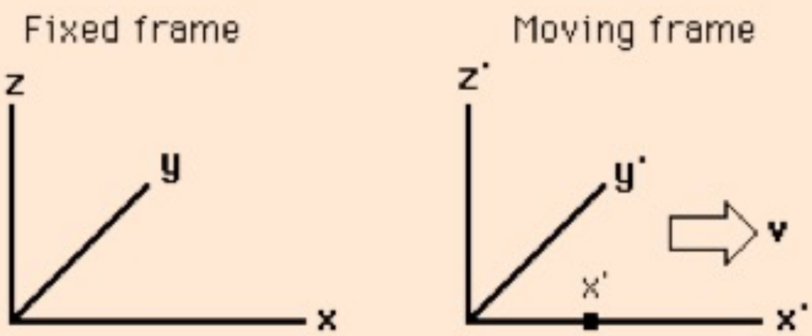
Impact of the M-M results

- Major mystery ("crisis") in 19th century physics – two highly successful theories seemed incompatible!
  - Mechanics – Galilean Relativity and Newton's laws
  - Electromagnetism – Maxwell's equations

Einstein enters the picture…

- Albert Einstein (1879-1955)
  - Knew that Maxwell's equations were invariant under "*Lorentz* transformation" of space and time a transform is the formula for the conversion of coordinates and times of events in different frames.
- But Newton's Laws are invariant under a *Galilean* transform
- Problem: if Maxwell's laws are 'correct' Newton's are not ?

Lorentz Transformation



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

The reverse transformation is:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Much of the literature of relativity uses the symbols β and γ as defined here to simplify the writing of relativistic relationships.

Evaluation of symbols

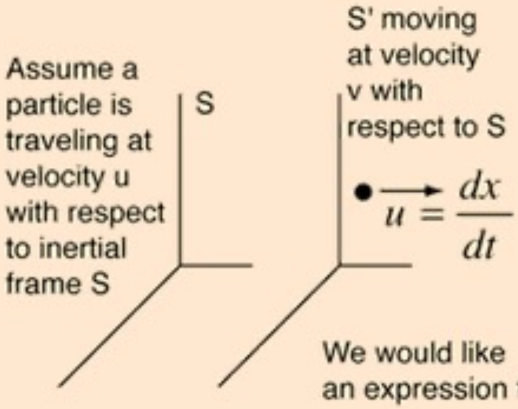
this changes the coordinates and the measurement of time (!!)

let v get very small then $\mathrm{sqrt}(1-v^2/c^2)=1$ and we get the usual t'=t and x=x'+vt

Hyperphysics web page

Relativistic Velocity Transformation

No two objects can have a relative velocity greater than c! But what if I observe a spacecraft traveling at 0.8c and it fires a projectile which it observes to be moving at 0.7c with respect to it!? Velocities must transform according to the Lorentz transformation, and that leads to a very non-intuitive result called Einstein velocity addition.



Lorentz transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We would like an expression for $u' = \frac{dx'}{dt'}$ which is the velocity as measured in the moving reference frame S'

Just taking the differentials of these quantities leads to the velocity transformation. Taking the differentials of the Lorentz transformation expressions for x' and t' above gives

$$\frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma\left(dt - \frac{vdx}{c^2}\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v\frac{dx}{dt}}{c^2}}$$

Putting this in the notation introduced in the illustration above:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

