Class 8 :
Schwarzschild Black Holes

ASTR350 Black Holes (Spring 2022)
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THIS CLASS- Black Holes At Last

- The General Relativistic view of black holes
- “Schwarzschild” black holes- the simplest kind
    - View of an external observer
    - View of an infalling observer
    - Spaghettification

Muddiest points

Any astro questions?

RECAP

- Special and General Relativity
  - Highlights importance of frames of reference
  - Measurements of time affected by motion
  - Time dilation necessarily implies length contraction. Space and time get "mixed together" when changing reference frame… instead, think of *space-time*.
  - Idea of light cones, the past/future, and causality (Lec 5)

- General Relativity
  - Measurements of time affected by gravity/acceleration
  - Gravity can be made to (locally) vanish by going to a free-falling reference frame

More Recap

- Laws of physics same in all inertial reference frames
- Special relativity tells us how to relate measurements in different reference frames
- Einstein tower- gravity affects light
- Strong equivalence principle-free-falling frames are inertial frames- in such a frame can make local effects of gravity "disappear"
- GR- free-falling matter (and light) follow geodesics in space-time
- Predictions and effects of GR- bending of light, precession of Mercury...

Einstein DID NOT get the Nobel prize for Relativity

- Instead, for the photoelectric effect (important stepping stone on the way to quantum mechanics)

I : Schwarzschild

- Karl Schwarzschild (1873-1916*)
  - Solved the Einstein field equations for the case of a “spherically-symmetric” point mass. (<1 year after Einstein's paper!)
  - First exact (non-trivial) solution of Einstein’s equations
  - Describes a non-spinning, 'spherical',non-charged black hole… a *Schwarzschild* black hole

*died in WWI from infection

Black Holes: Newtonian

Remember escape velocity?

By making M larger and R smaller, V_{esc} increases- in Newtonian physics to arbitrary values

Idea of an object with gravity so strong that light cannot escape was first suggested by Rev. John Michell in 1783

Black Holes General Relativity

Karl Schwarzschild (1916)
First solution of Einstein's equations of GR
Describes gravitational field in (empty) space around a non-rotating mass

Features of Schwarzschild's solution:

Yields Newton's law of gravity, with flat space, at large distances (Large R)

