Class 7 :
General Relativity II

ASTR350 Black Holes (Spring 2022)

Cole Miller

This class: General Relativity II

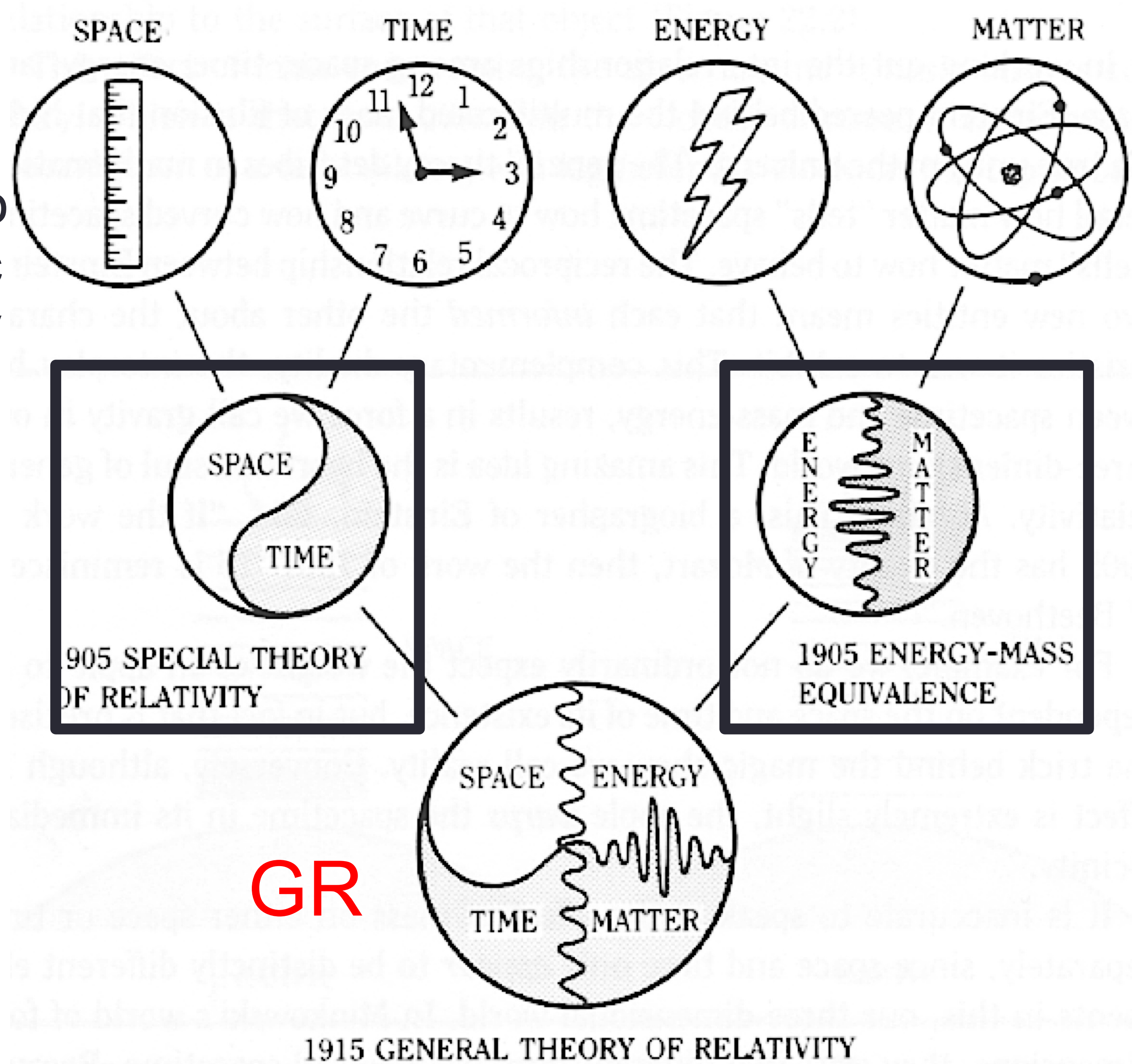
- Like special relativity, the general theory predicts phenomena which differ significantly from those of classical physics, especially concerning
  - the passage of time,
  - the geometry of space,
  - the motion of bodies in free fall,
  - the propagation of light.
- Examples of such differences include gravitational time dilation, gravitational lensing, the gravitational redshift of light, and gravitational time delay (discussed in last lecture).

Muddiest points

Any astro questions?

RECAP

- Einstein and gravity
  - Cannot simply add Newtonian Gravity to a Special Relativistic framework (causality violation, frame-dependency, mass-energy, gravitational redshift etc etc)



RECAP

- Einstein Tower Thought Experiment
  - Energy conservation demands the gravitational redshifting of light
  - Strong Equivalence Principle (inertial frames of reference are free-falling)
  - A consistent theory of gravity cannot be constructed within the framework of special relativity- need something else.

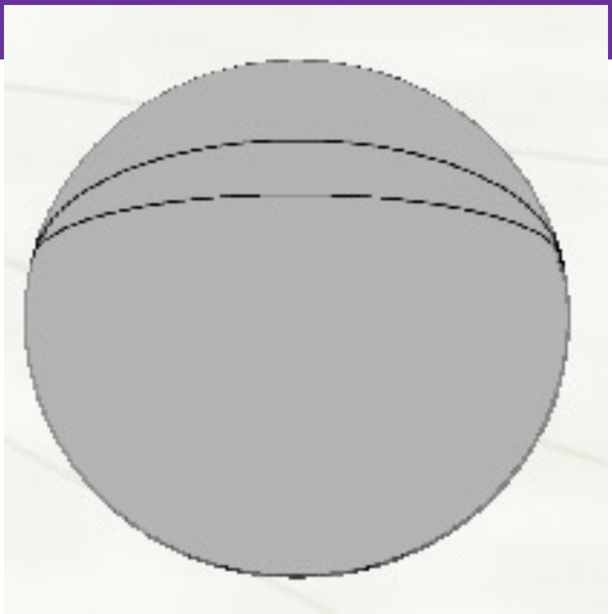
I: CURVED SPACE-TIME

- Einstein pondered several things…
  - Success of Special Relativity showed that space & time were closely linked
  - The "tower thought experiment" suggested that free-fall observers are (locally) free of effects of gravity
  - Is gravity an illusion caused by the fact that we live in an accelerating frame?
  - … but there is no *single* accelerating frame that works! Somehow, you need to stick together frames of reference that are accelerating in different directions

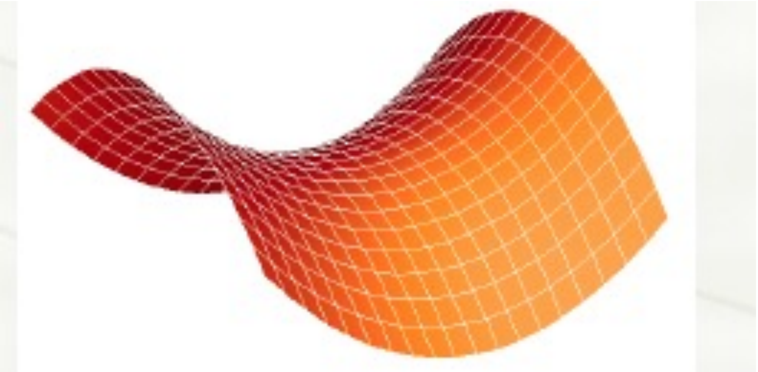
CURVED SPACE-TIME

- Einstein's suggestion
  - Free-falling objects move on "geodesics" (generalizations of straight lines) through ***curved space-time***.
    - 4-dimensional space-time is curved
    - Matter and energy causes space-time to be curved.
- What is a geodesic?
  - Shortest path between two points on a surface
  - E.g. path flown by aircraft
  - Geodesics that start parallel can converge or diverge

Einstein’s proposal



- 4-dimensional space-time can be “curved,” not necessarily flat (like Euclidian space)
  - Examples:
    - surface of sphere is curved (2D space);
    - surface of football field is flat (2D space)



