Class 5 : Special Relativity II

ASTR350 Black Holes (Spring 2022)

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This class

- More time dilation
- Length contraction
- Relativity of simultaneity
- Causality
- New velocity addition law
- Mass and Energy

Muddiest points

Any astro questions?

RECAP

- Maxwell's equations say that speed of light is the same in any (inertial) frame of reference… experimentally verified by Michelson-Morley experiment
- Einstein's postulates
  - Laws of physics same in any inertial frame
  - Speed of light same in any inertial frame
- Time dilation is one of the consequences
  - Moving clocks run more slowly as speed increases

How Do We Know That These Strange Predictions are True???

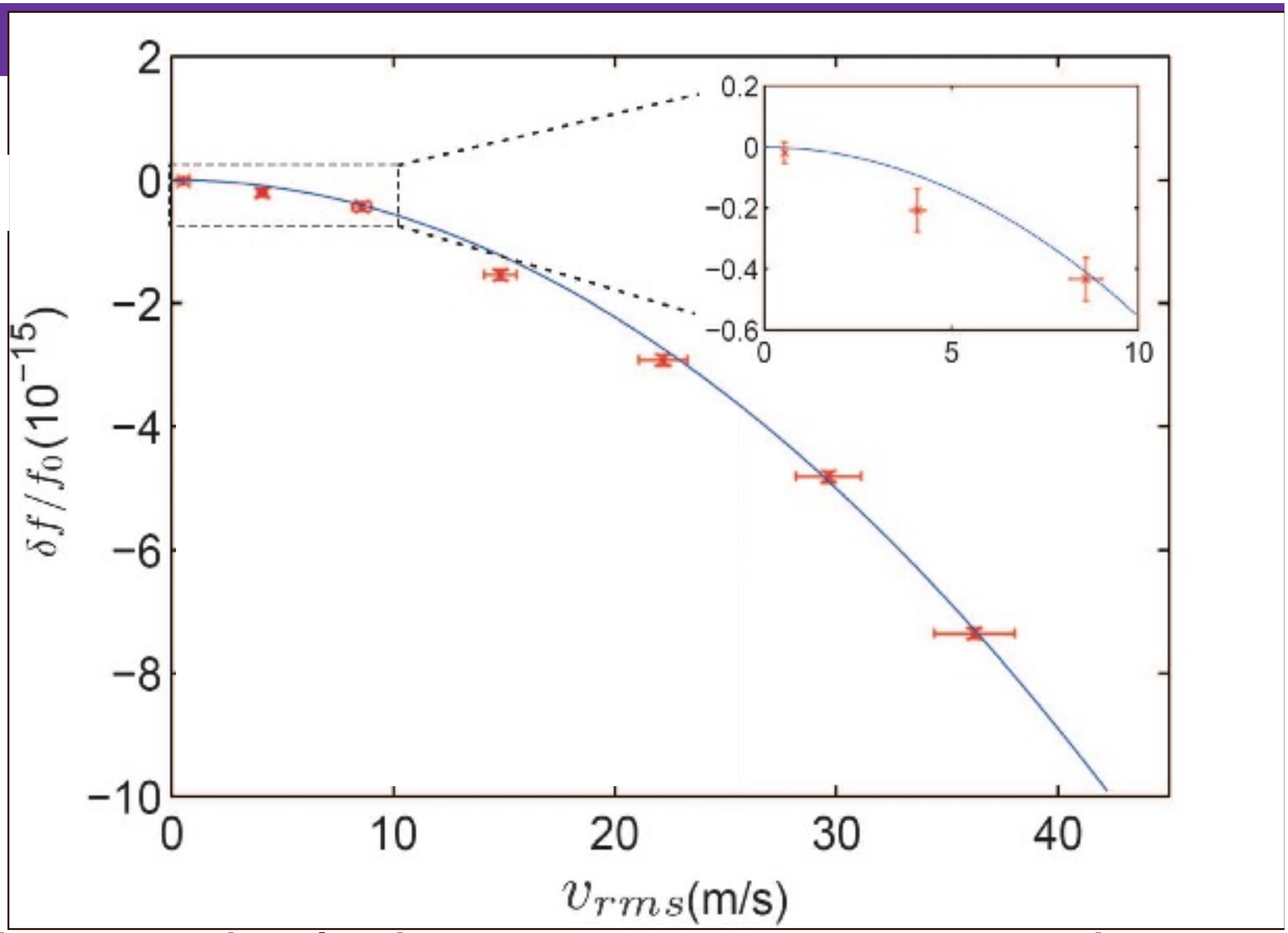
Experiments and measurement

look at the predictions and perform precise experiments
and measure them accurately that test these predictions

- If the predictions and the measurements agree the theory passes
- The more accurate the measurements the stronger the test
- As technology advances stronger and stronger tests are possible –sometimes the theory now fails the stronger tests and changes are required

Time Dilation at Normal Speeds

time dilation in units of 1 part in 10^{15}



Using an ultra-precise atomic clock one can measure even tiny amounts of time difference.

Comparison of observed (red) vs. predicted time dilation at normal speeds. Predictions of special relativity confirmed (Science V329 pg 1631-2010

10m/s=22 mph

Conclusion

- To the best of our ability to measure, time dilation is real and has the value calculated by Einstein
- A strong test of his theory given the level of precision of the tests.
    - Special relativity agrees with results better than Galilean relativity.

Muons … again!

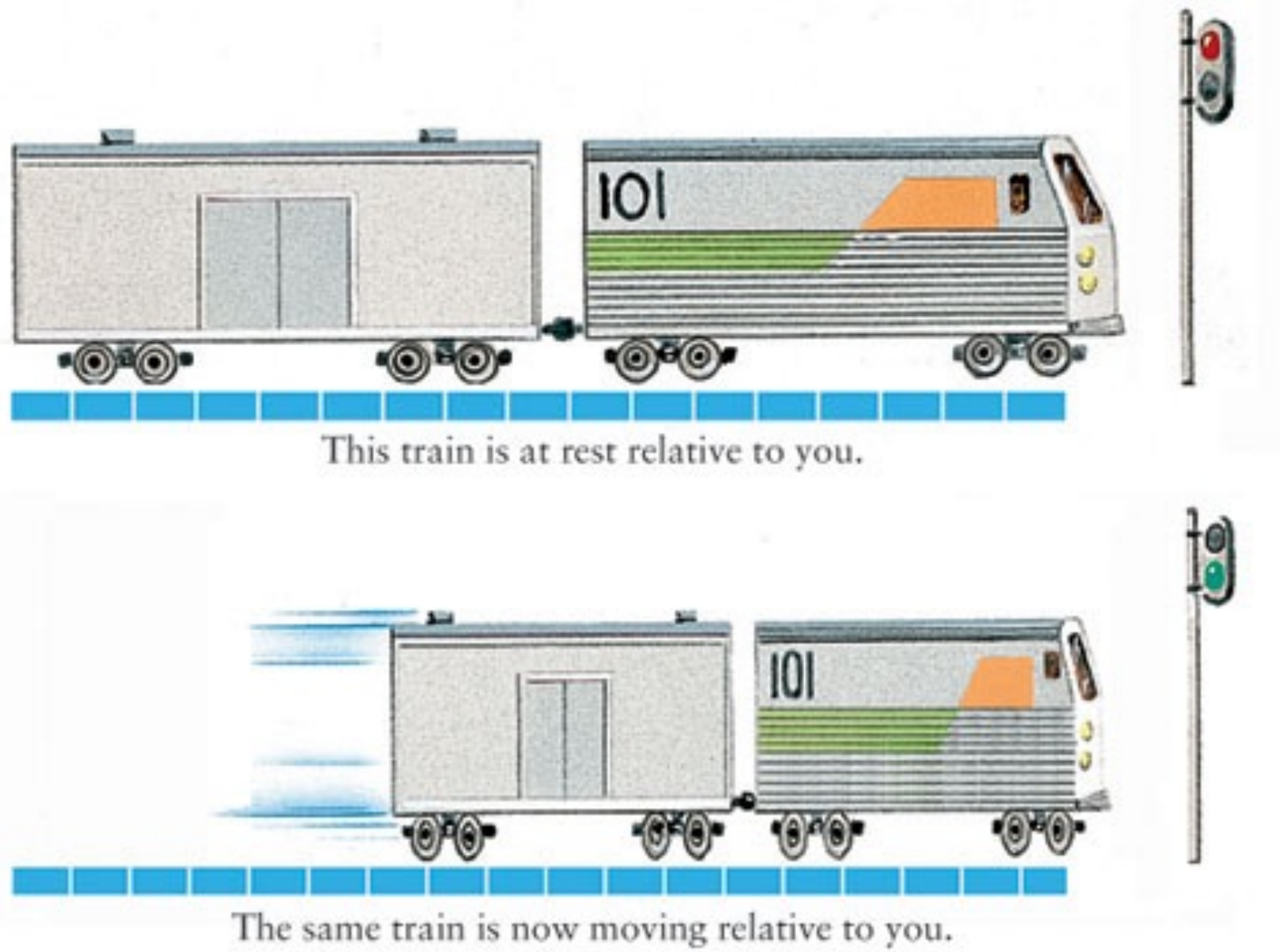
- Consider atmospheric muons again, *this time from point of view of the muons* i.e. think in frame of reference in which muon is at rest
- Decay time in this frame is 1.6 μs (1.6/1,000,000 s)

How do they get from top of the atmosphere to sea level before decaying?