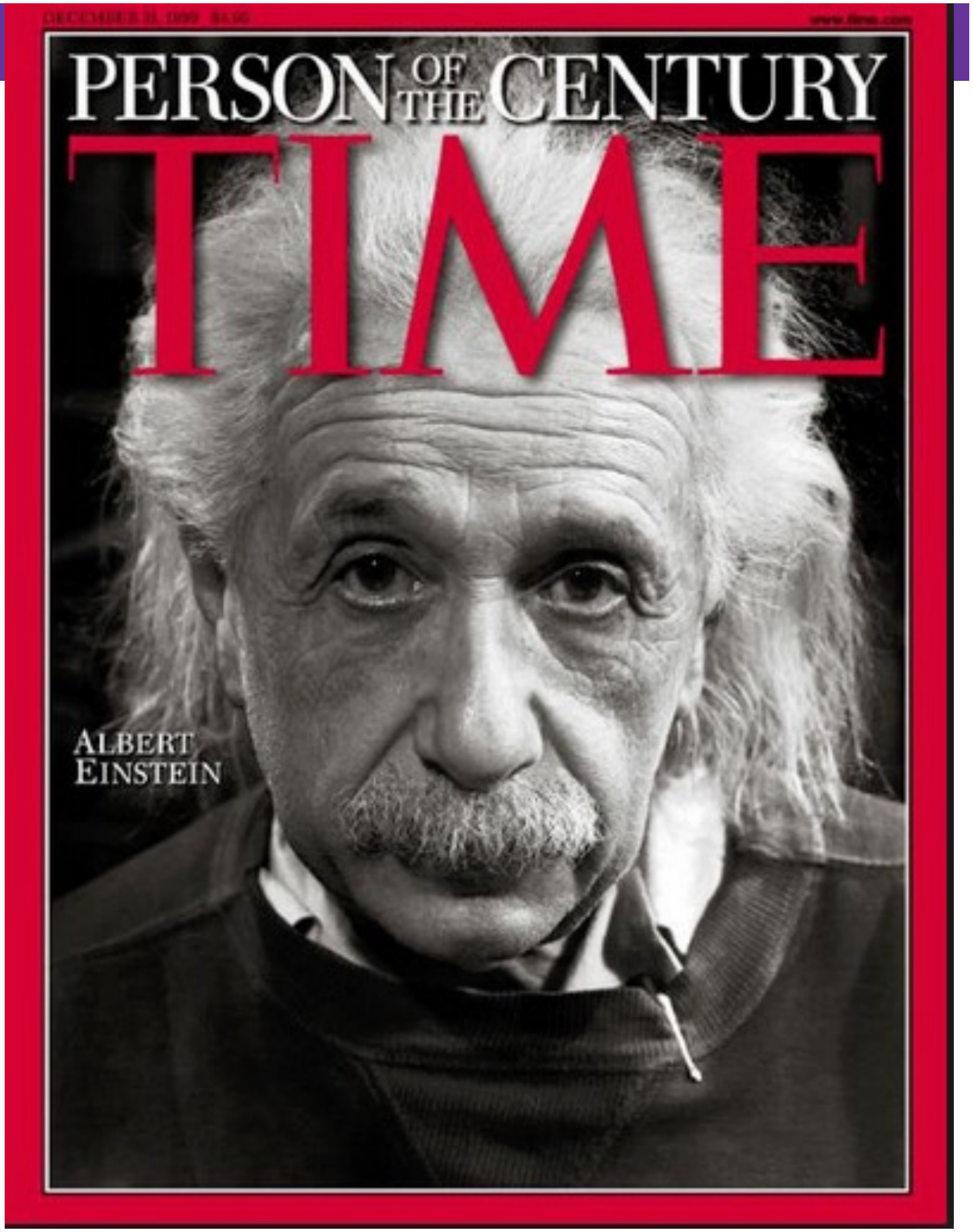
HyperPhysics © 2016C.R. Nave Georgia State University

Lorentz Velocity Addition Law

Let's let v get small... then $uv \ll c^2$ and we get the old Galilean transform $u' = u - v$ (or $u + v = u$)

Albert Einstein

- Over a 25 year period (1905-1930) came the most explosive physics ideas of the century. They set in motion a reappraisal of every premise and postulate of modern natural science, a physical revolution whose end is far from sight.
- His ideas, like Newton's and Darwin's, reverberated beyond science, influencing modern culture from painting to poetry.



II : Einstein’s Postulates of Special Relativity

- Albert Einstein
  - Didn’t like the idea of the Aether
    - Any aether would need to be massless, incompressible, entirely transparent, continuous, devoid of viscosity and nearly infinitely rigid and thus not like any known substance.

II : Einstein's Postulates of Special Relativity

- **Postulate 1** – The laws of nature are the same in all inertial* frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all inertial frames of reference.

The second postulate is necessary to allow Maxwell's equations to follow from Postulate 1

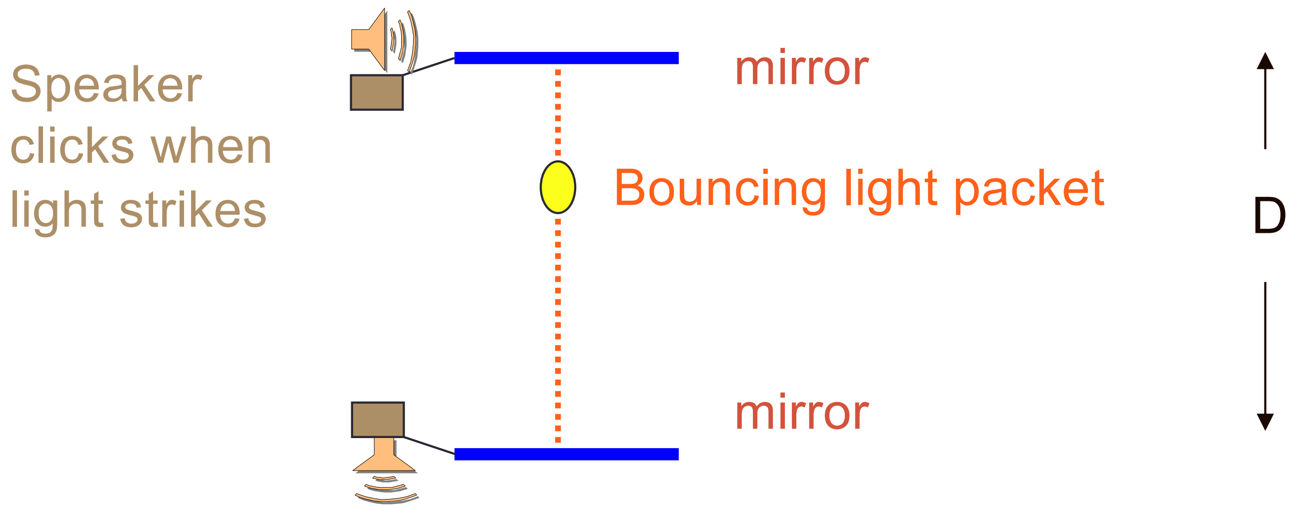
* inertial frame of reference- no acceleration (no forces on object, no rotation)