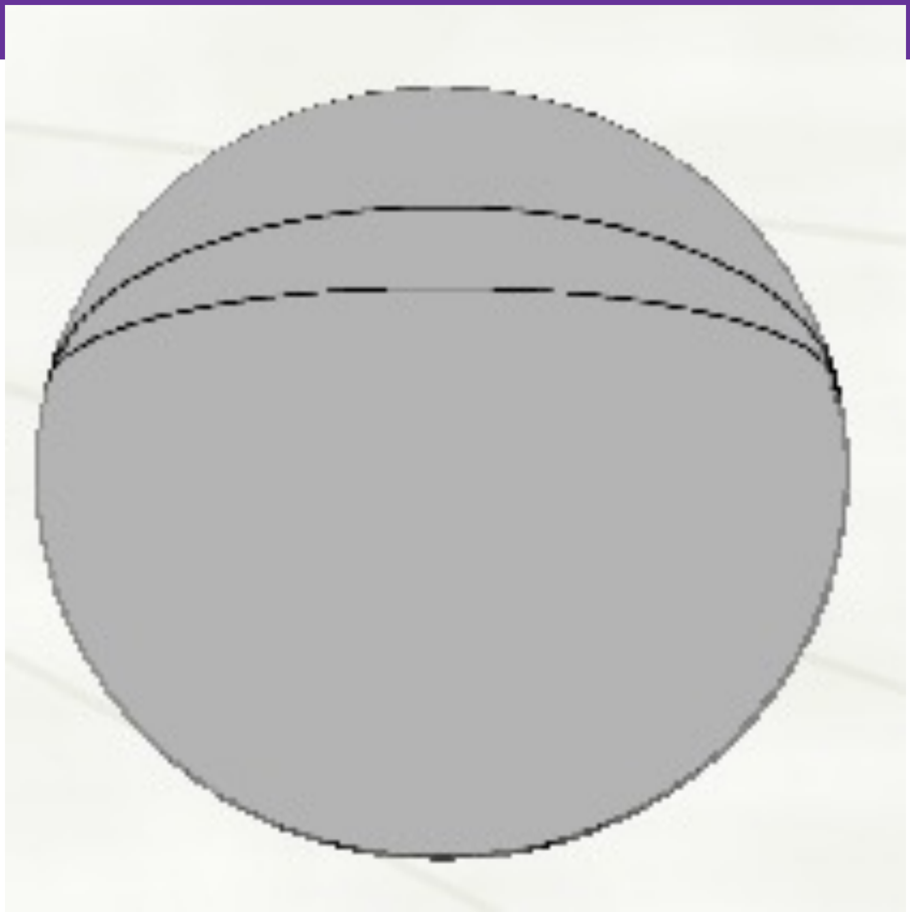
- Free-falling objects move on geodesics* through curved space-time

*a geodesic is the generalization of a straight line in flat space to curved space
It is the shortest path between two points on a surface; for instance, the path flown by an aircraft between cities on the globe
Unlike straight lines in flat space, geodesics that start as parallel can converge or diverge (or even cross on a curved surface

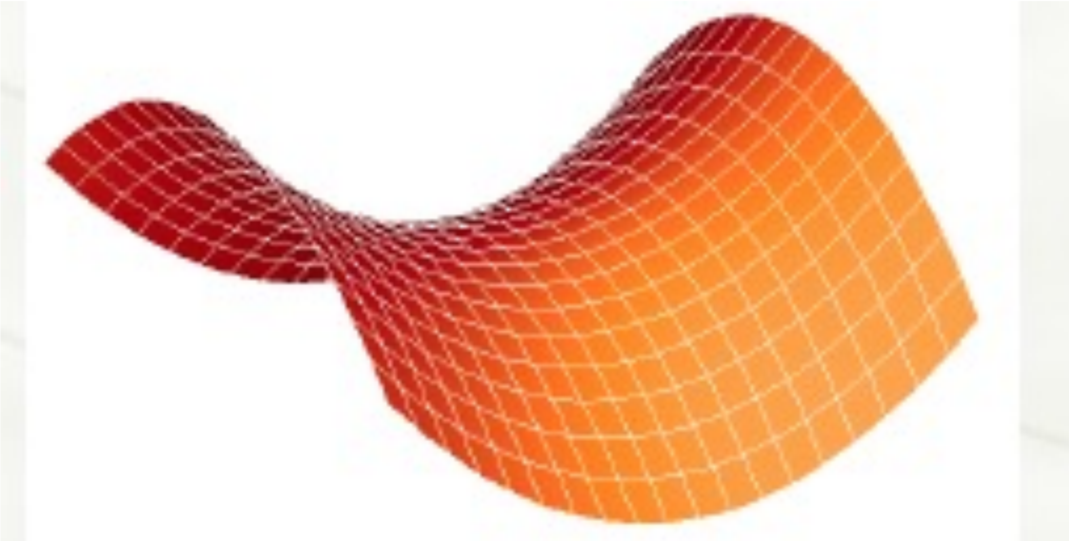
Einstein's proposal

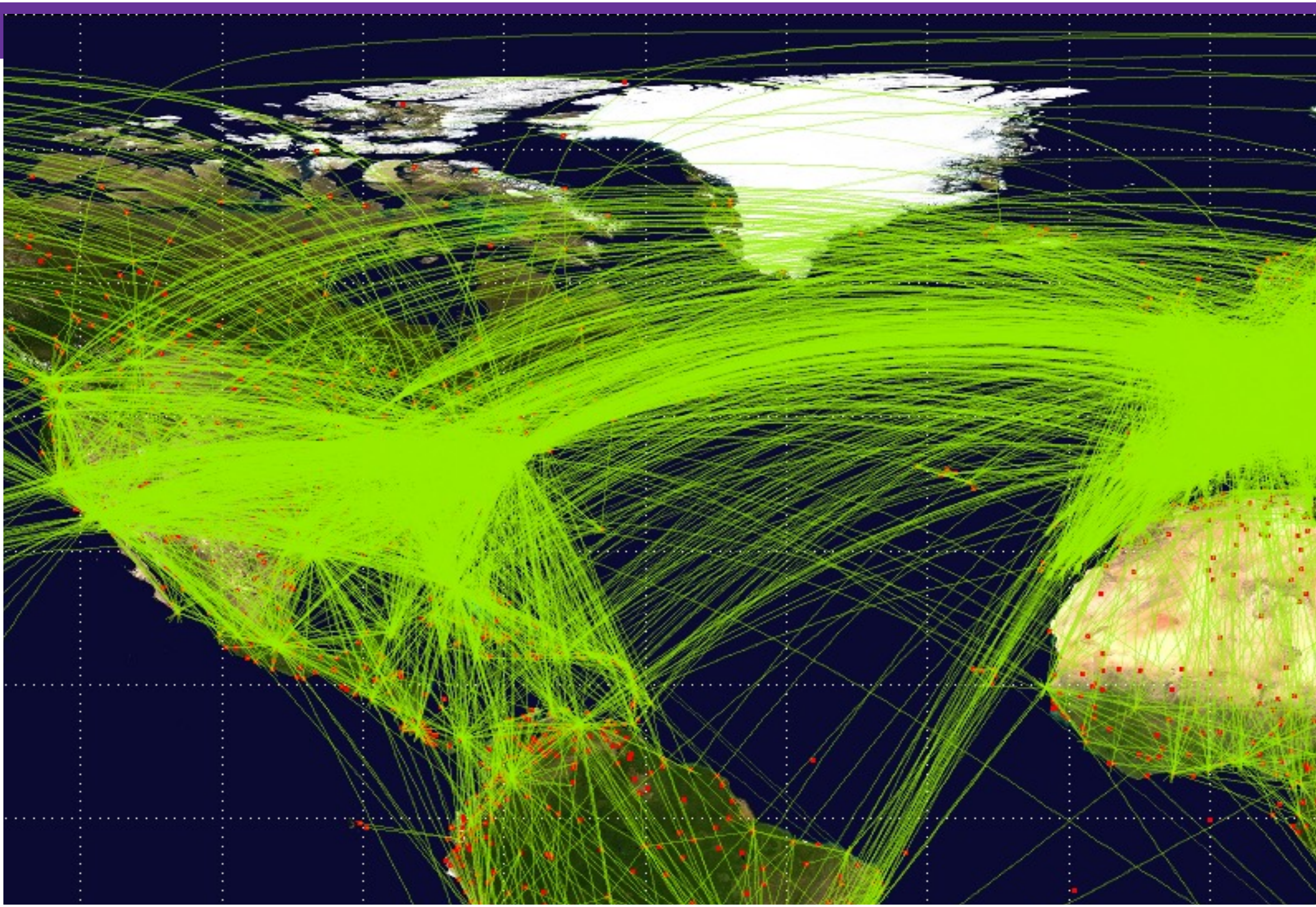
- The curvature (bending) of space-time is produced by matter and energy

- **"Space-time curvature tells matter/energy how to move. Matter/energy tells space-time how to curve."**