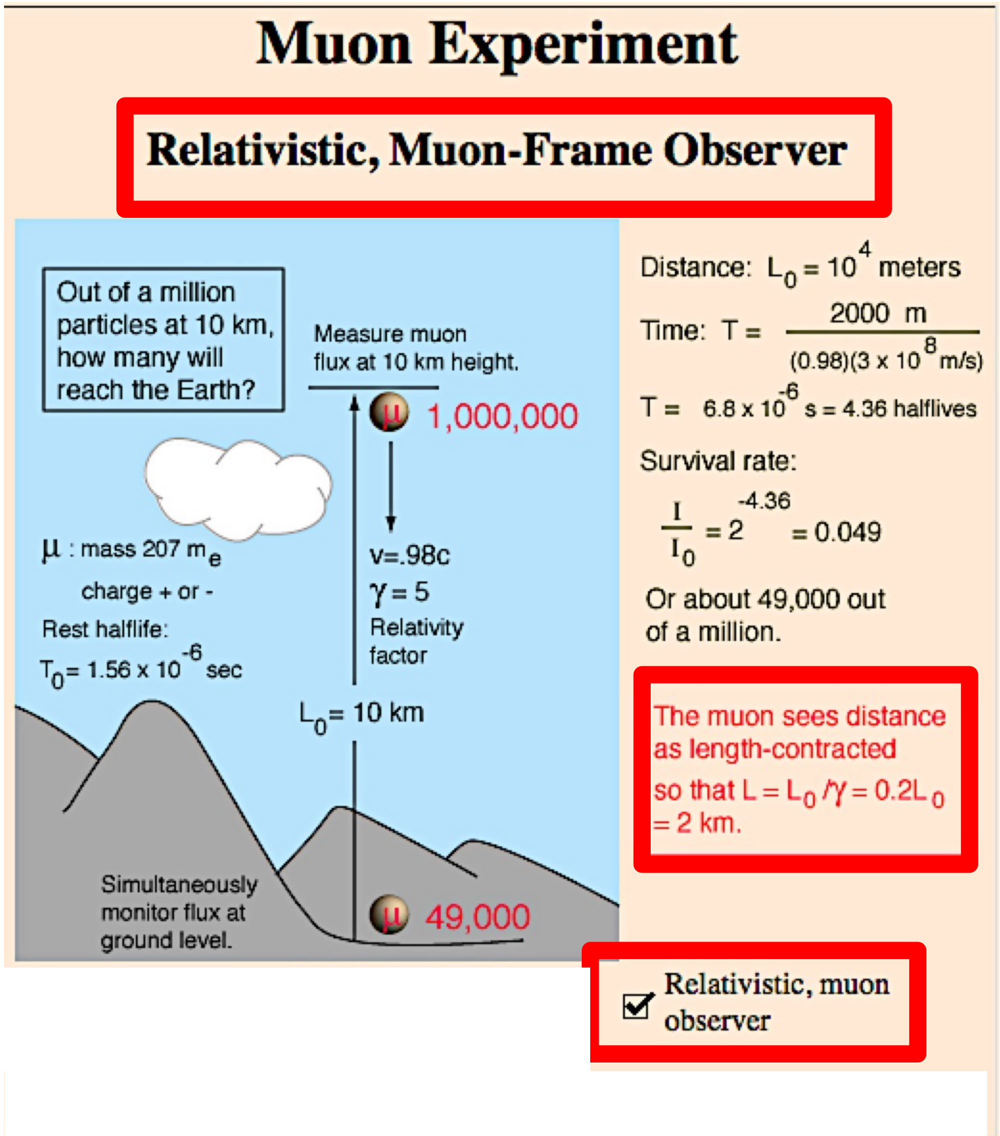
- **From point of view of muon, the atmosphere's height *contracts* by a factor of γ**
- Muons can then travel the reduced distance (at almost speed of light) before decaying.

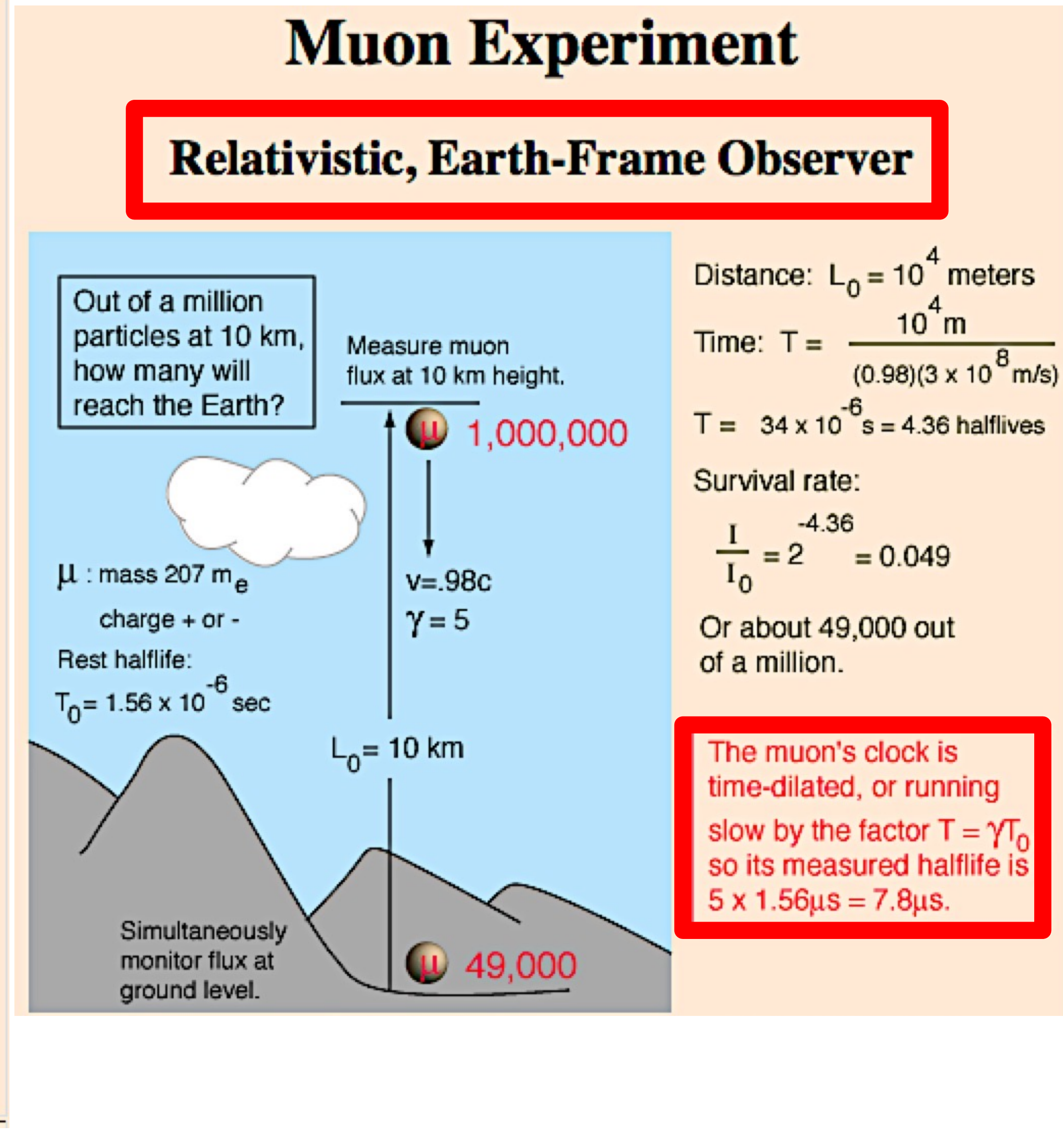
Note that there is *no contraction of lengths* that are **perpendicular** to the direction of motion

FITZGERALD Length CONTRACTION



(a) Length contraction





These are equivalent representations of reality

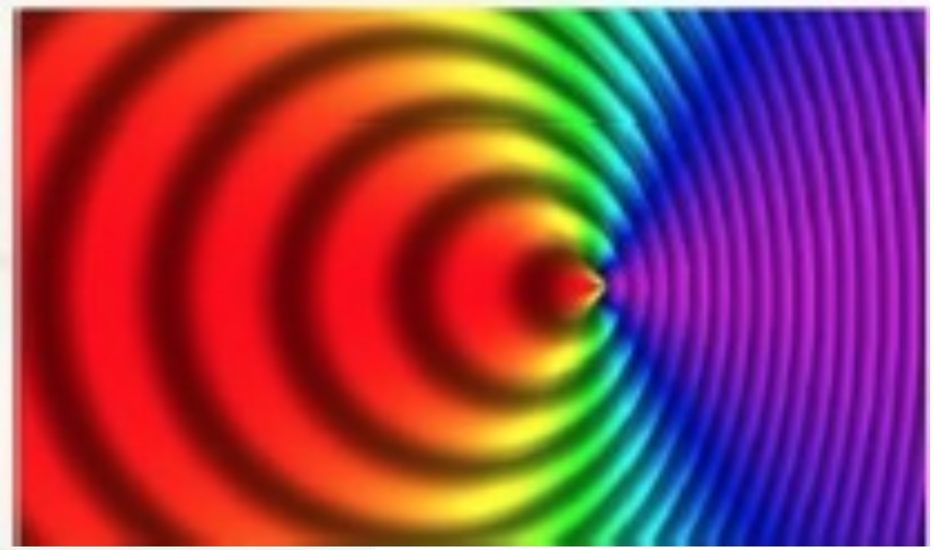
"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

Doppler shift

$\nu_{obs}=\nu_{emit}*\mathrm{sqrt}((1+v/c)/(1-v/c))$ for the observer in front of the source and