Consequences of EINSTEIN'S POSTULATES OF RELATIVITY

- Let's start to think about the consequences of these postulates.
- We will perform "thought experiments" (Gedankenexperimenten) to think of what observers moving at different speeds will think
- For now, we will ignore effect of gravity – we suppose we are performing these experiments in the middle of deep space (or, as it turns out, in free fall)

III: TIME

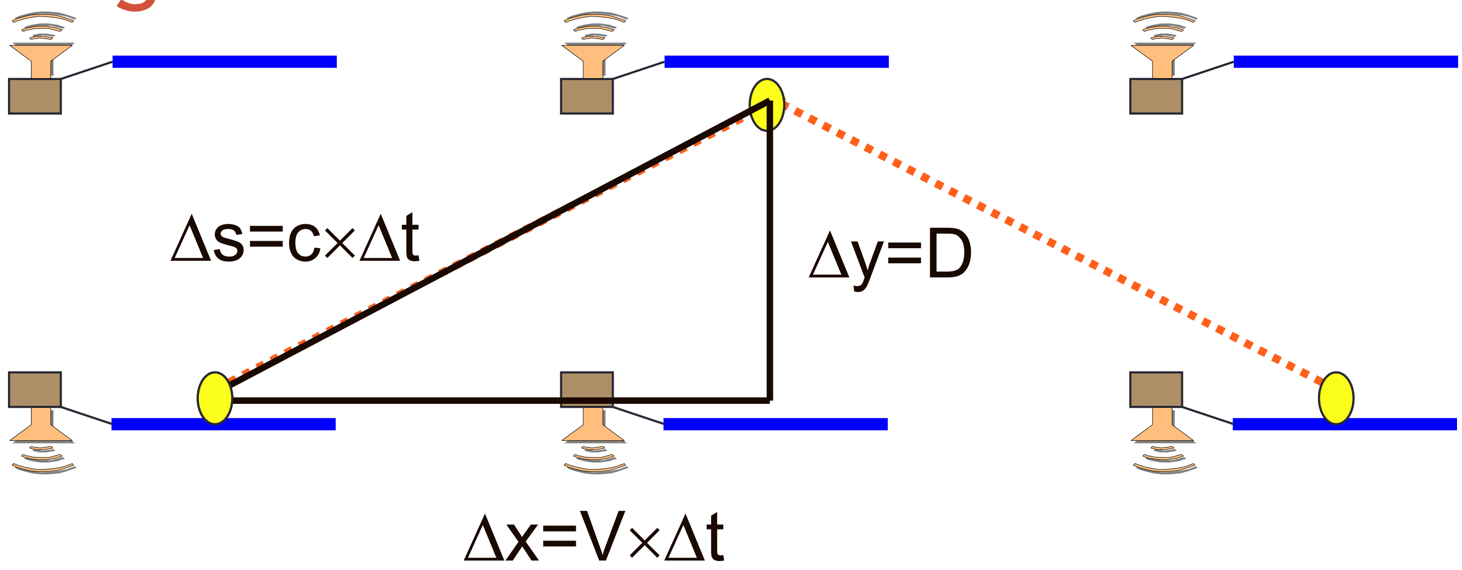
- Imagine building a clock using mirrors and a light beam.



- One “tick” of the clock is the time it takes for light to travel from one mirror to the other mirror.

$$\Delta T = \frac{D}{c}$$

Moving clock



- Now suppose we put the same "clock" on a spaceship that is cruising (at constant velocity, V) past us.
- How long will it take the clock to "tick" when we observe it in the moving spacecraft? Use Einstein's postulates…

- Total distance travelled by light beam is $\Delta s = c \times \Delta t$
- Therefore time $\Delta t = \Delta s / c$

$$\Delta s = c\Delta t = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(V\Delta t)^2 + D^2}$$

- By Pythagorean theorem,
- Can solve to obtain $\Delta t = (D/c) \div (1 - V^2/c^2)^{1/2} > D/c$
- **Clock appears to run more slowly!!**

Now change the point of view…

- For ground-based observer, clock on spaceship takes longer to "tick" than it would if it were on the ground
- But, suppose there's an astronaut in the spacecraft
  - the inside of the spacecraft is also an inertial frame of reference – Einstein's postulates apply…
  - So, the astronaut will measure a "tick" that lasts

$$\Delta T = \frac{D}{c}$$

  - This is just the same time as the "ground" observers measured for the clock their own rest frame
- So, different observers see the clock going at different speeds!