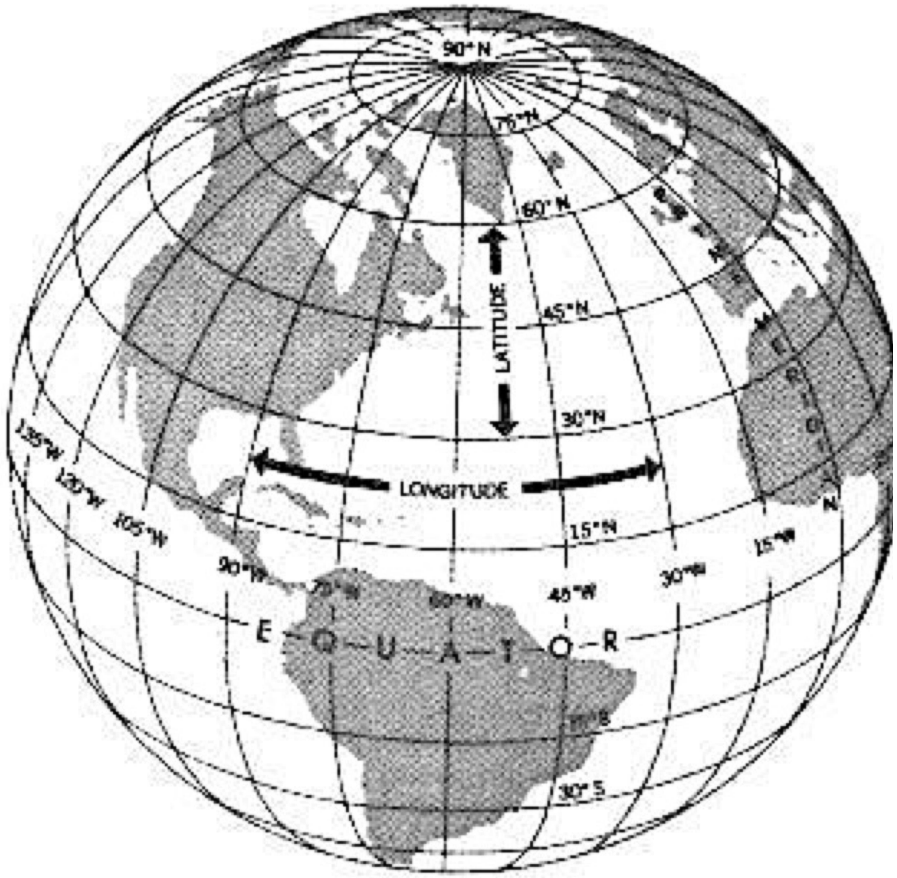


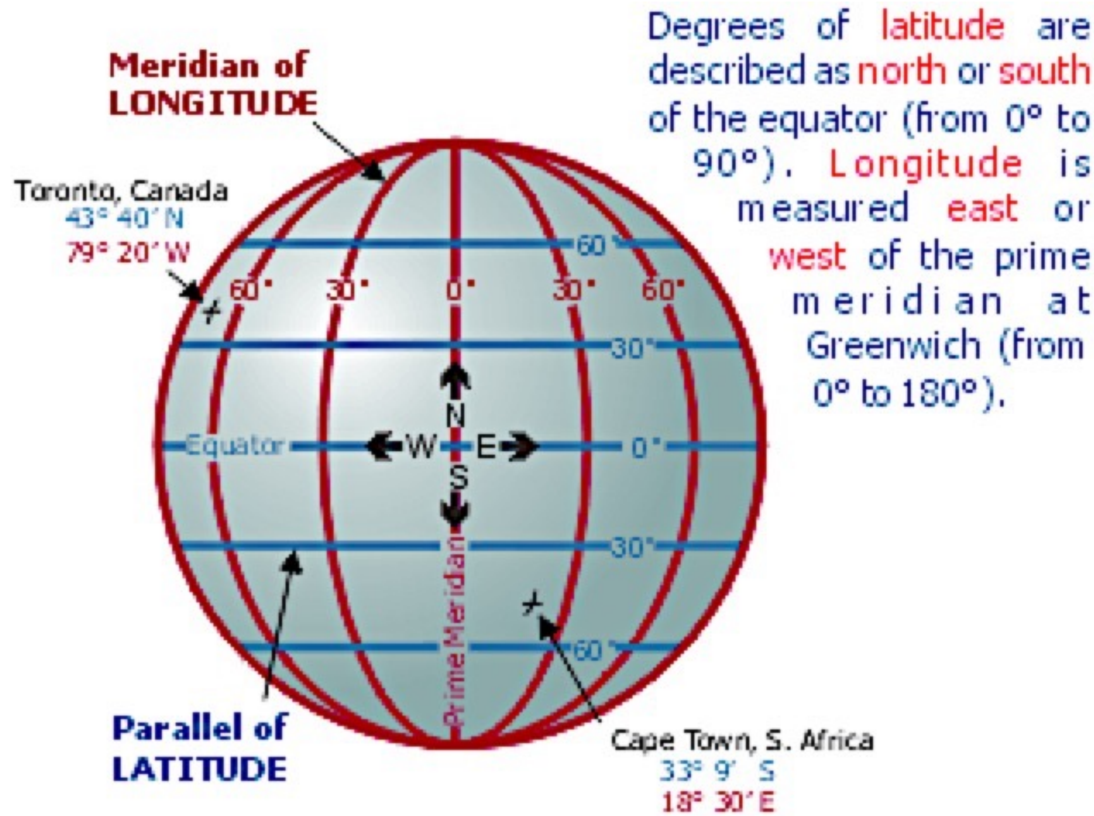
2 examples of curved surfaces





Lines appear to be curved due to mapping of sphere to flat surface



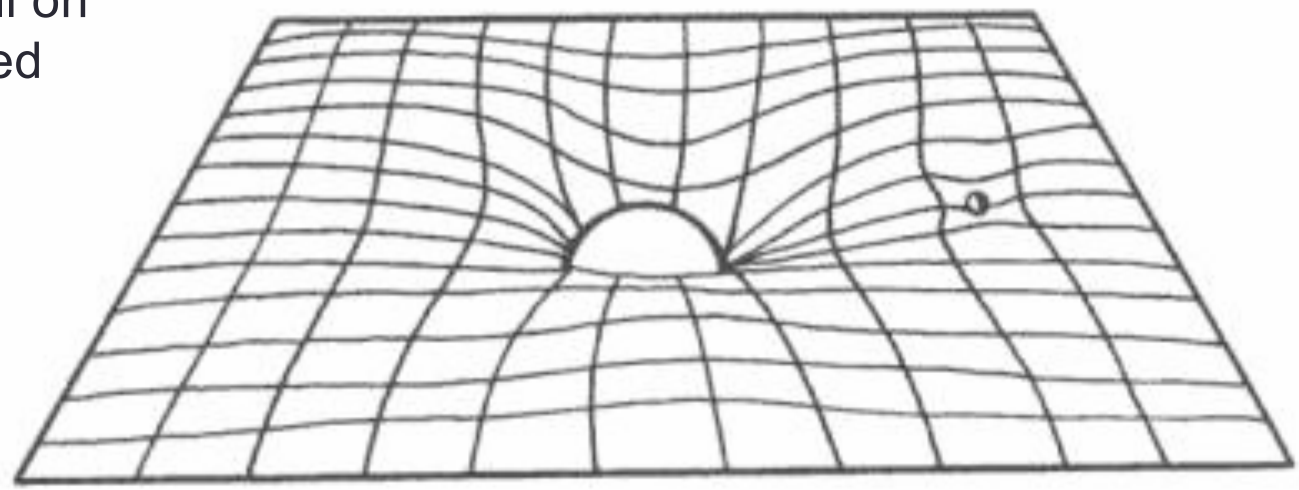


- Constant-longitude lines (meridians) are geodesics
- On the Earth, geodesics are Great Circles, the shortest distance between two points on the surface.
- Constant-latitude lines (parallels) are **not geodesics**

Effect of matter on coordinates

- Lines that would be straight become curved (*to external observer*) when sheet is “weighted”

place a large leaden ball on a rubber sheet suspended between four poles.



The rubber sheet will sag from the weight of the lead ball.
If you spin a marble on the sheet it will circle the lead ball, just as the earth circles the sun.

The reason is not that there is any attraction between the earth and the sun but that the earth follows the straightest path it can follow in curved space-time.

II: THE GENERAL THEORY OF RELATIVITY

- Free-falling particles/observers move on geodesics through curved space-time
- The distribution of matter and energy determines how space-time is curved.
  - Within a free-falling frame, the Special Theory of Relativity applies.

"Space-time curvature tells matter/energy how to move.
Matter/energy tells space-time how to curve."

Structure of GR

- **Gravity is geometry**, not a normal force!!!!