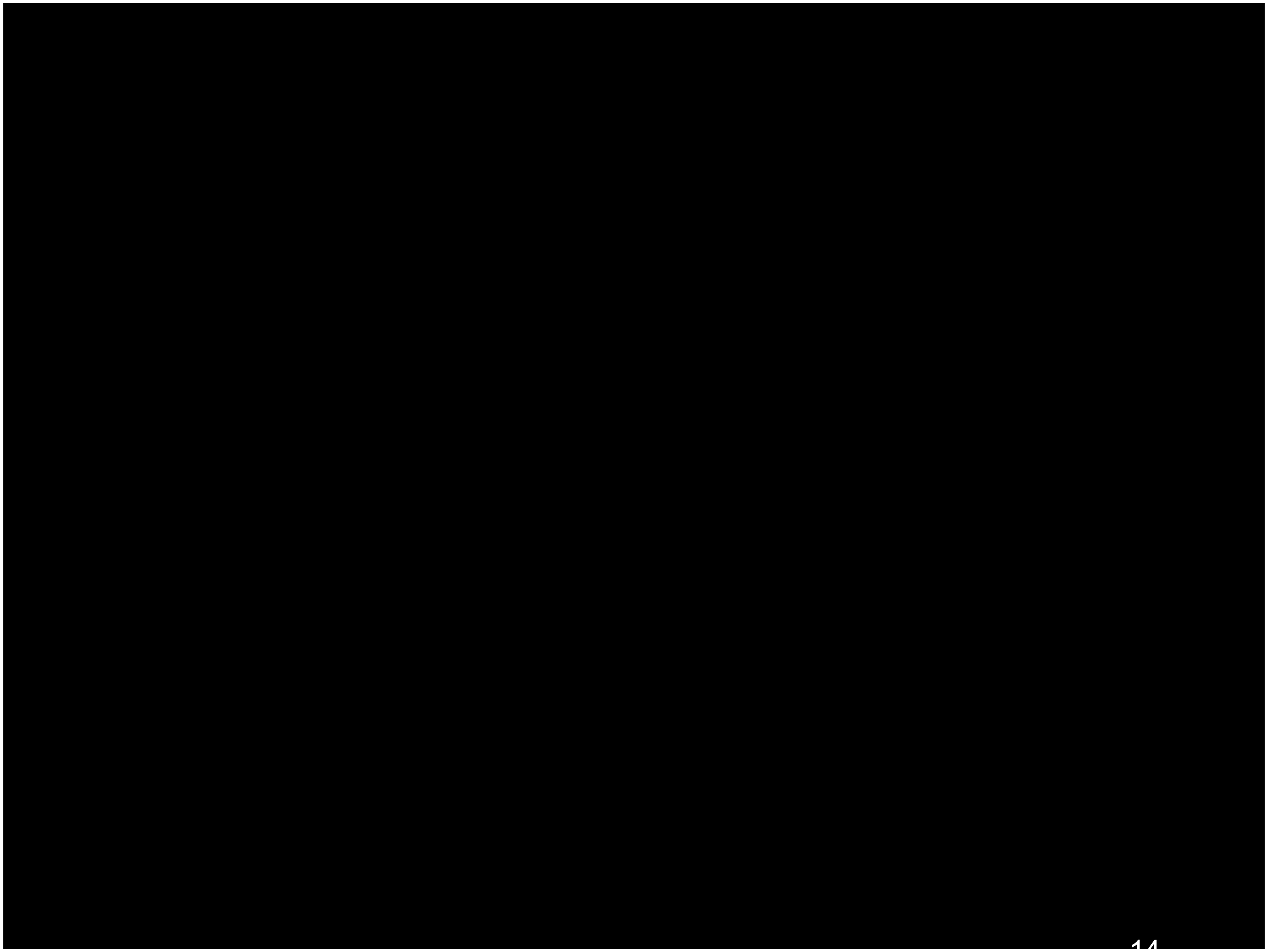
$\nu_{obs}=\nu_{emit}*\mathrm{sqrt}((1-v/c)/(1+v/c))$ for the observer in back

result is due to contraction of length (change in wavelength) or time dilation (change in frequency) of the wave



Observer in front sees contraction one in back expansion

II: Relativity of Simultaneity

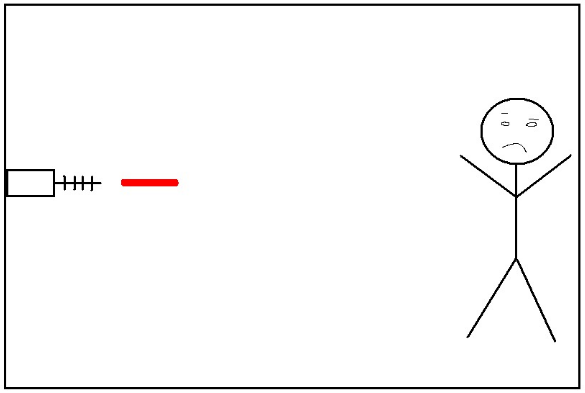


Simultaneity

- Time can be different for two observers
- whether two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame.
- reference frame that is moving relative to the first will generally assign different times to the two events

The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.

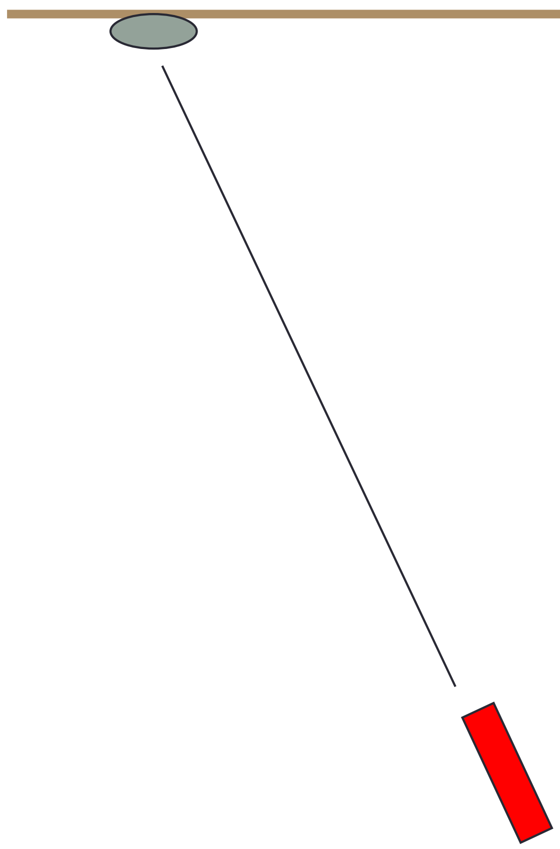


- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

- This is a question of **causality**.
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- It is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.

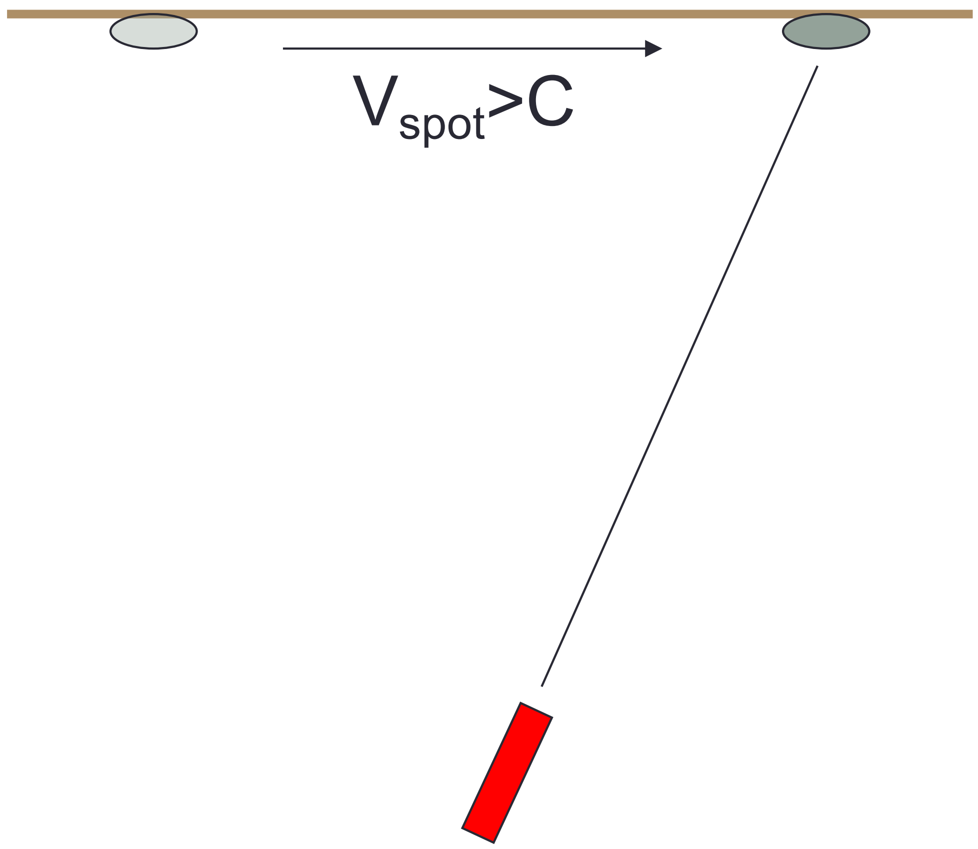
- This is true provided that
  - The laser blast does not travel faster than the speed of light
  - We do not change to a frame of reference that is going faster than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe. Anything? Well, **anything that transmits any information**.

Faster than light ?



- E.g., light spot on a distant screen

Faster than light ?