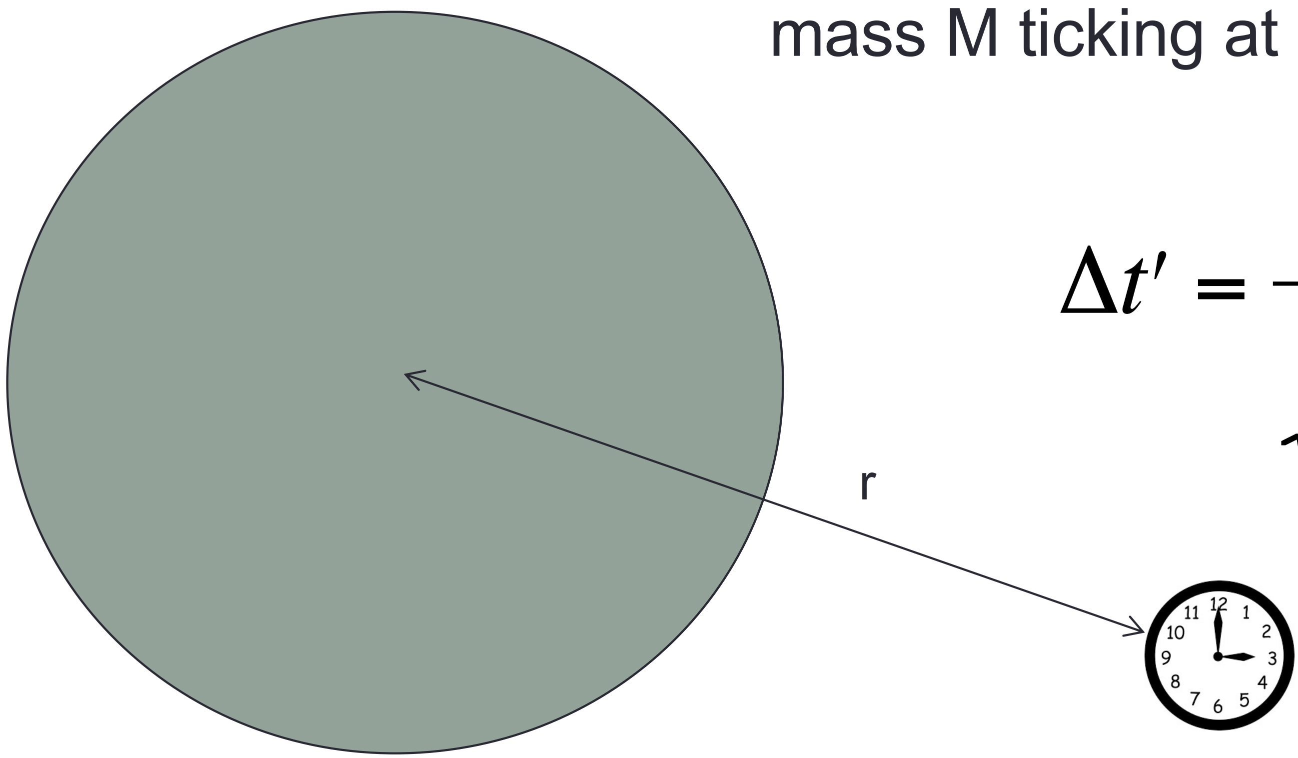
Space-time curvature becomes infinite at center (R=0; this is called a **space-time singularity**)

- Gravitational time-dilation effect becomes infinite on a spherical surface known as the **event horizon** Big confusion about this!
- Radius of the sphere representing the event horizon is called the **Schwarzschild radius,** $R_s = 2GM/c^2$

The view of a distant observer

Distant observer sees a (stationary) clock at a distance r from a body of mass M ticking at a rate $\Delta t'$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$



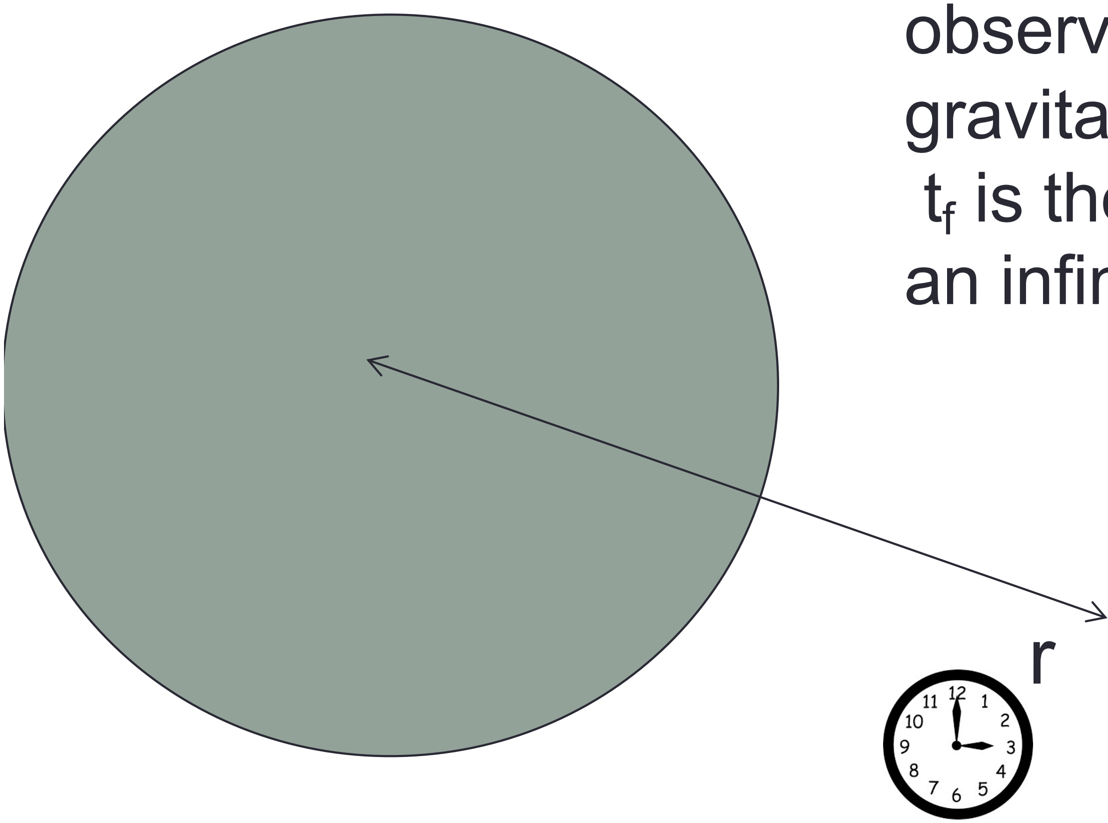
Δt is what it ticks at in the distant observer's frame