- Free-fall particles follow geodesics "straight-lines" through the 4-d spacetime (Strong Equivalence Principle)…

spacetime curvature tells matter/light how to move

- Distribution of matter and energy determines curvature of spacetime (Einstein Field Equations).

matter/energy tells spacetime how to curve

Master Equation of GR

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

you will not need to know this equation for a test

- Notes:
  - The Einstein curvature *tensor* "**G**" is mathematical object describing curvature of 4-D space-time.
  - The Stress-Energy *tensor* "**T**" is mathematical object describing distribution of mass/energy.
  - Newton's constant of gravitation "G" and the speed of light "c" appear as fundamental constants in this equation.
  - This is actually a horrendous set of 10 coupled non-linear partial differential equations!!
- For weak gravitational fields, this reduces to Newton's law of gravitation.

Master Equation of GR

$$\underline{\underline{\mathbf{G}}} = \frac{8\pi G}{c^4}\underline{\underline{\mathbf{T}}}$$

This equation relates the local *spacetime* *curvature* (expressed by the Einstein tensor G) with the local energy and momentum within that spacetime (expressed by the stress–energy tensor T).

- The Einstein Field Equations determine the spacetime geometry resulting from the presence of mass–energy and momentum, that is, they determine the metric tensor of spacetime for a given arrangement of stress–energy in the spacetime.

consistent with the local conservation of energy and momentum

Einstein Field Equations (EFE)

The nonlinearity of the EFE distinguishes general relativity from many other fundamental physical theories. – Hard to solve them!

For example, Maxwell's equations of electromagnetism are linear in the electric and magnetic fields, and charge and current distributions (i.e. the sum of two solutions is also a solution); another example is Schrödinger's equation of quantum mechanics which is linear in the wavefunction

The EFE reduce to Newton's law of gravity in both the weak-field approximation and the slow-motion approximation and uses the same constant G

General relativity is consistent with the local conservation of energy and momentum

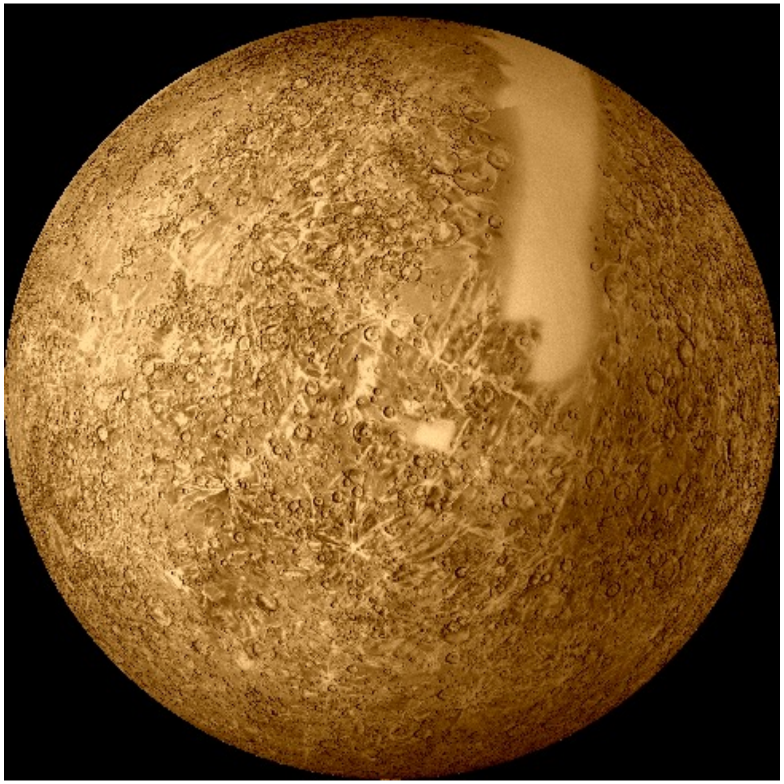
Einstein Field Equations

Solutions to the EFE are descriptions of spacetime.

These describe the structure of the spacetime including the inertial motion of objects

III: GR EFFECTS IN THE SOLAR SYSTEM

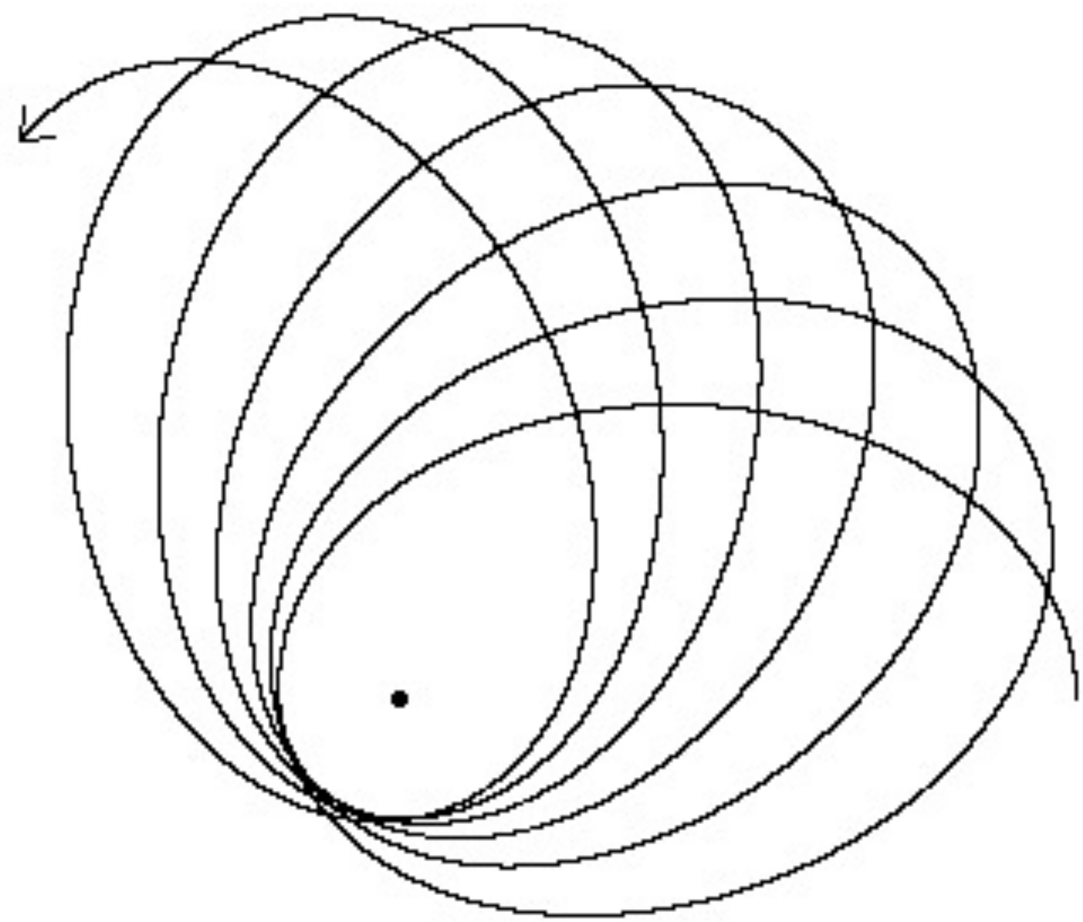
- Have already heard about bending of star light by the Sun (detected by Eddington) and Shapiro delay
- **Orbit of Mercury:**



Mercury's Precession

- Under Newtonian physics, a (small) object orbiting a spherical mass would trace out an ellipse with the spherical mass at a focus (Kepler's laws) but ...
  - Gravitational effect of other planets
  - Oblate deformation of the Sun (Sun is not a perfect sphere)
- Leads to "precession of perihelion" even for Newtonian physics…
  - But, Newton wouldn't have worried about non-inertial nature of Earth's frame!

Precession of Perihelion
Mercury's Orbit 'Rotates' with time



the precession of Mercury's orbit is 5600 seconds of arc per century (one second of arc=1/3600 degrees).

Newton's equations, taking into account all the effects from the other planets predicts a precession of 5557 seconds of arc per century.