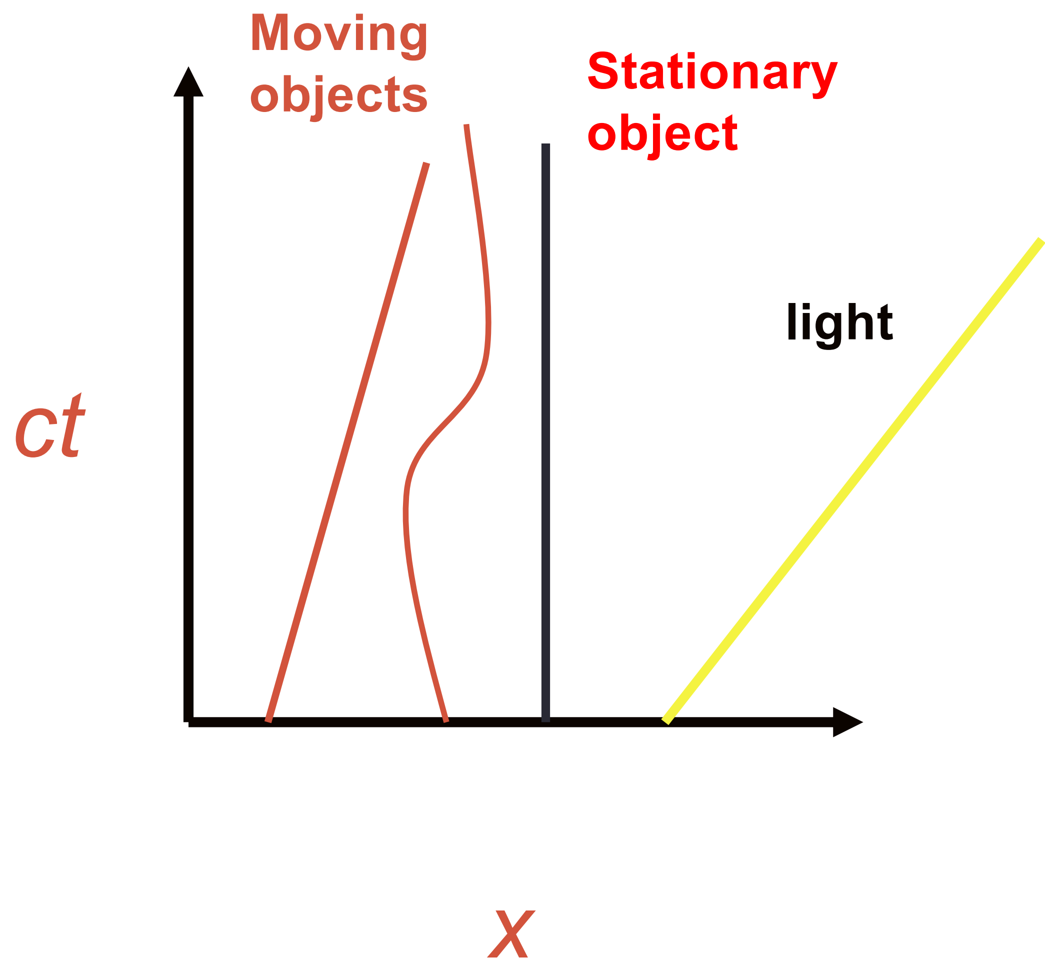


- E.g., light spot on a distant screen
- But no information or energy travels at this speed

III Space-time diagrams

- Because space and time are "mixed up" in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.

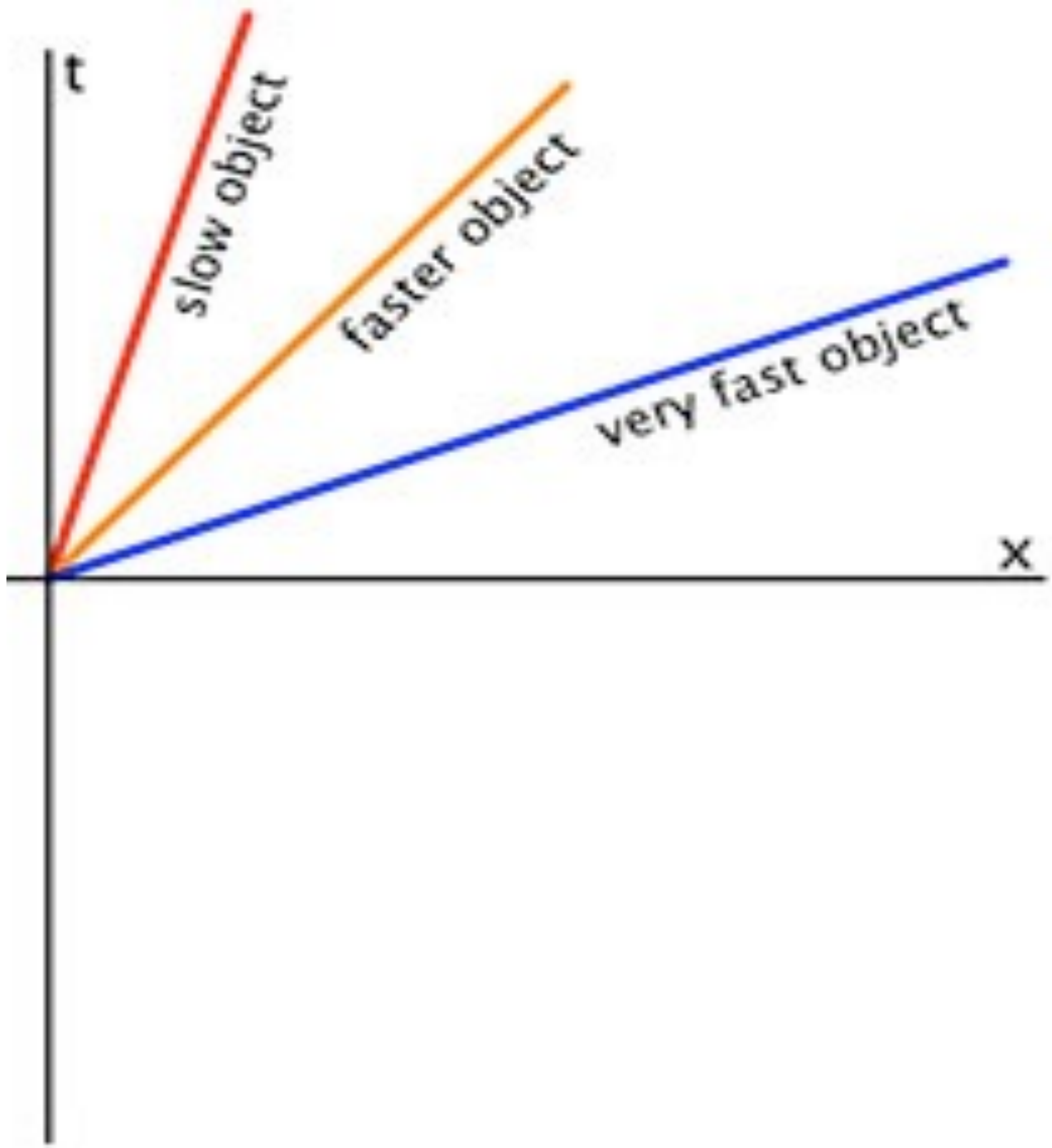
- This is simplest to do for events that take place along a line in space (*one-dimensional space*)
  - Plot as a 2D graph
  - use two coordinates: ***x*** **and** ***ct (same units of distance)***



World lines of events

Space-time diagrams

- If moving at constant velocity, *slope* of line is how fast you are moving (maximum is speed of light)
- *Notice units- length is ct*



Space Time Diagrams

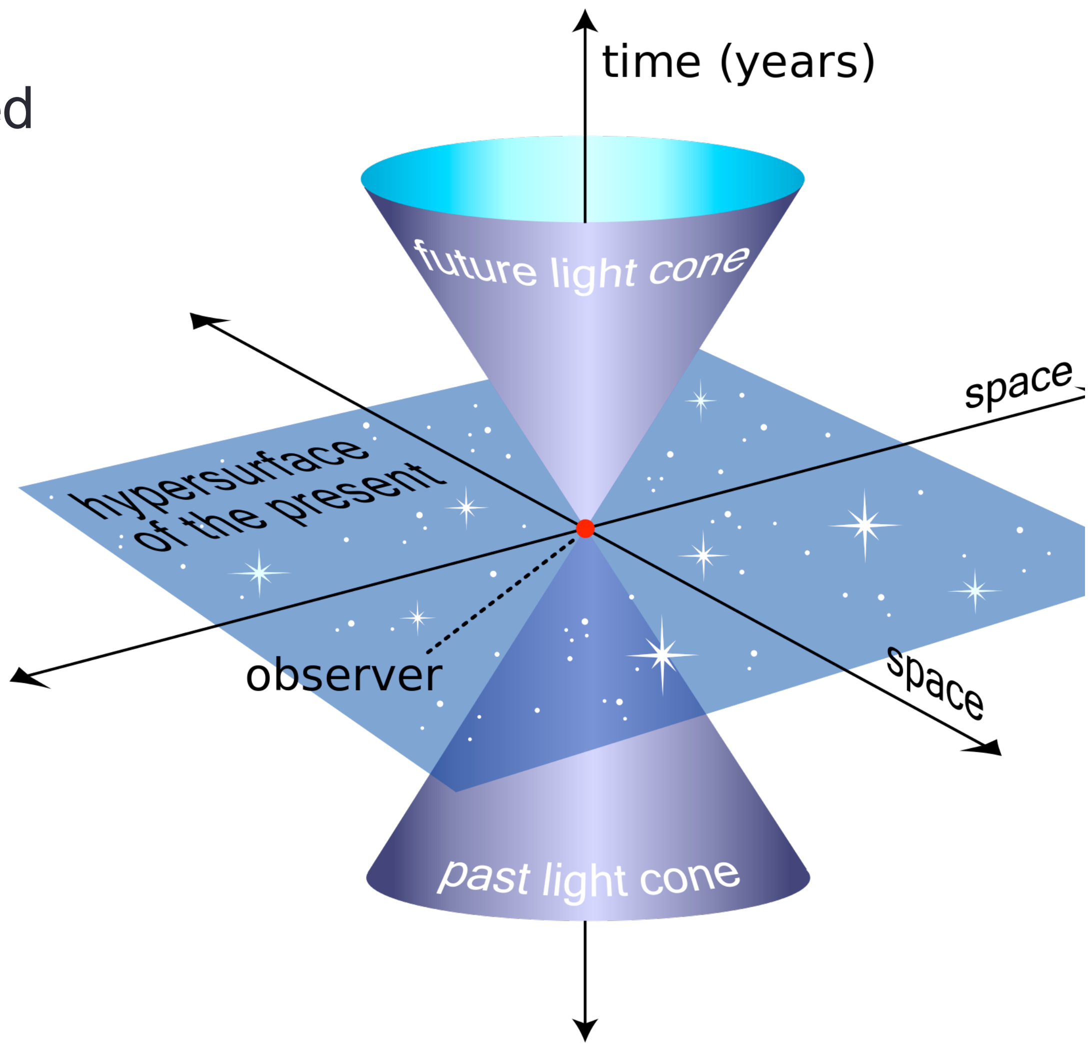
Can't get there from here

the light cone

The **future** of the given event is formed by all events that can be reached traveling slower than light