Time slows down in a gravitational field

The view of the infalling observer

t_0 is the time measured when observer and event are in the same gravitational potential

t_f is the time as measured when at an infinite distance from any mass.

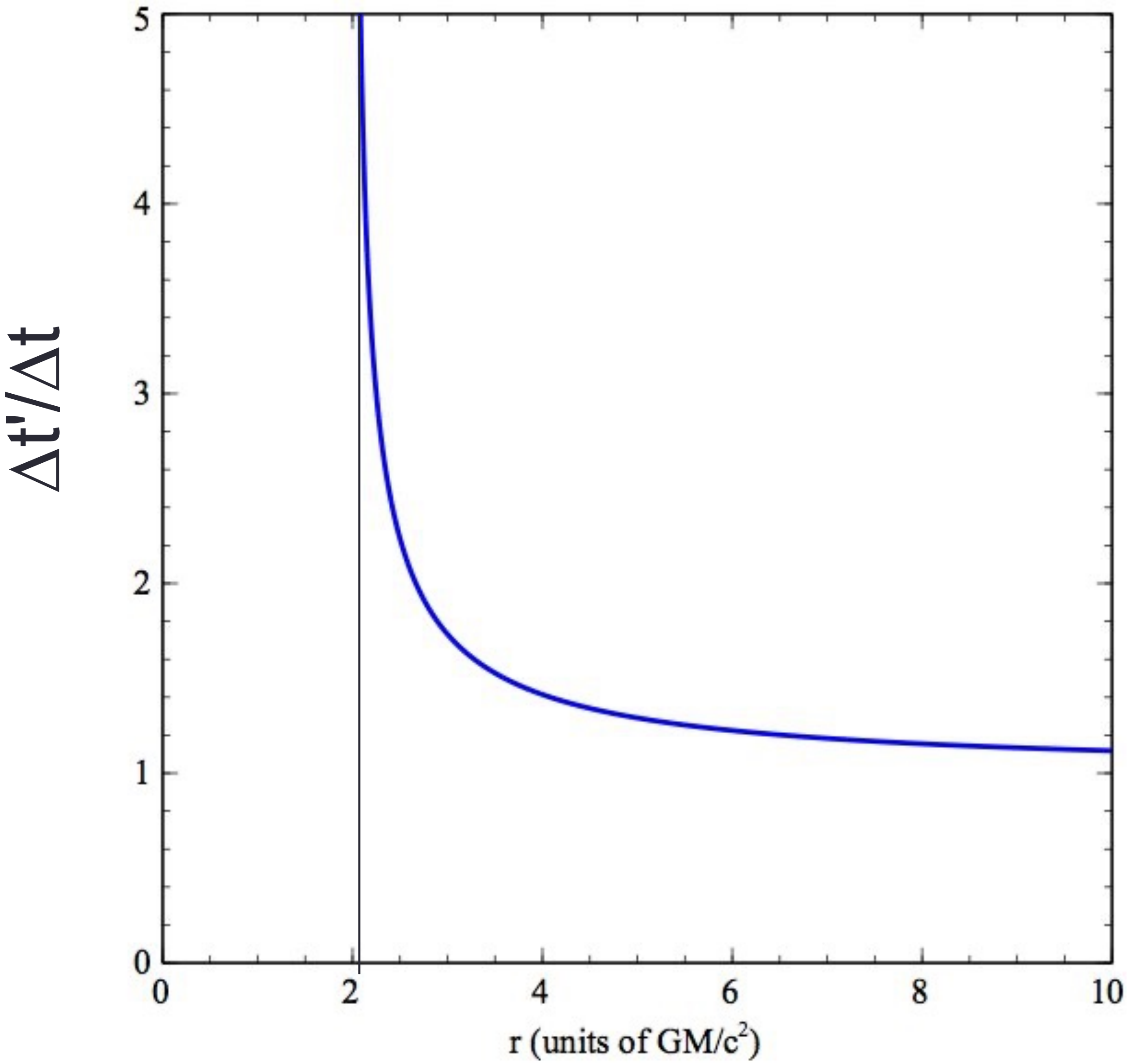


$$t_0 = t_f \sqrt{1 - \frac{2GM}{rc^2}}$$

Δt is what it ticks at in the distant observer's frame

Time slows down in a gravitational field

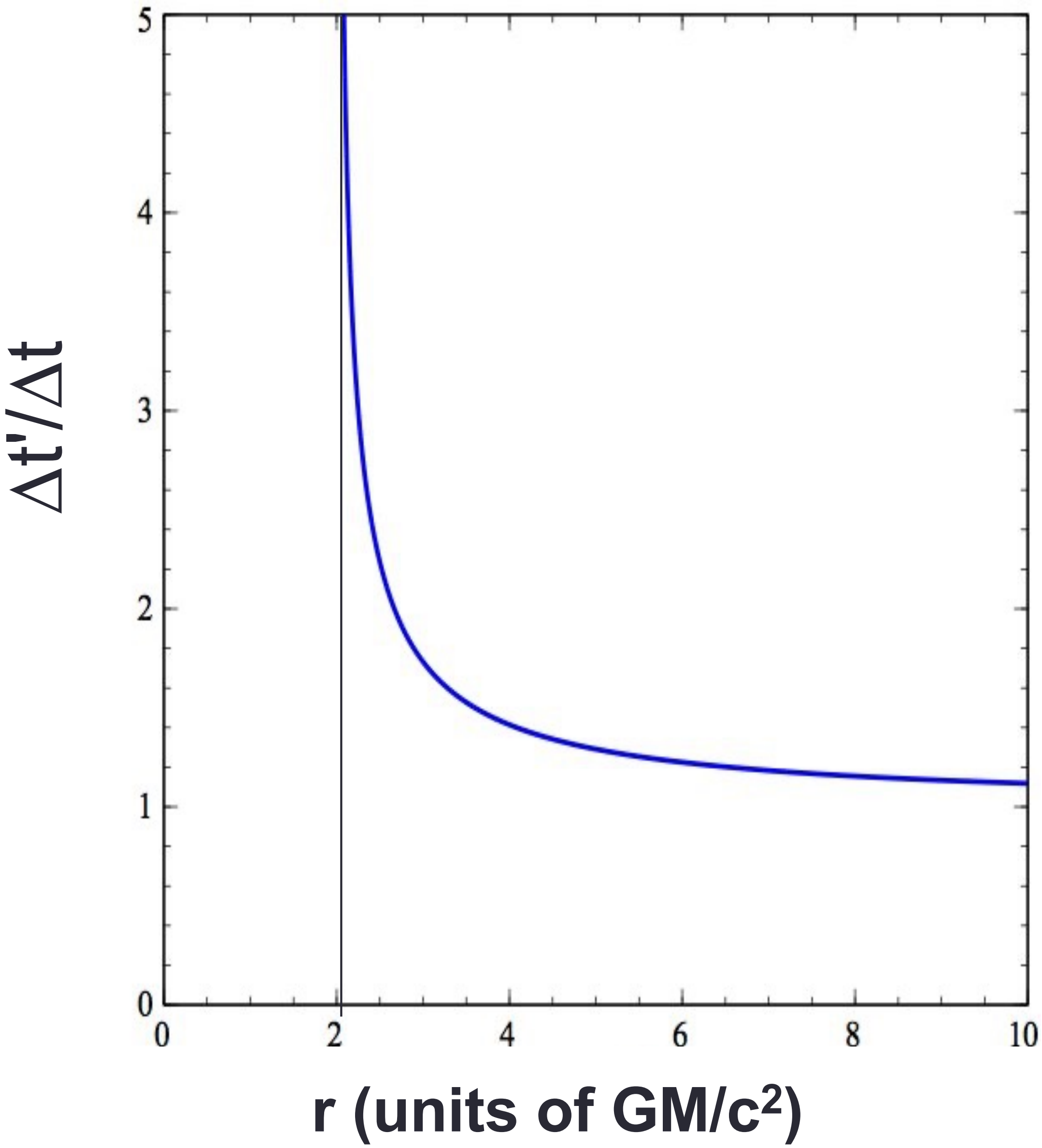
Time 'stops' as r gets closer and closer to $2GM/c^2$