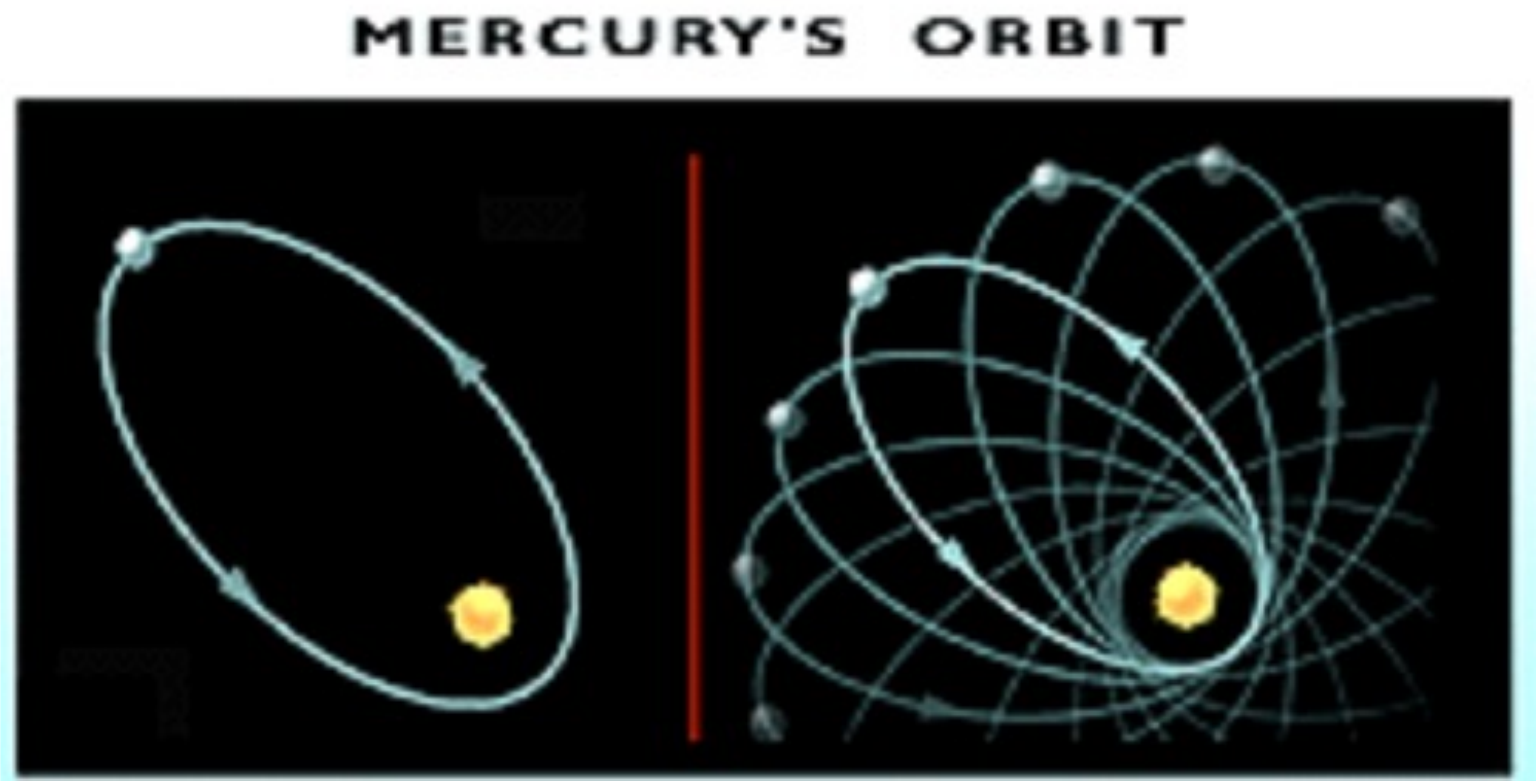
There is a discrepancy of 43 seconds of arc **per century** (0.7% effect)

Importance of Accurate Measurements

Astronomers were so confident of their measurements and of Newtonian physics that a 0.7% difference over a century was considered a serious problem !

This requires a lot of confidence.

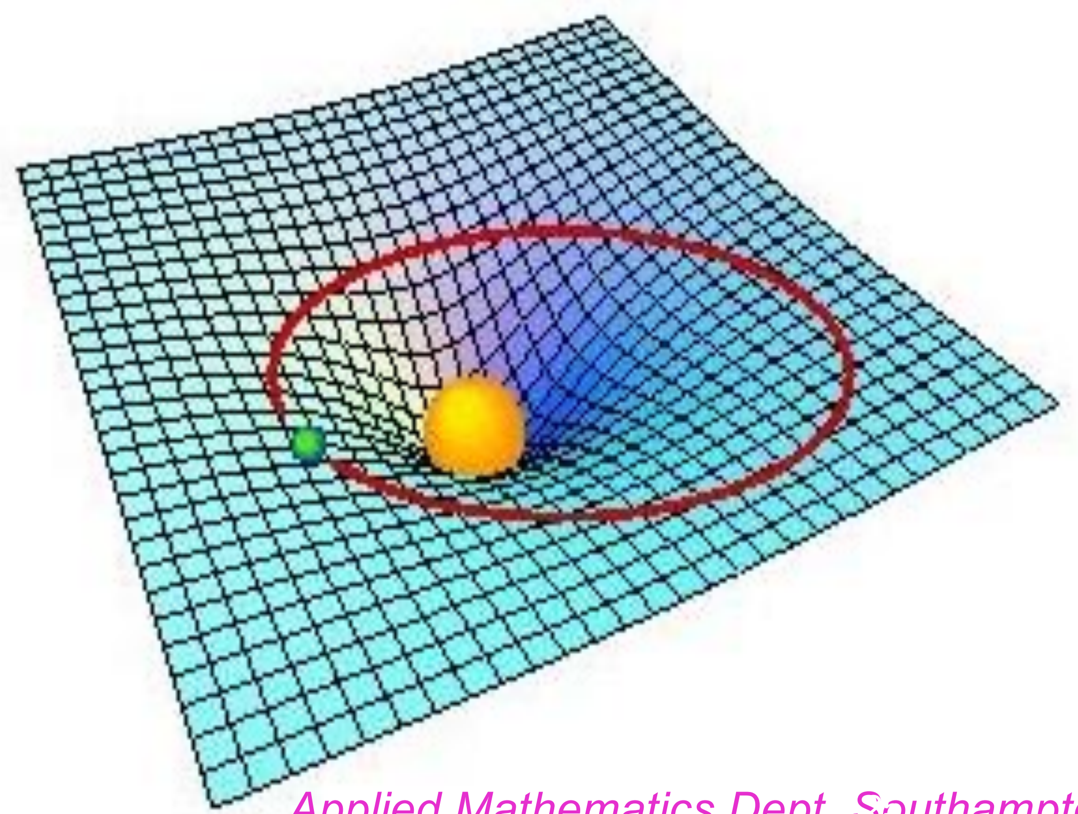
Orbits in Curved Spacetime



http://www.fourmilab.ch/gravitation/orbits/

How are orbits of planets affected?

- Green marble would follow straight line if yellow weight were not there
- Marble's orbit becomes curved path because mass warps space



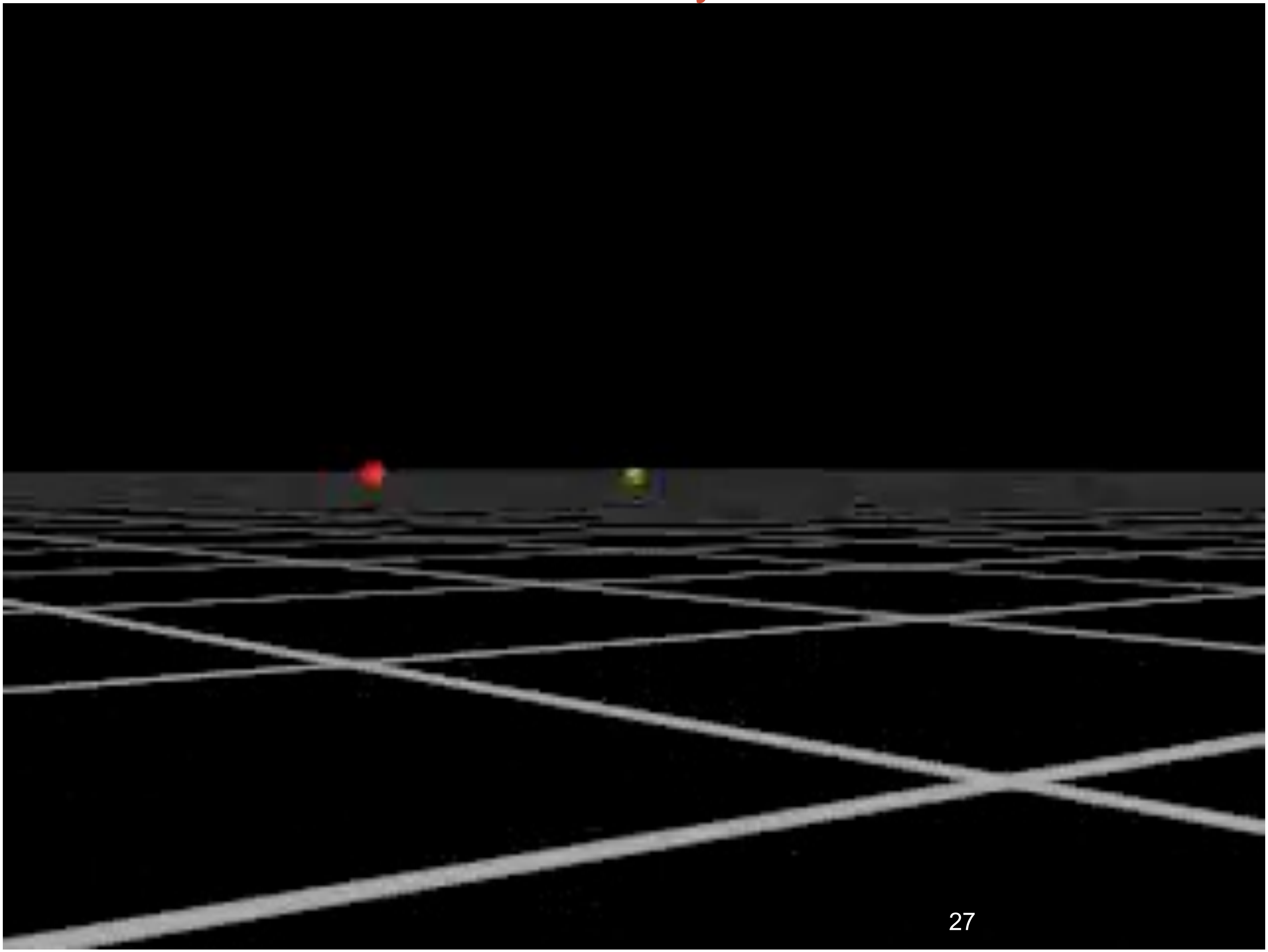
The orbit precesses because of the curvature of spacetime