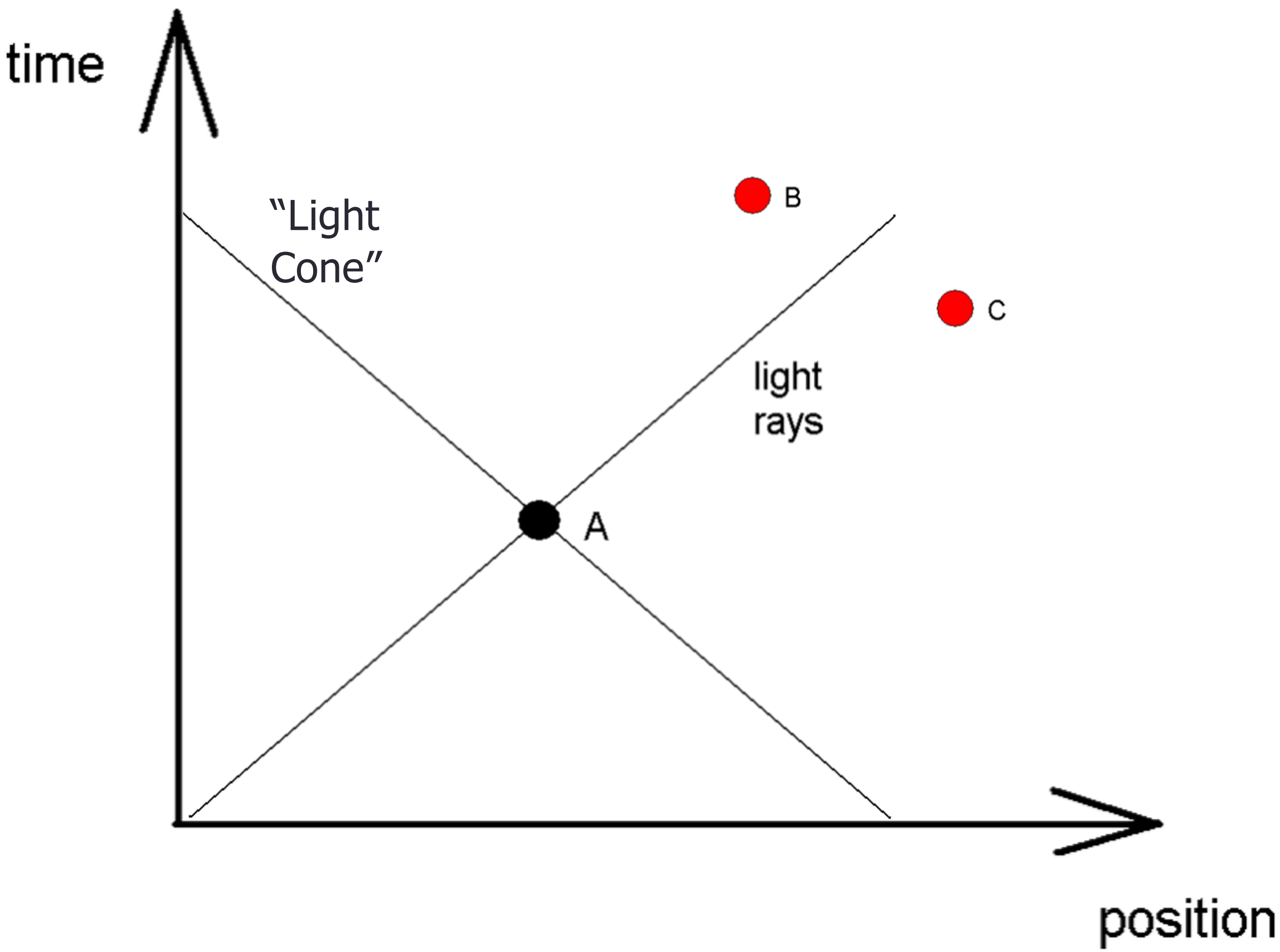
The **past** of the given event is formed by all events that can influence the event

The **light cone** at the given event is formed by all events that can be connected through light rays with the event. When we observe the sky at night, we basically see only the past light cone.

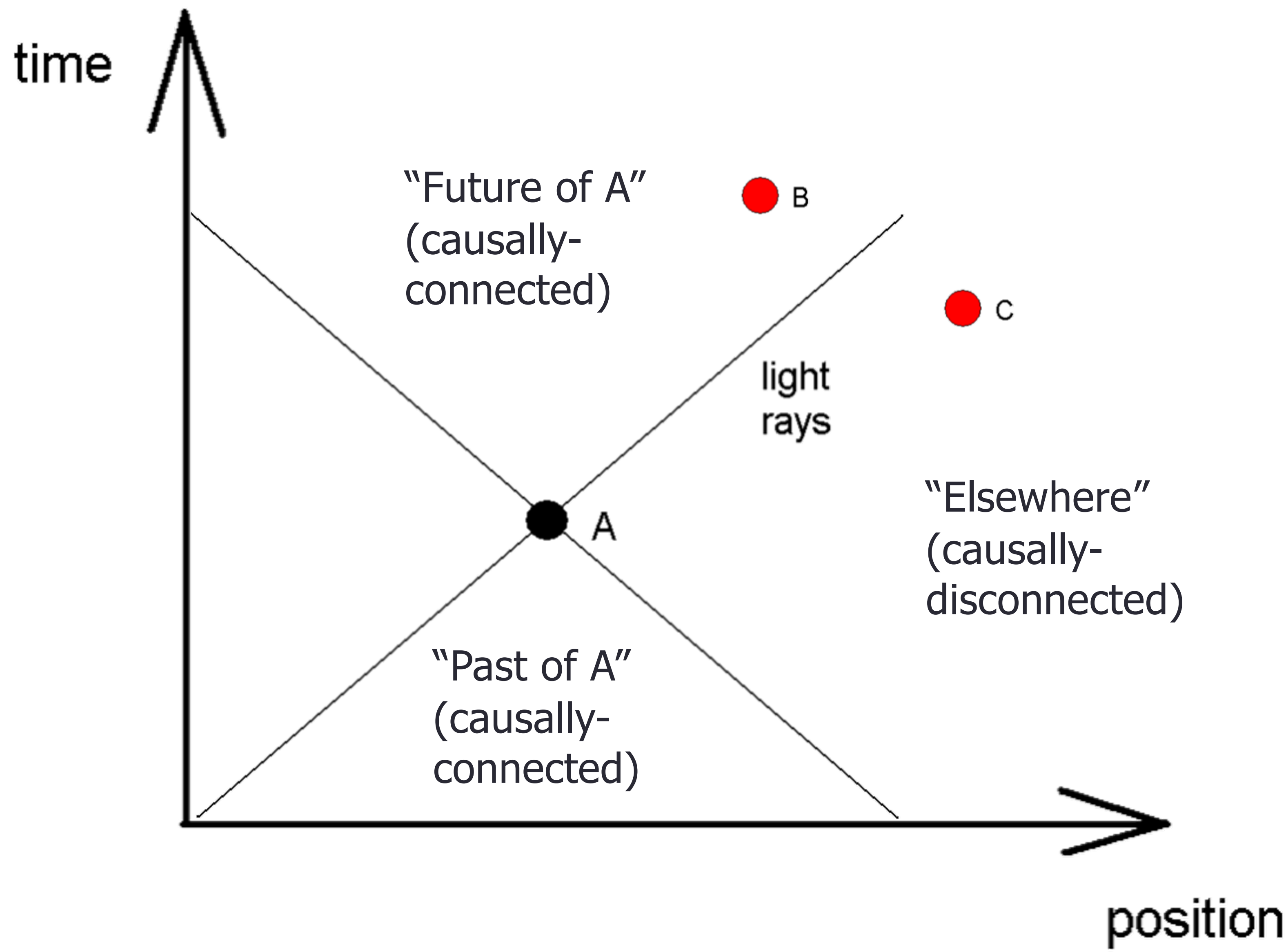


https://en.wikipedia.org/wiki/World_line#/media/File:World_line2.svg

Light cone for event “A”

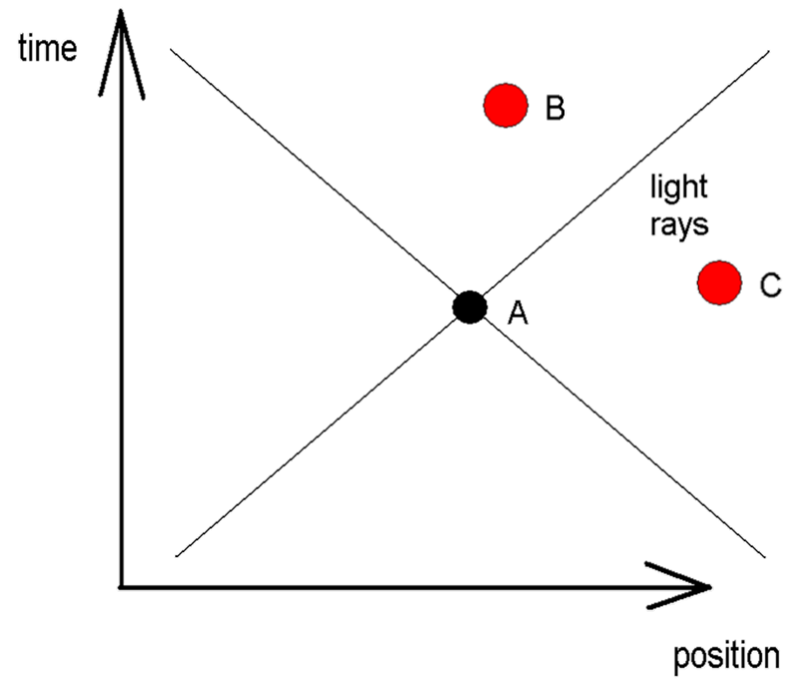


Past, future and “elsewhere”.