So time is not absolute!!

It depends on your point of view…

Time dilation

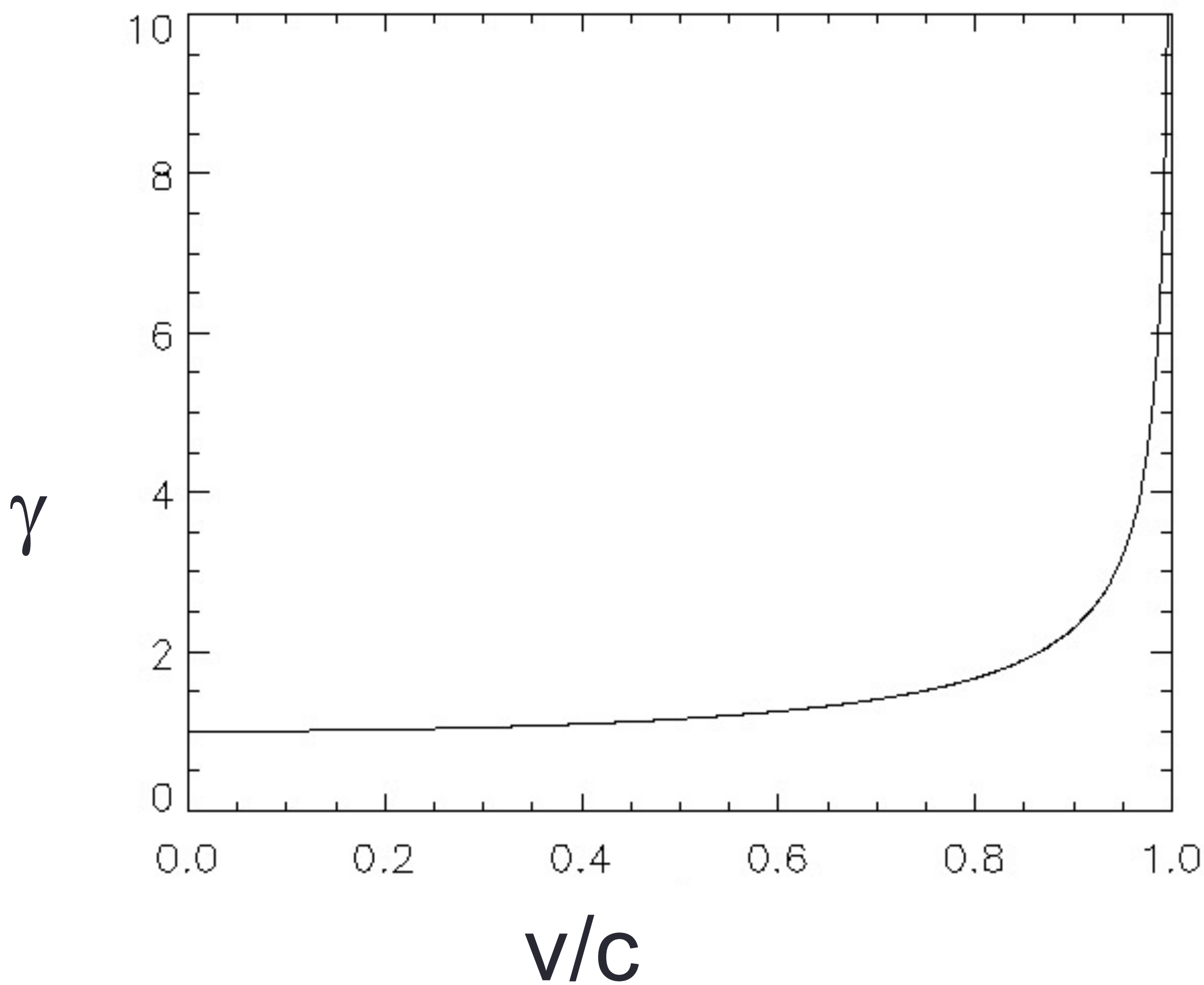
- This effect is called **Time Dilation**.
- Clock always ticks most rapidly when measured by observer in their *own* rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta T = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

- This slowing factor is called the **Lorentz factor,** γ

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Lorentz factor



A 1% effect at v = 0.14 *c*, or 42,000 km/s

Apparent length of a moving thing is contracted, by the exact same Lorentz factor

Lorentz factor goes to infinity when V→c!