. When Einstein calculated the magnitude of this effect for Mercury he got precisely the previously unexplained 43".

- He correctly took the view that this was an important confirmation of his theory.

- Einstein's theory also correctly accounts for a smaller discrepancy of 8.6" per century in the precession of the perihelion of Venus. The value is smaller than that of Mercury because Venus is further from the Sun and thus the curvature of spacetime is less.

Precession of the orbit of Mercury



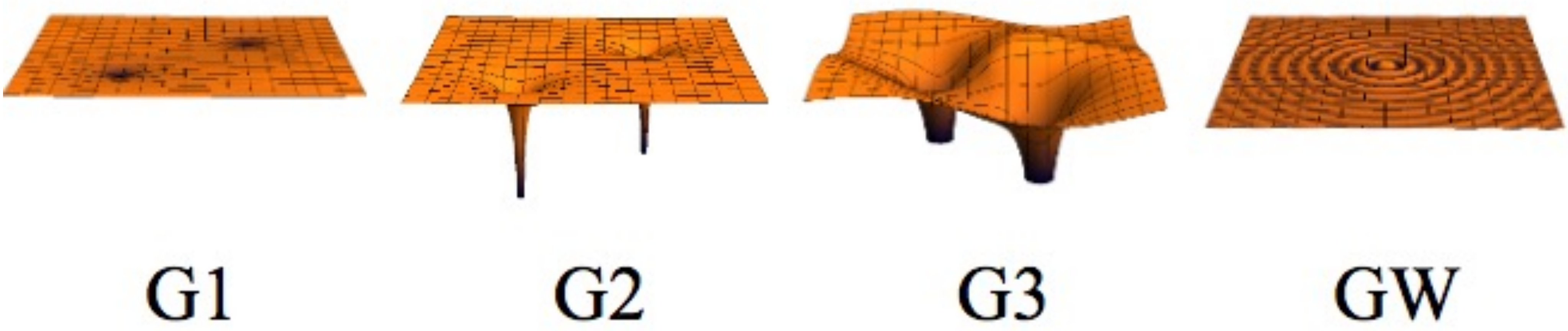
Limits of being 'relativisitic"

G1 Quasi-stationary: The motion of the masses is slow compared to the speed of light (v<<c) and spacetime is only very weakly curved- [Mercury around sun]

G2 Quasi-stationary strong-field regime: the motion of the masses is slow compared to the speed of light
(v/c<<1), but one or more bodies of the system are strongly self-gravitating,- [binary neutron stars]

G3 Highly-dynamical strong-field regime: Masses move at a significant fraction of the speed of light (v~c) and spacetime is strongly curved [black holes and merging neutron stars]

How Strongly Curved is Space

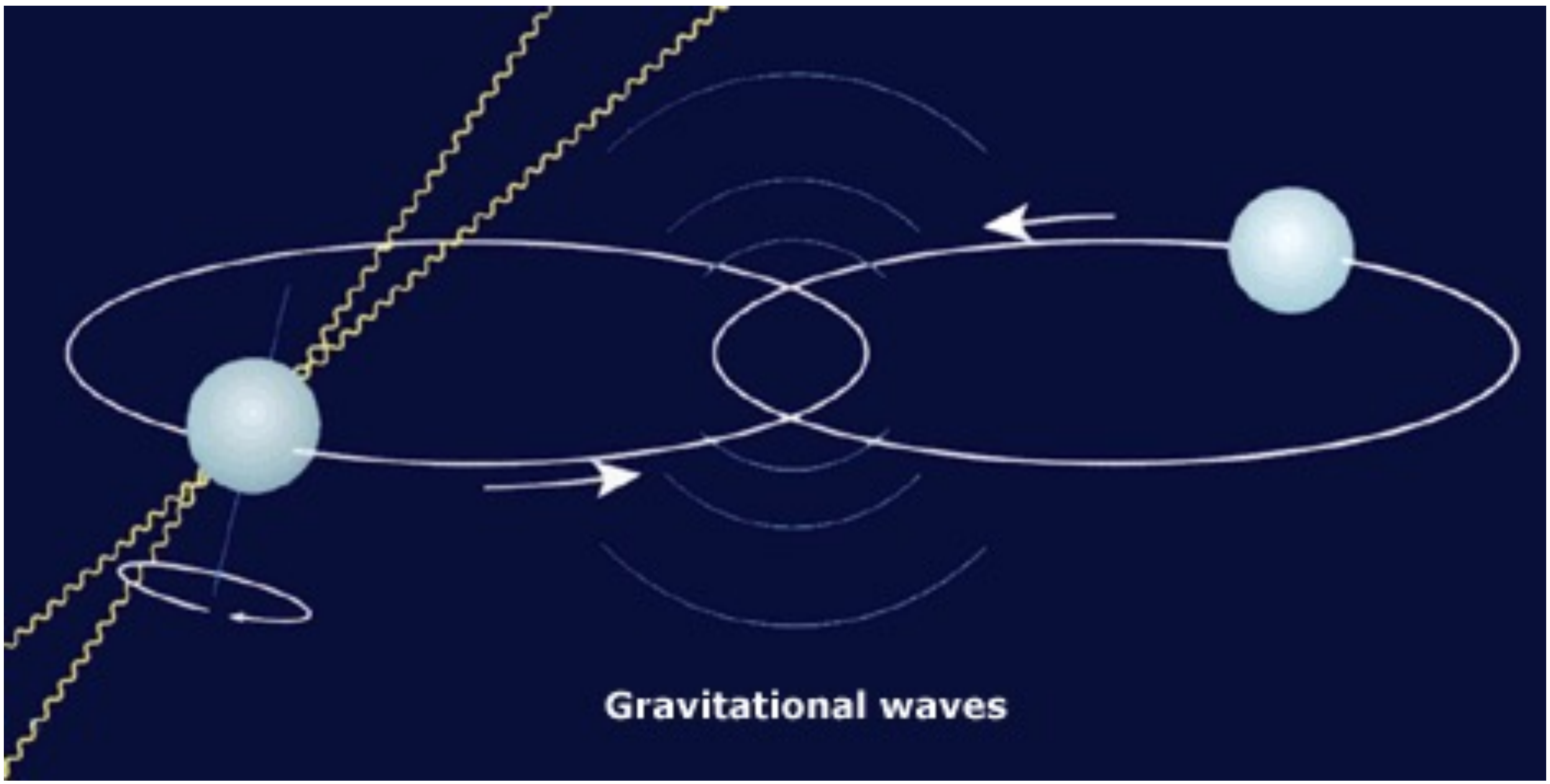


regime G1 is well tested in the Solar system.
Binary pulsar experiments test G2
Black holes and neutron stars for G3

Testing Relativistic Gravity with Radio Pulsars Norbert Wex

The binary pulsar (PSR 1913+16)