IV Causality

- Events A and B…
  - ***Cannot*** change order of **A** and **B** by changing frames of reference.
  - **A** can also communicate information to **B** by sending a signal at, or less than, the speed of light.
  - This means that **A** and **B** are causally-connected.
- Events A and C…
  - ***Can*** change the order of **A** and **C** by changing frame of reference.
  - If there were any communication between **A** and **C**, it would have to happen at a speed faster than the speed of light.
- If idea of **cause and effect** is to have any meaning, we must conclude that no communication can occur at a speed faster than the speed of light.



IV : Causality

- Can causality be proved?
  - No, it is an axiom of physics
  - However, it is consistent with experiments!
- What if causality doesn't hold?
  - Then the Universe returns to being random, unconnected events that can't be understood or predicted.
  - But we would then have to deal with it
- So we will *insist* on causality as we continue to explore relativity; not because we want it that way, but because that's what we see!

Distances in space

- Two events A and B separated by distance Δs in space (x, y, z):

  $$\Delta s = [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{1/2}$$

  (Thanks, Pythagoras!- remember your analytic geometry)

  where $\Delta x = x_A - x_B$, $\Delta y = y_A - y_B$, $\Delta z = z_A - z_B$

Everyone in the same frame of reference agrees about the distance even if they have different axes x, y, and z; the distance is an *invariant*

But observers in *different* frames of reference do *not* agree about the distance defined in this was

Distances in time and space

- Two events A and B separated by distance Δs in time (Δt), but the same place in space:

  $\Delta s = [(c\Delta t)^2]^{1/2}$

  where $\Delta t = t_A - t_B$, and we've multiplied by c to make the units of Δs come out as a distance

- Two events A and B separated in x and t:

  $\Delta s = [(c\Delta t)^2 - (\Delta x)^2]^{1/2}$

  one space dimension+time

Any inertial observer moving in x direction sees the same Δs even if they disagree about Δt and Δx!

- For a light ray since $\Delta x = c\Delta t$

$\Delta s^2 = \text{sqrt}((c\Delta t)^2 - (\Delta x)^2)) = 0$

- Not like Euclidean space

if $(\Delta x)^2 > (c\Delta t)^2$ the events are separated by a 'spacelike' interval - can't get from here to there or more formally

not enough time passes between their occurrence for there to exist a causal relationship crossing the spatial distance between the two events at the speed of light or slower

V: NEW VELOCITY ADDITION LAW

- Einstein's theory of special relativity was partly motivated by the fact that Galilean velocity transformations (simply adding/subtracting frame velocity) give incorrect results for electromagnetism

- Once we've taken into account the way that time and distances change in Einstein's theory, there is a new law for adding velocities

- For a particle measured to have velocity V_p by a observer moving at velocity V_s to a stationary observer ,the particle's velocity as measured by the observer is

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

NEW VELOCITY ADDITION LAW

$$V = \frac{V_p + V_s}{1 + V_p V_s / c^2}$$

- Notice that if V_p and V_s are much less than c, the extra term in the denominator is very small and therefore $V \sim (V_p + V_s)$
- Thus, the Galilean transformation law is *approximately correct* when the speeds involved are small compared with the speed o flight
- This is consistent with everyday experience
- Also notice that if the particle has V_s=c in the moving frame, then it has V_p=c in the stationary frame
- the speed of light is frame-independent!

(algebra e.g $V_p + V_s = 2c$; $V_p * V_s = c^2$; $2c/(1+1)=c$)

VI : Mass and Energy

- Einstein reworked Newton's laws of mechanics using his new relativistic formulae
- The relationships between mass (M), velocity (v), momentum (**p**) and energy (E) **are different than those found in Newtonian mechanics**

$$\mathbf{p} = \gamma M \mathbf{v}$$

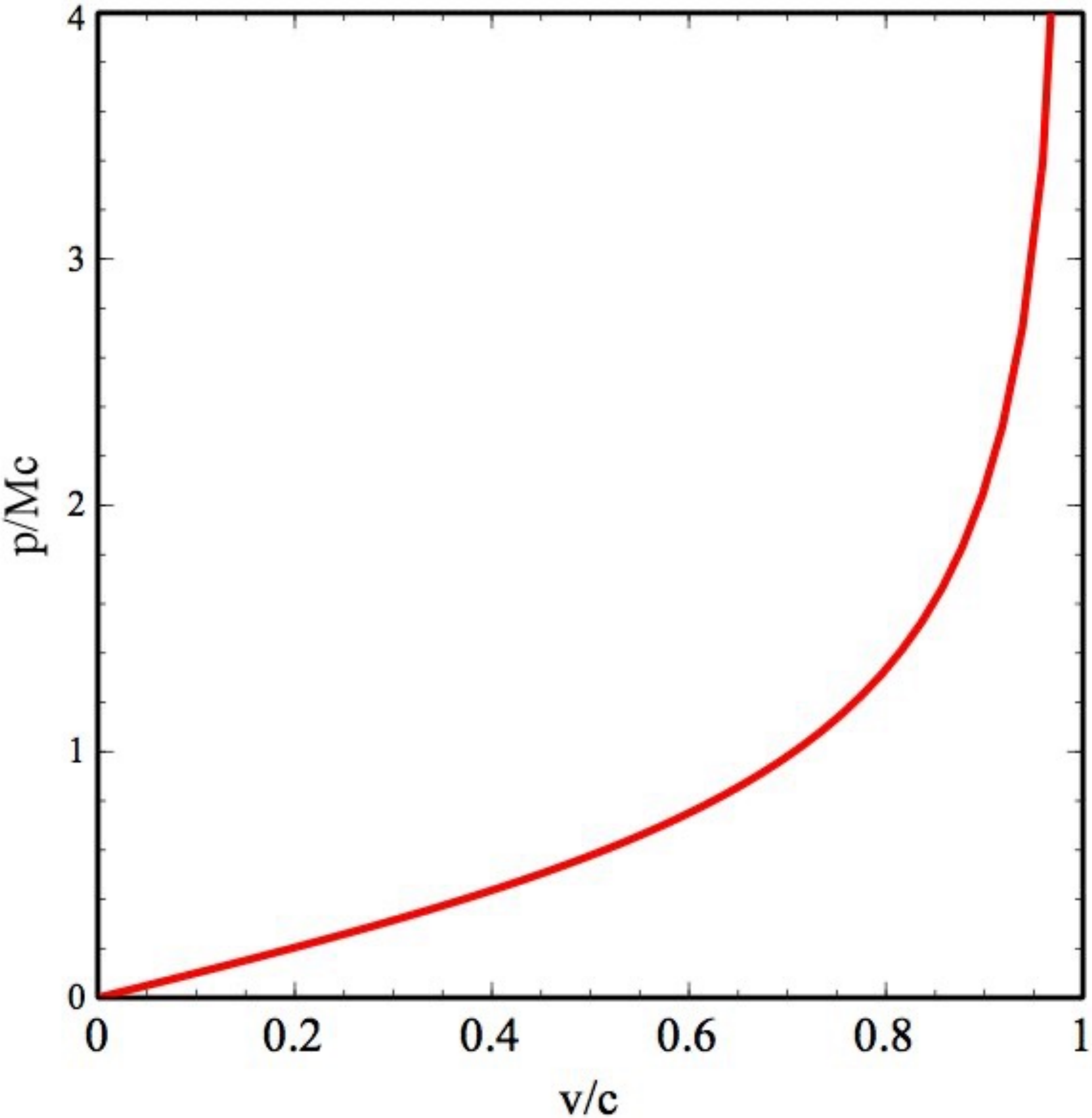
$$E = \sqrt{(Mc^2)^2 + (\mathbf{p}c)^2}$$

E=sqrt((Mc^2)^2+(**p**c)^2)