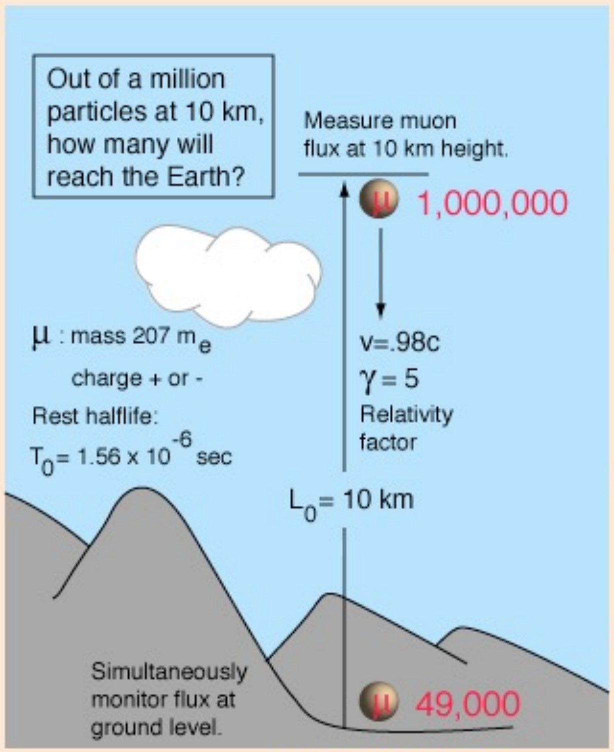
But it is very close to 1 for V/c small; most of our experience!

Do Frequent Fliers Age More Slowly?

- https://scienceline.org/2010/10/do-frequent-fliers-age-more-slowly/
- For small velocities at which the relativity factor is very close to 1, then the time dilation can be expanded in a binomial expansion to get the approximate expression:
- $\gamma = (1+v^2/2c^2)$; so for 200m/s (speed of a jet plane) the difference in time is only $2.2\text{x}10^{-7}$ or 7 sec per year of flying

Muon Experiment

Relativistic, Muon-Frame Observer



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{2000 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 6.8 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$\dfrac{I}{I_0} = 2^{-4.36} = 0.049$

Or about 49,000 out of a million.

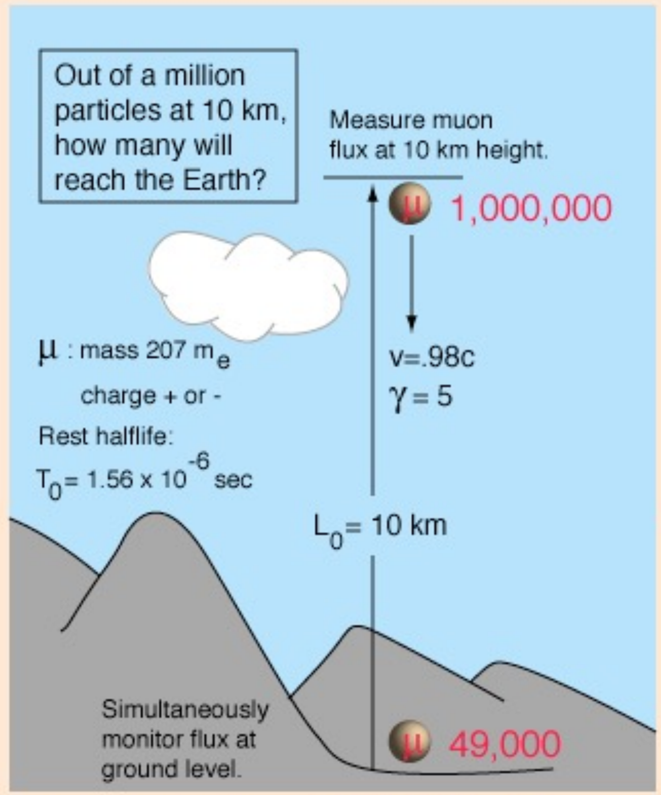
The muon sees distance as length-contracted so that $L = L_0/\gamma = 0.2L_0$ = 2 km.

☐ Non-relativistic ☐ Relativistic, Earth observer ☑ Relativistic, muon observer

☐ Comparison ☐ Comments on comparison ☐ Vary parameters

Muon Experiment

Relativistic, Earth-Frame Observer



Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4 \text{ m}}{(0.98)(3 \times 10^8 \text{ m/s})}$

$T = 34 \times 10^{-6}$ s = 4.36 halflives

Survival rate:

$\dfrac{I}{I_0} = 2^{-4.36} = 0.049$

Or about 49,000 out of a million.

The muon's clock is time-dilated, or running slow by the factor $T = \gamma T_0$, so its measured halflife is 5 x 1.56μs = 7.8μs.