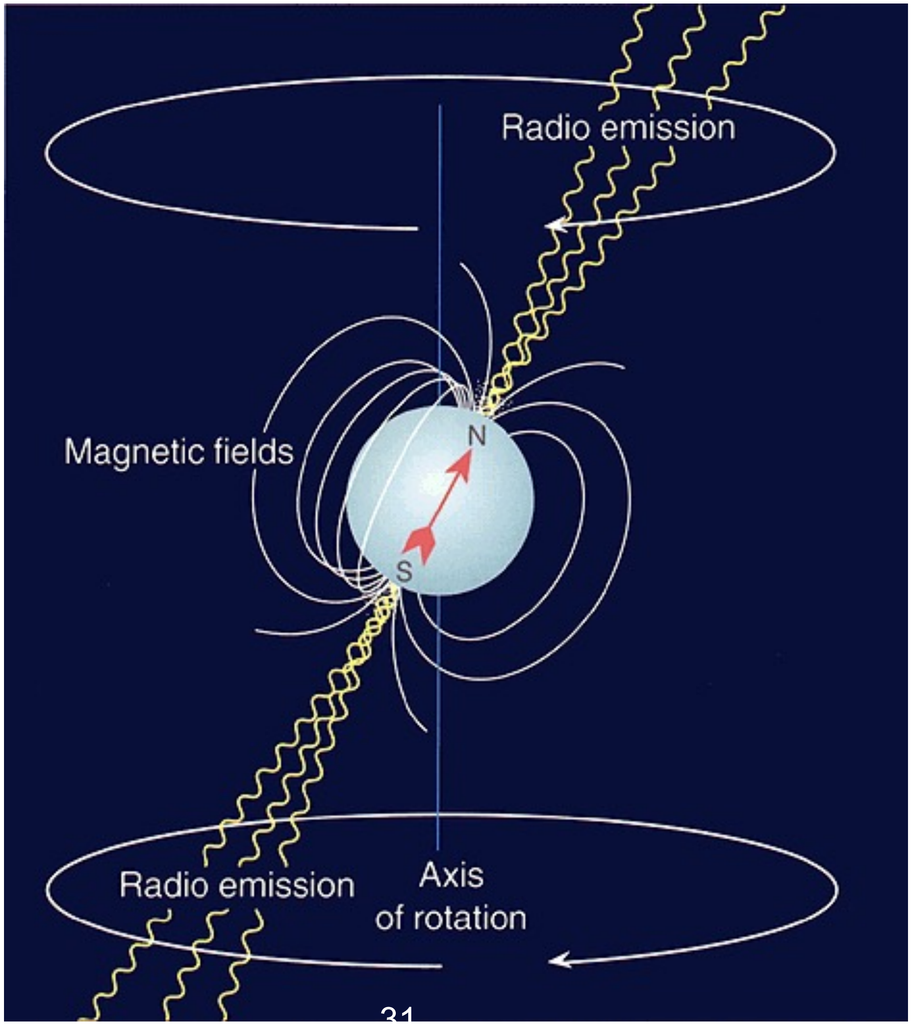
- Russell Hulse & Joseph Taylor (1974)
  - Discovered remarkable double star system
  - Nobel prize in 1993



From Nobel Prize website

Hulse-Taylor system

- Two **neutron stars** orbiting each other- v~300km/sec~0.1%c, 7.75 hr period
- One neutron star is a pulsar –
  - Neutron star is spinning on its axis (period of 59ms)
  - Emits pulse of radio towards Earth with each revolution
  - Acts as a very accurate clock!
- Strong gravity- good place to test GR
  - Orbit precesses 4 deg/year!
  - Orbit is shrinking due to emission of gravitational waves



Binary Pulsar

To maximize the "precession of perihelion" effect want objects of high mass close to each other

- two neutron stars in orbit around each other
- size of effect $\sim R^{-5/2}M^2$

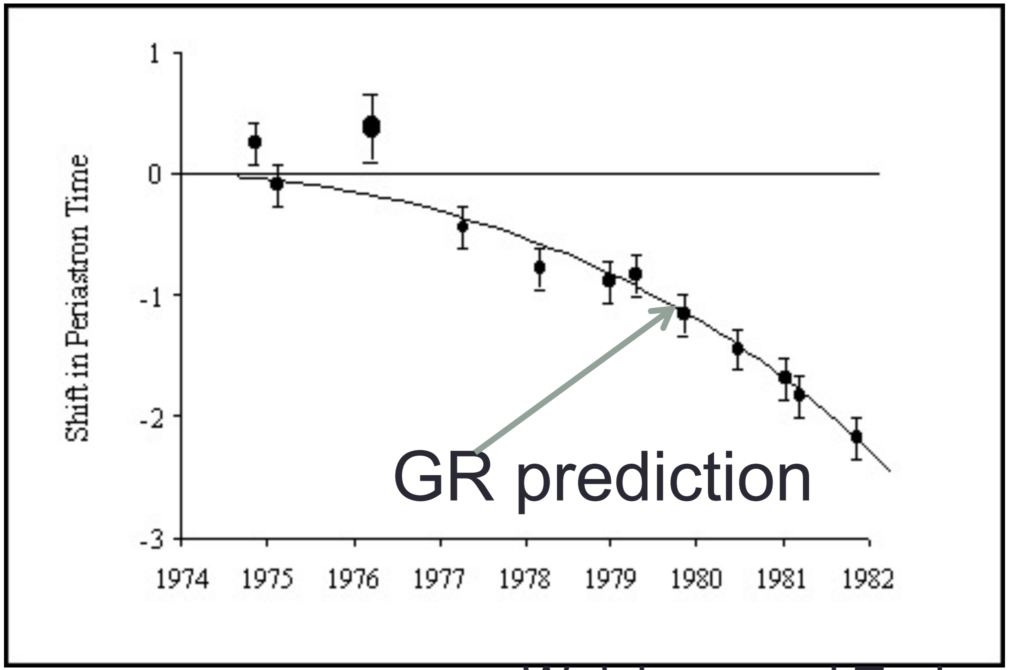
where R is the separation and M is the mass- e.g. want system to be massive and in close orbit (M/R large).

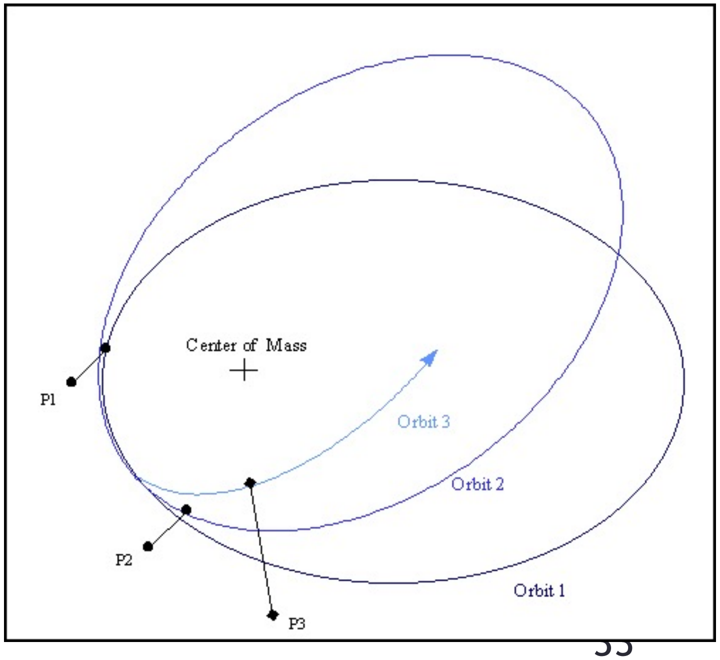
The binary pulsar PSR 1913+16 precession is 4.2 degrees per year (compared to **43 arc-seconds per century for Mercury,35,000x more)**

A measure of "how relativistic you are" is $(2\phi/c^2)$ where ϕ is the Newtonian potential (GM/r)
At the surface of the sun $2\phi/c^2$ is 2×10^{-6} while close to a neutron star it is 0.2

Precise test of certain aspects of GR

- When pulsar is approaching Earth, pulse frequency increases (Doppler shift); when pulsar is receding, pulse frequency decreases -- orbit of pulsar can therefore be "mapped"
- Orbit seen to be precessing (same physics as for Mercury) and shrinking (loss of energy due to gravitational waves) at exactly the rate predicted by Einstein's theory