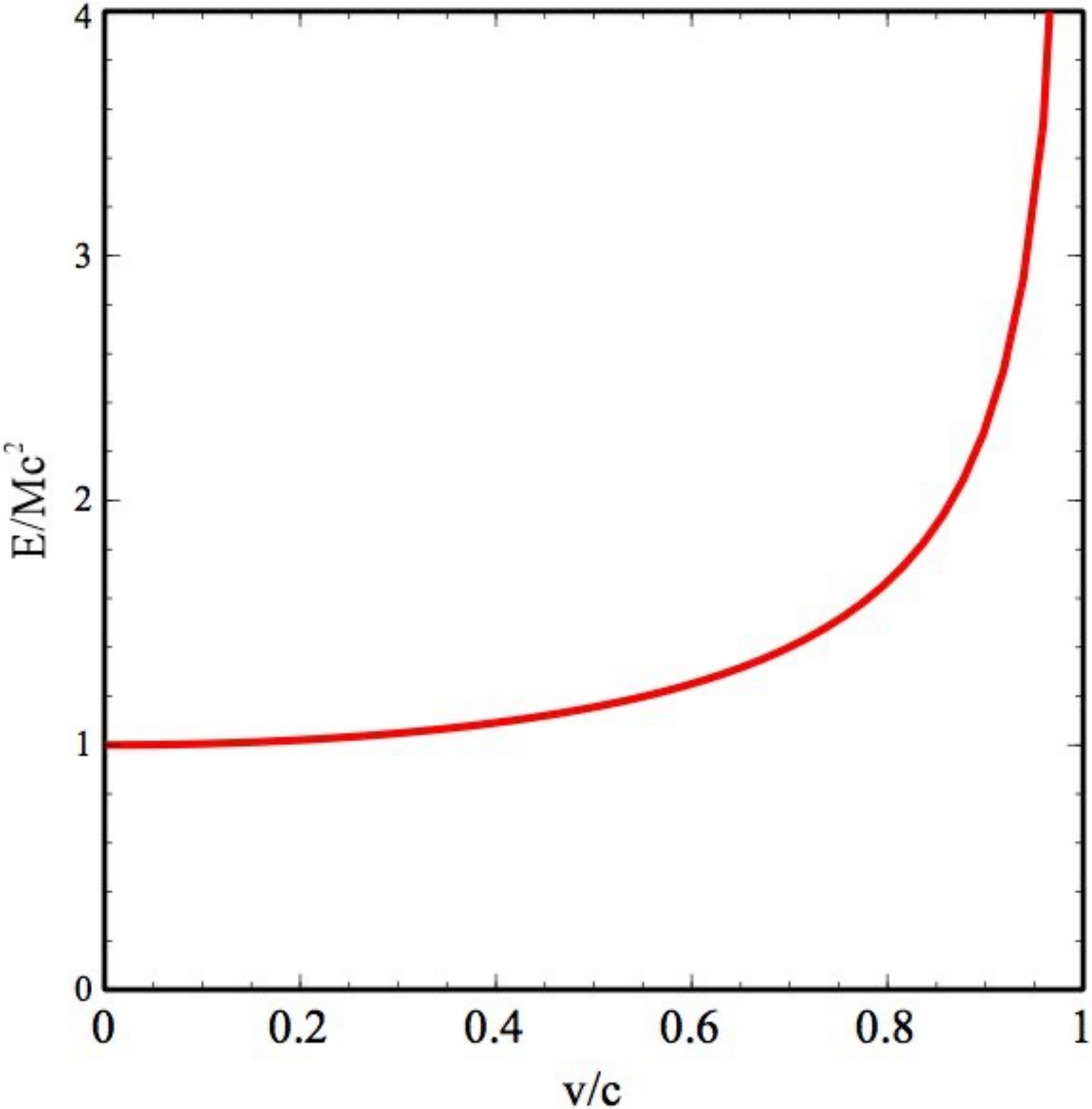
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Thus energy increases as the speed increases, and energy would become infinite if V = c;

γ is the Lorentz factor



p/Mc
v/c
0
1
2
3
4
0
0.2
0.4
0.6
0.8
1



E/Mc^2
v/c
0
1
2
3
4
0
0.2
0.4
0.6
0.8
1

Newton was only a little bit wrong

- What about objects moving at “small velocity”?
- It can be shown that: (binomial expansion)

$$E \approx mc^2 + \frac{1}{2} m V^2$$

  - The ½mv^2 is the Newtonian expression for the **kinetic energy** of a moving object.
- What counts as “small velocity”?
  - For car going at 30mph, approximate formula is wrong by 1 part in 10^{30}
  - For rocket going at 30,000mph, this approximate formula is wrong by 1 part in 10^{18}
  - So, Newtonian approximation is fine for all velocities experienced in everyday life.

Rest mass energy

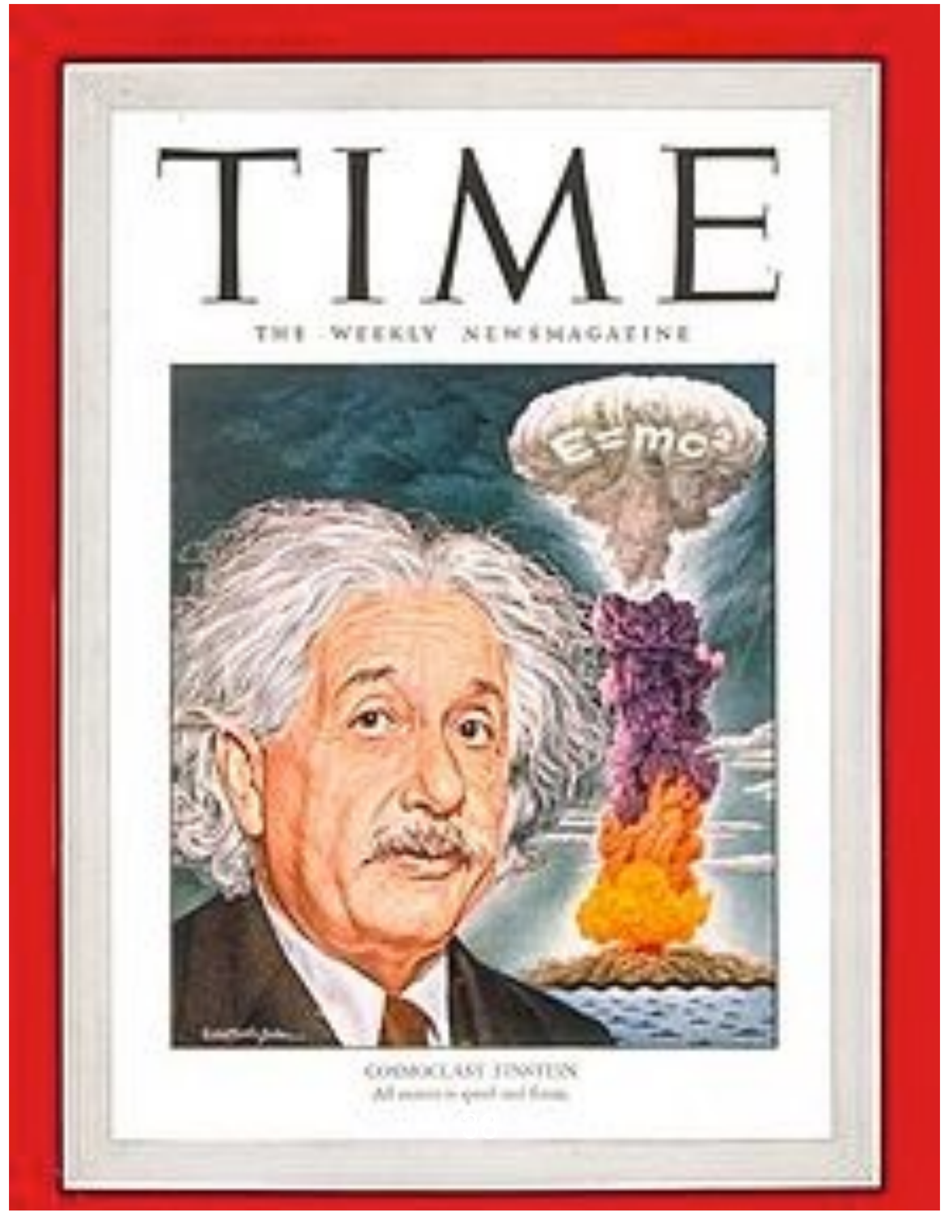
- Einstein's energy formula at zero velocity

$$E = mc^2$$

- What does this mean?
  - Can this energy can be accessed? In other words, maybe mass can be turned into "usable" energy? **It turns out that this is correct!**
  - Also, this can go the other way – energy can be turned into mass!

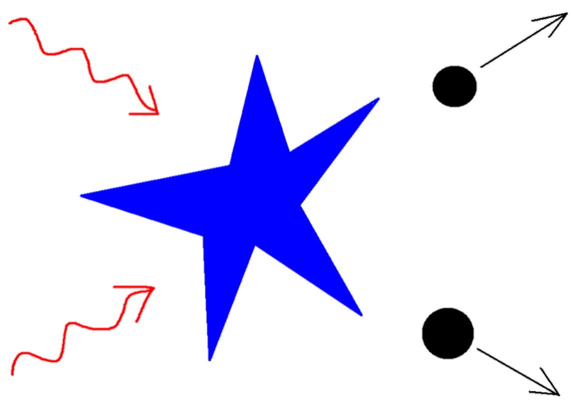
Einstein and the Atomic Bomb

- When Einstein learned that the Germans might figure out how to split the atom [actually, it is nuclear energy that is tapped], he wrote to President Franklin Roosevelt with his concerns. Einstein's 1939 letter helped initiate the U.S. effort to build an "atomic" bomb



EXAMPLES OF CONVERTING ENERGY TO MASS

- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy(e.g.,gamma-rays) can produce particle/anti-particle pairs



  - **Very fundamental process in Nature…**this process, operating in the early universe, is responsible for all of the mass that exists today!

conservation of energy sets a minimum photon energy required for particle creation : this threshold energy must be greater than the total rest energy of the particles created.

To create an electron-positron pair the total energy of the photons must be at least $2m_ec^2$=2x0.511KeV=1.022MeV

Some Consequences of Special Relativity

There is no absolute time or absolute space.

It is impossible for two events to be simultaneous for all possible observers.

There are pairs of events which will happen in one order for some observers and in the other order for other observers.

The kinetic energy of massive moving bodies increases without bound as the velocity of the body approaches the speed of light.

The same holds for the momentum of massive moving bodies: it increases without bounds as the velocity approaches the speed of light

No massive object can travel at or faster than the speed of light. All massless objects can only travel at the speed of light.