☐ Non-relativistic ☑ Relativistic, Earth observer ☐ Relativistic, muon observer

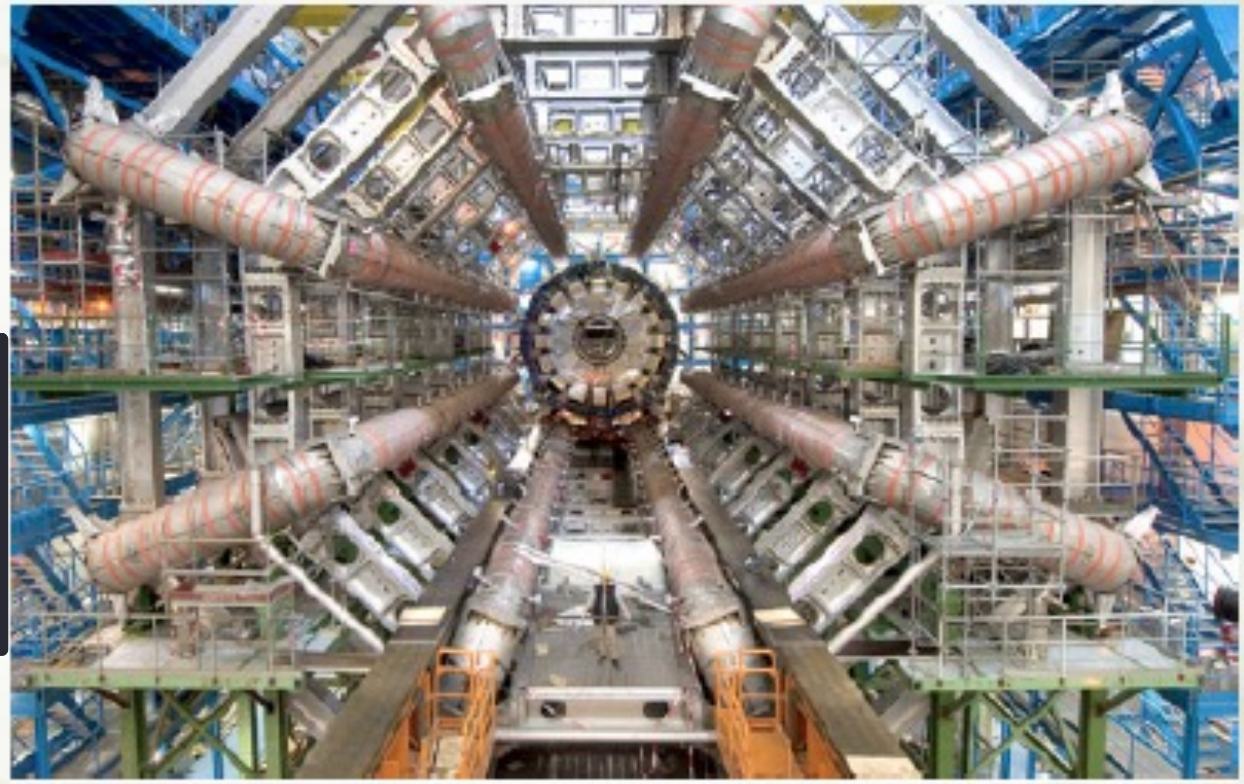
☐ Comparison ☐ Comments on comparison ☐ Vary parameters

LHC

- When the power of this machine is discussed, the energy of each proton is often mentioned: The protons each have an energy of 7 TeV. What does that mean?
- With E = 7 TeV: $E=\gamma mc^2$

the Lorentz factor has a value of about 7460 corresponding to v = 0.999999991 times the speed of light

- time passes 7460 times more slowly for the particles than it does for us observers.



A clock traveling at that speed from Earth to Proxima Centauri would measure a journey time of under 5 hours, while an observer who would remain on Earth would have aged over 4 years (Proxima Centauri is about 4.243 light-years away from us).

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

- Newton's laws are clearly powerful. But they also led to some puzzles, particularly relating to **reference frames.**

- We have already come across idea of **frames of reference that move with constant velocity**. In such frames, Newton's laws (esp. N1-**Newton's 1st law**

($\underline{V}$ = *constant* if $\underline{F}$ = *0*) hold. These are called **inertial frames of reference**.

Fictitious Forces- Examples

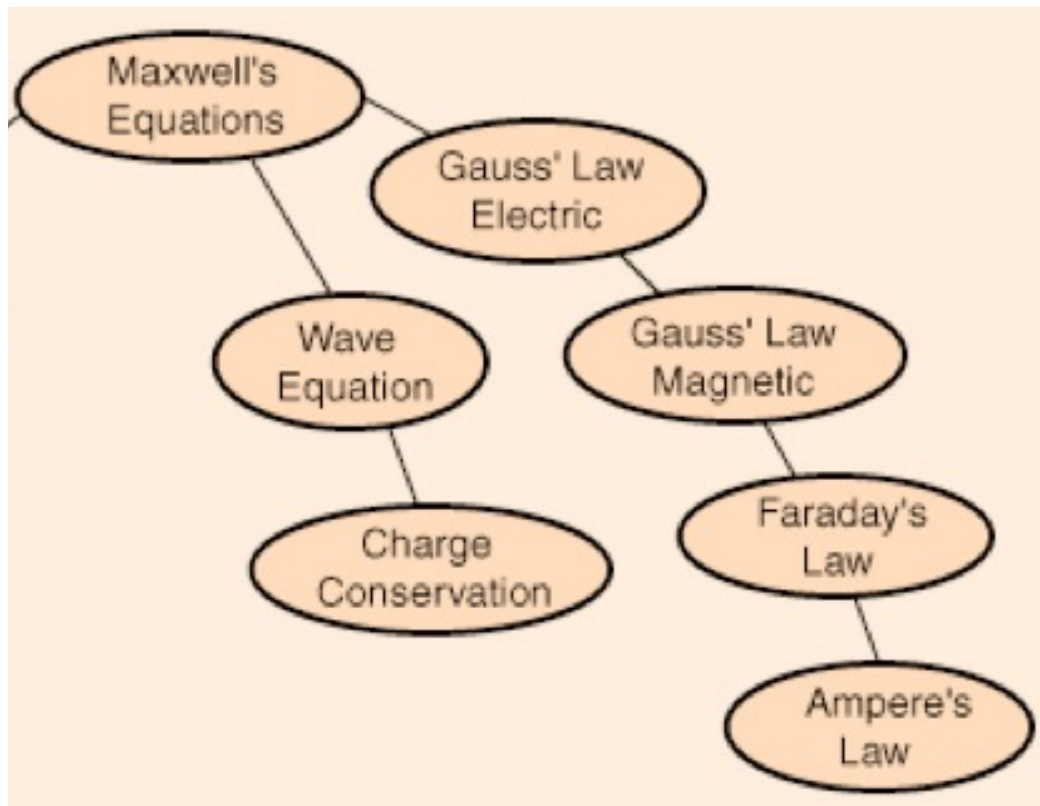
- In a passenger vehicle that is *accelerating* in the forward direction – the passengers perceive that they are acted upon by a force in the rearward direction pushing them back into their seats.
- An example in a *rotating reference frame* is the force that appears to push objects outwards towards the rim of a centrifuge. These apparent forces are examples of fictitious forces.