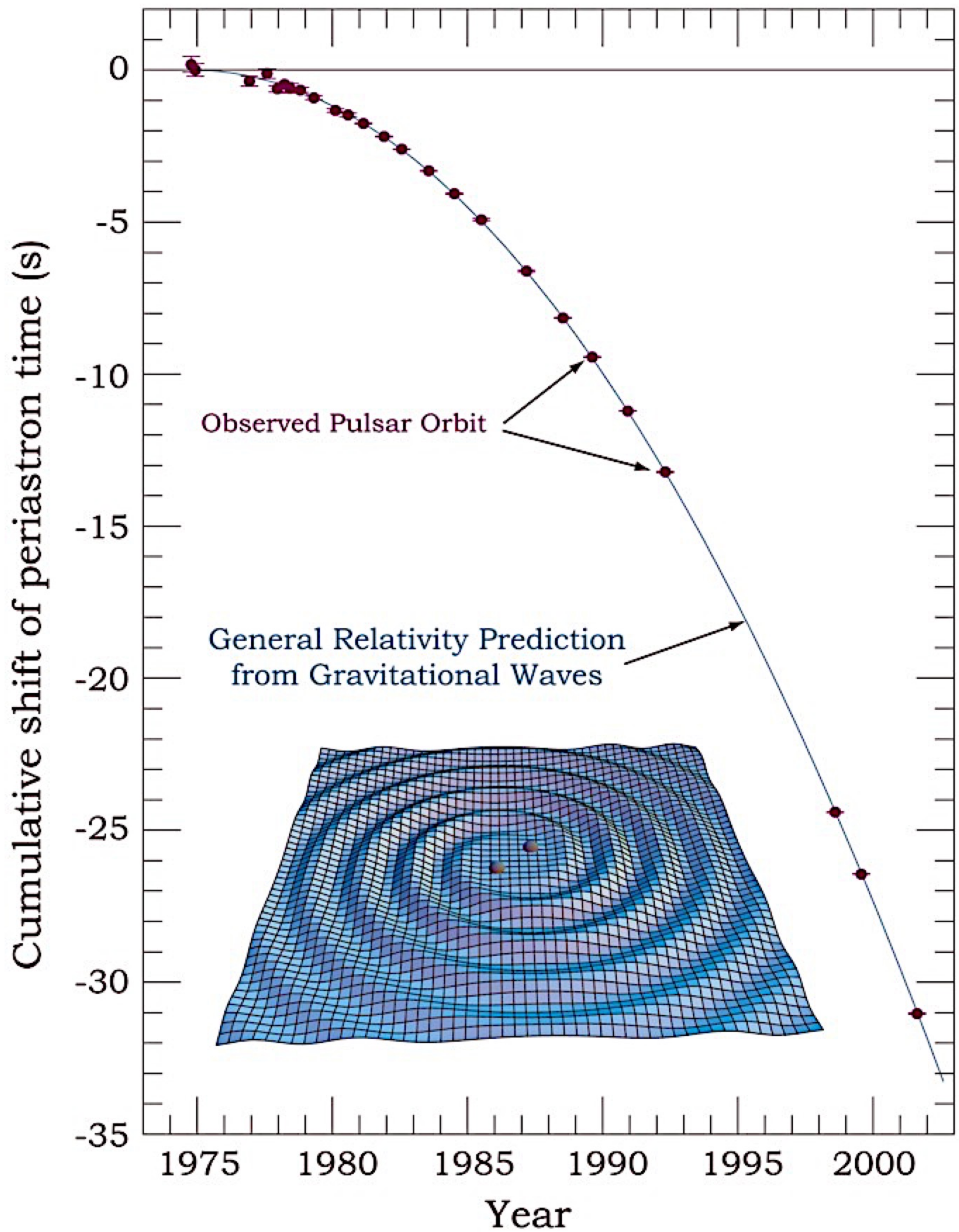


Weisberg and Taylor (2004)

As data have gotten better the agreement with General Relativity has become more and more precise
Kramer+ 2021: 0.013%!

A strong theory survives stronger and stronger tests

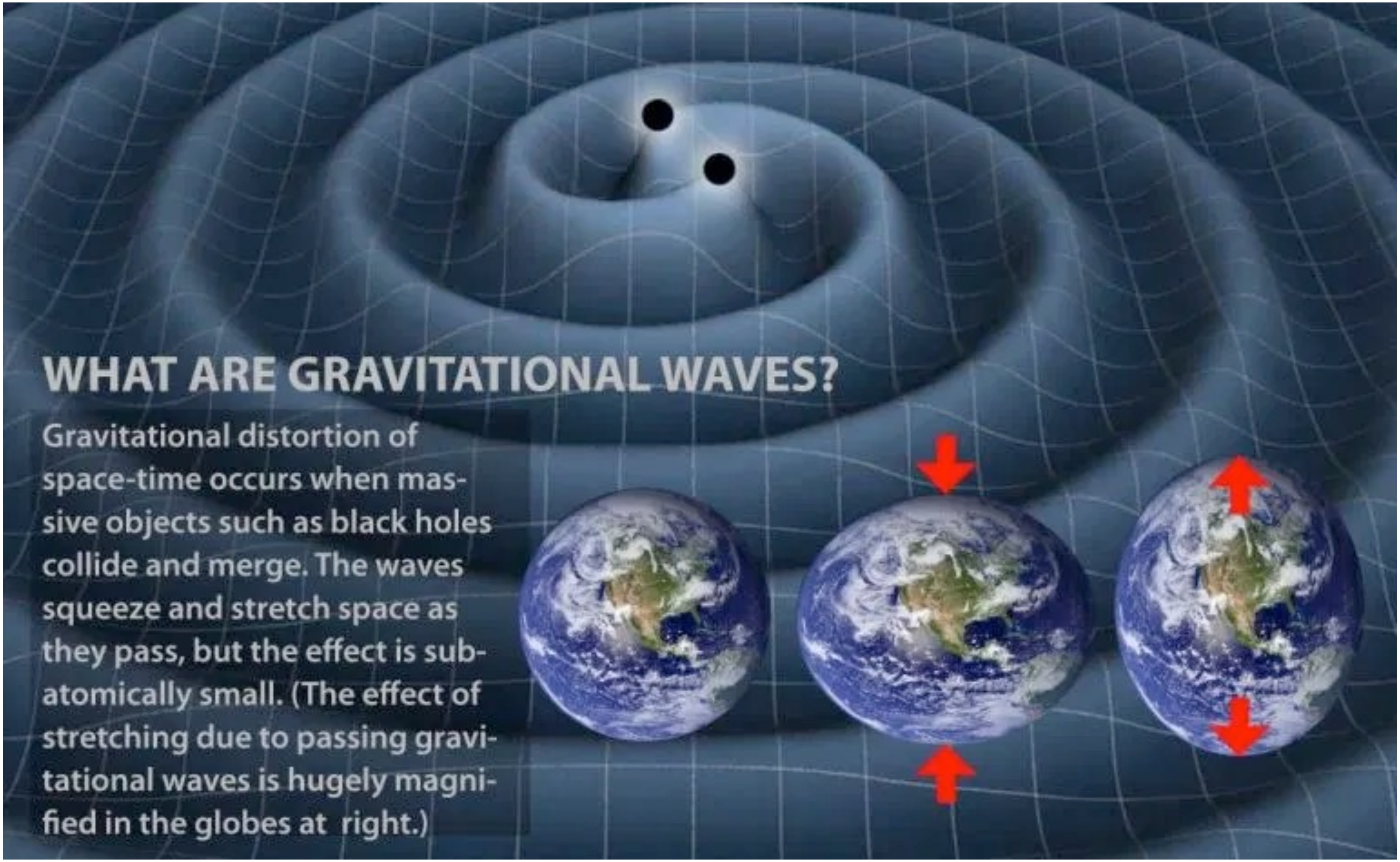
Binary pulsars were the first unambiguous detection of the effects of gravitational radiation



GRAVITATIONAL WAVES-Another Prediction of GR and a Test

- Accelerating masses produce continual changes in space-time geometry
- Periodically-moving bodies (e.g. orbiting stars) create *ripples* in space-time curvature
- Ripples travel at speed of light through space-time
- These are called **gravitational waves**
- Usually **VERY** weak unless LARGE masses are experiencing LARGE accelerations- merger of black holes
- Detected by Advanced LIGO 6 years ago !!!

Gravitational waves



GRAVITATIONAL WAVES- Much More later

General Relativity states that any accelerating masses will produce gravitational waves.
But to be able to detect the waves the objects have to be extremely massive and moving very quickly.

LIGO researchers discovered a wave that stretched space by one part in 10^{21}, making the 4 km detector expand and contract by <1/100 the size of a proton(!!!)

https://www.ligo.org/science/Publication-GW150914TestingGR/index.php