Philosophical Consequences

Special relativity produced major changes in the way reality was perceived

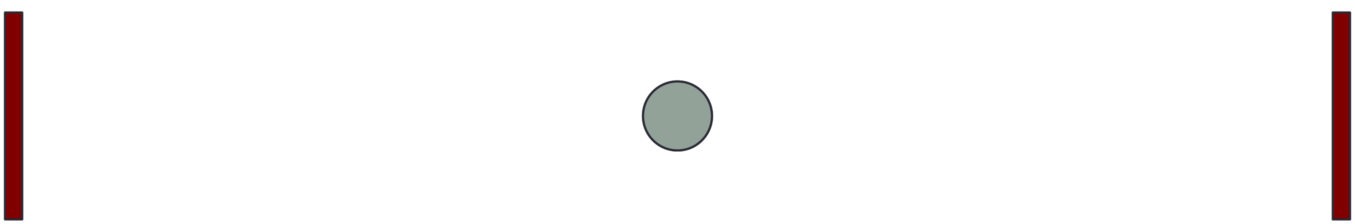
Space and time are not absolutes and are inextricably linked

There is no such thing as a perspective-independent “present.” Time is relative

Relativity fueled postmodernism and philosophic relativism. Prior to relativity, philosophers such as Aristotle, Kant, and Mill argued that there was an absolute truth and an absolute way of approaching various aspects of life.. with relativity, facts are no longer absolute, but instead dependent upon your viewpoint,

II: Relativity of Simultaneity

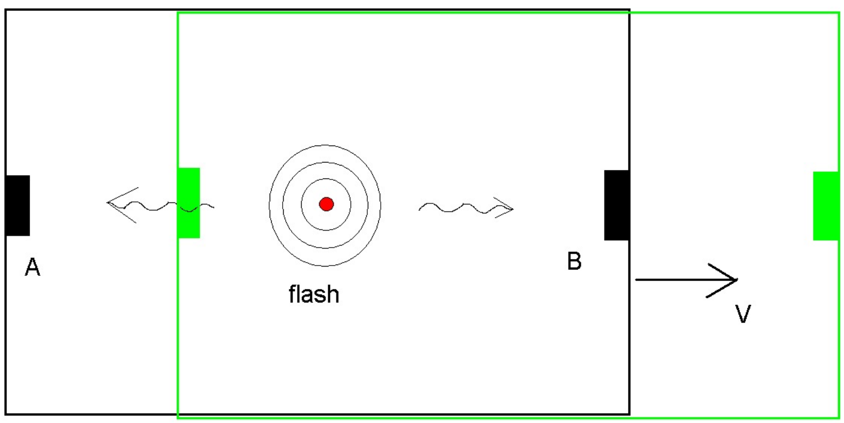
- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



- **Since speed of light is constant**, light rays will hit opposite walls at precisely the same time. Call these events A and B.

Change frames…

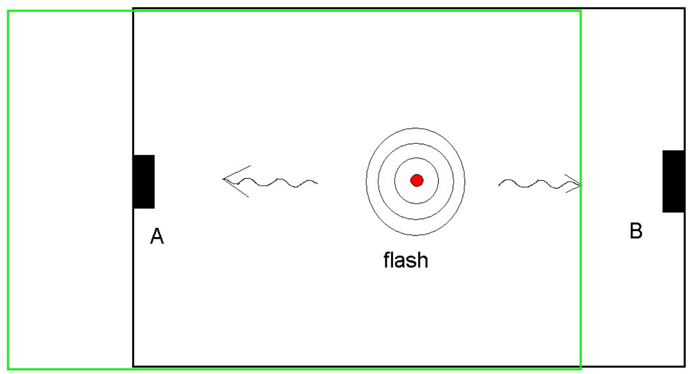
- Imagine performing same experiment aboard a moving spacecraft, and **observing** it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (*since the walls are moving!*). Event A will happen before event B.



- *But astronaut in spac* .
- **Concept of "events being simultaneous" (i.e. simultaneity) is different for different observer**

Change frames again!

- What about perception of a 3rd observer who is moving faster than spacecraft?



- 3rd observer se
- So, **order** in which events happen can depend on the frame of reference.

Length (Fitzgerald) contraction

- Fitzgerald contraction…
  - A moving object **contracts** by a factor γ (the same Lorentz factor) in the direction of motion

This is really a contraction of space itself… the object does not experience forces or stresses that make it contract

Again, everything is relative… if someone watches you travel past them at high speed, **you will appear** to be contracted in the direction of motion

Length contraction

- So, moving observers see that objects contract *along the direction of motion*.
- **Length contraction**… also called

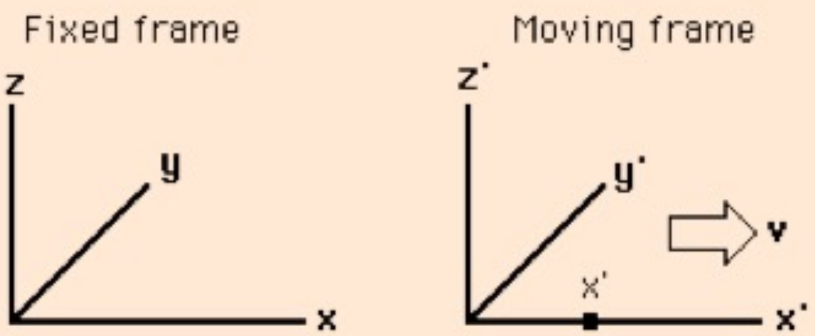
  Lorentz contraction

  FitzGerald contraction

Note that there is *no contraction of lengths* that are **perpendicular** to the direction of motion

- *Recall M-M experiment: results consistent with one arm contracting*

Lorentz Transformation



$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame. The reference frames coincide at t=t'=0. The point x' is moving with the primed frame.

The reverse transformation is:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Much of the literature of relativity uses the symbols β and γ as defined here to simplify the writing of relativistic relationships.

Evaluation of symbols

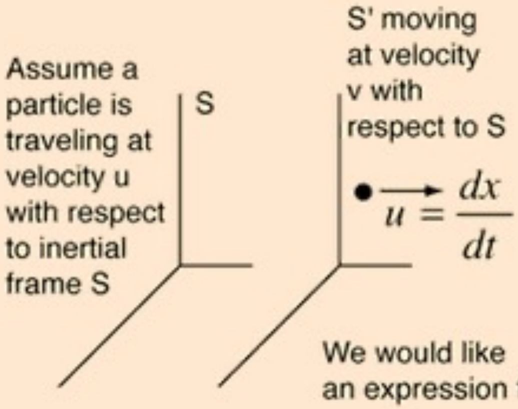
This changes the coordinates and the measurement of time (!!)

let v get small then $\mathrm{sqrt}(1-v^2/c^2) \sim 1$ and we get the usual t'=t and x=x'+vt

Hyperphysics web page

Relativistic Velocity Transformation

No two objects can have a relative velocity greater than c! But what if I observe a spacecraft traveling at 0.8c and it fires a projectile which it observes to be moving at 0.7c with respect to it!? Velocities must transform according to the Lorentz transformation, and that leads to a very non-intuitive result called Einstein velocity addition.



Lorentz transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{xv}{c^2}\right)$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We would like an expression for $u' = \frac{dx'}{dt'}$ which is the velocity as measured in the moving reference frame S'

Just taking the differentials of these quantities leads to the velocity transformation. Taking the differentials of the Lorentz transformation expressions for x' and t' above gives

$$\frac{dx'}{dt'} = \frac{\gamma(dx - vdt)}{\gamma\left(dt - \frac{vdx}{c^2}\right)} = \frac{\frac{dx}{dt} - v}{1 - \frac{v\frac{dx}{dt}}{c^2}}$$

Putting this in the notation introduced in the illustration above:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Lorentz Velocity Addition Law

Let's let v get small... then $uv \ll c^2$ and we get the old Galilean transform u'=u-v (or u+v=u)

Summary

Galilean Transformation

Galilean Transformation

$x' = x - vt$

$y' = y$

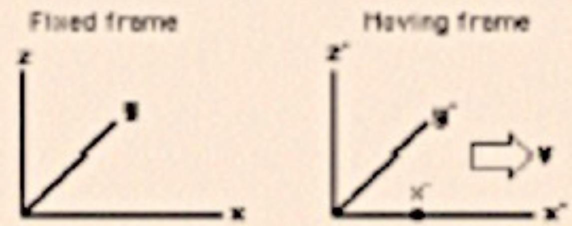
$z' = z$

$t' = t$

The primed frame moves with velocity v in the x direction with respect to the fixed reference frame.
The reference frames coincide at t=t'=0.
The point x' is moving with the primed frame.
The Galilean transformation gives the coordinates of the point as measured from the fixed frame in terms of its location in the moving reference frame.

The Galilean transformation is the common sense relationship which agrees with our everyday experience.

Lorentz Transformation



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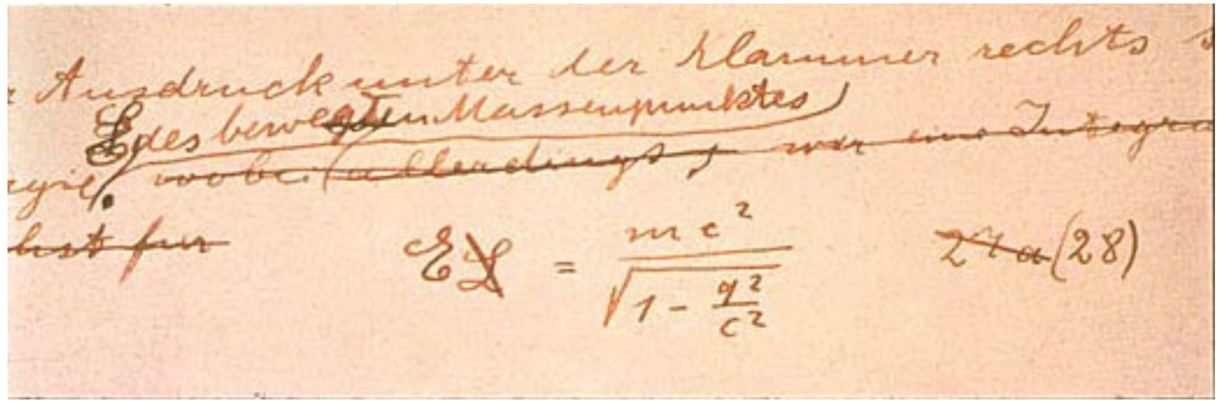
http://hyperphysics.phy-astr.gsu.edu/hbase/relativ/relcon.html#c1

Einstein's Insight

- Einstein speculated $E=mc^2$ was not simply an academic exercise; he believed that it might explain how an ounce of radium could emit 4,000 calories of heat per hour indefinitely (Marie Curie), seemingly violating the first law of thermodynamics-



Once again, relativity forced a major revision in classical physics.
Before, the first law of thermodynamics, which states that the total amount of energy can never be created or destroyed.

Now the total combined amount of **matter and energy** is the conserved quantity.

adapted from text by Michio Kaku