A little bit more detail?

- Maxwell's equations showed the waves speed is equal to the speed of light **from a purely theoretical argument**
- Thus light is an electromagnetic ***wave*** (and that there must be other such waves with different wavelengths).
- characterized by a wavelength (λ), a frequency (ν) and a speed (c). It carries energy E.

Maxwell's equations are central to an understanding of electricity, magnetism and light. but require a level of mathematical sophistication to understand

The speed of light is related to 2 constants that come out of the laws describing electric and magnetic forces



Maxwell's equations:

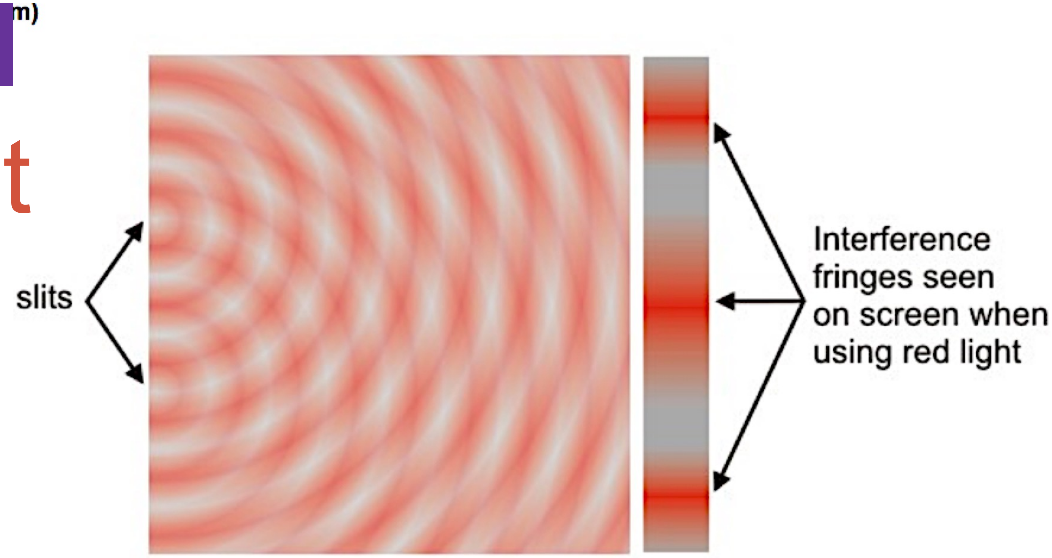
- Basic laws of physics governing electricity and magnetism
- Solutions are waves of electromagnetic energy – quickly realized that these were light waves!
- *The speed of light "c" appears as a fundamental constant in the equations* same in all frames (But what about Galilean transform ??)
  - c=299,792,458 km/s (1.0 feet/nanosecond).
  - According to Maxwell's equations, the speed of light in a vacuum is a universal constant. This violates Galilean invariance, a long-standing cornerstone of classical mechanics.

- **BUT, what frame of reference is this speed measured relative to?**
- **If Light is a Wave, What is Waving?**

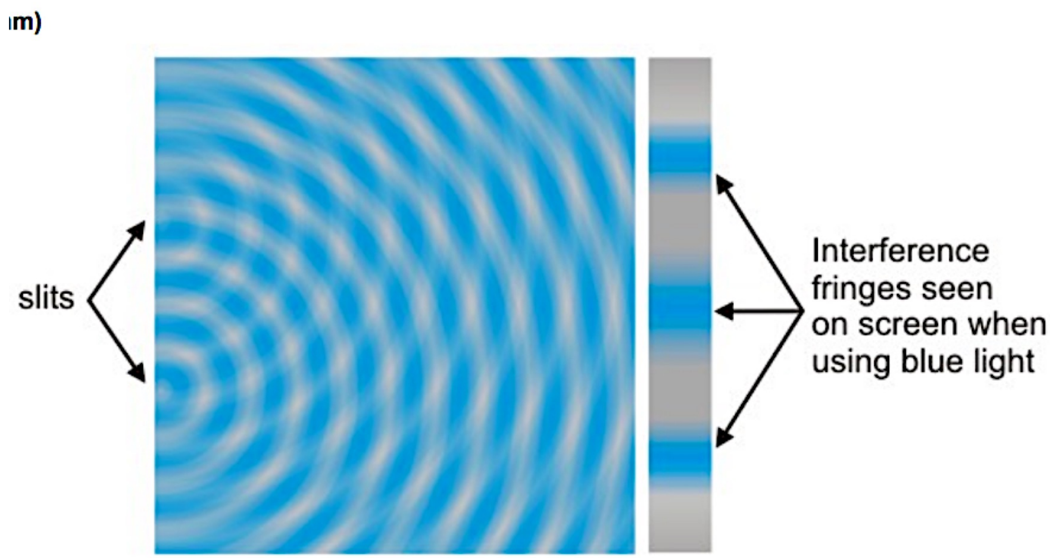
Basic Problem of How does Light Propagate

- According to Maxwell's equations light is a wave
- In the 19th century it was known that waves need a medium to move in (sound waves, ocean waves, earthquake waves, etc.)
  - sound waves are compressional waves in air.
- If light is a wave, just what is waving? It clearly isn't air, because light reaches us from the sun, and stars, and we know the air doesn't stretch that far

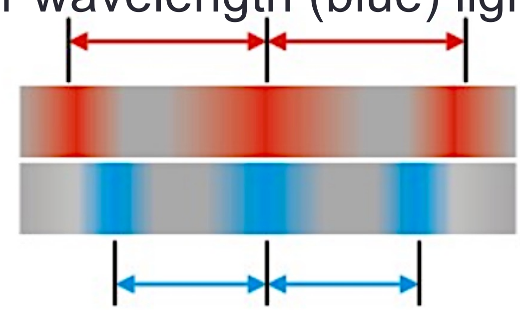
Proof that Light is a Wave



Interference pattern



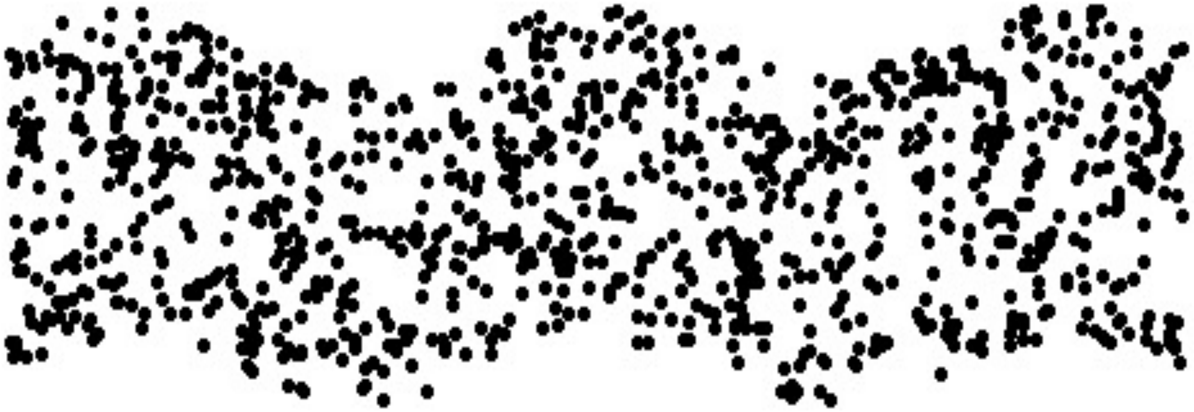
fringe spacing is smaller for shorter wavelength (blue) light



https://www.saburchill.com/physics/chapters2/0015.html

Waves

- Before Maxwell all waves (sound, pressure waves in earthquakes etc.) were known to be disturbances in the substance (water, air) through which they traveled
- sound waves propagate via air molecules bumping into their neighbors



http://www.kettering.edu/physics/drussell/Demos/waves/wavemotion.html

- Luminiferous Aether* (19th century)
  - *Hypothetical* substance that fills space – provides "medium" through which light travels.
  - Was presumed that "c" should be measured with respect to the rest frame of the Ether.
  - This would explain why the speed of wave propagation "c" is a constant in the equations

- Albert Michelson & Edward Morley attempted to measure motion of Earth through aether…Is the speed of light the same in all directions in a moving reference frame?

  serious problem with aether- invisible and infinite material with no interaction with physical objects.

  *"luminiferous", means "light-bearing"- invoked to explain the ability of the light wave to propagate through empty space, something that waves should not be able to do.