Clock appears to tick more slowly as it gets closer to the black hole… seems to stop ticking when it gets to **$r=2GM/c^2$**. But only as seen by distant observer! Locally, clock seems normal...

The **event horizon**… sphere on which the gravitational redshift is **infinite**

Time 'stops' as r gets closer and closer to $2GM/c^2$



$r=2GM/c^2=R_{sch}$

R_{sch} is the Schwarzschild radius

Event horizon

Point of no return… the location where the **escape velocity equals the speed of light**

- The **gravitational redshift becomes infinite** here (as seen by an outside static observer)
- Nothing occurring inside can be seen from outside (or have any causal effect on the external Universe!)
- So… as a practical matter, we never need concern ourselves with the Universe interior to the event horizon

$$R_{Sch} = \frac{2GM}{c^2} \approx 3\left(\frac{M}{M_\odot}\right)\,\mathrm{km}$$

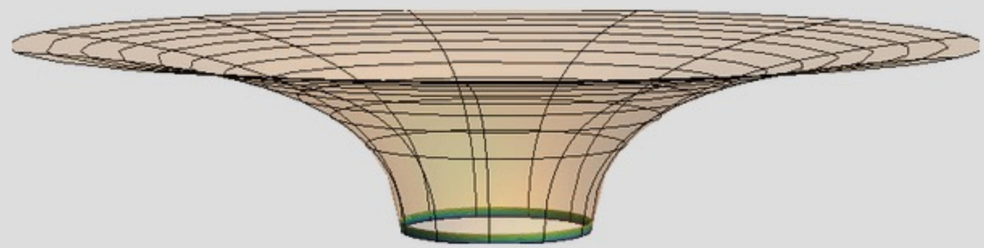
$\odot$symbol for sun, $M_\odot$ =solar mass

- Radius corresponding to event horizon for a non-spinning black hole is known as the **Schwarzschild radius R_{Sch}**

In general relativity we can make a black hole by putting a lot of matter together into a small region.

But can never 'unmake' the hole, since nothing that went in the hole can ever come out.

A typical star (the dark disc) creates a gentle curvature of spacetime



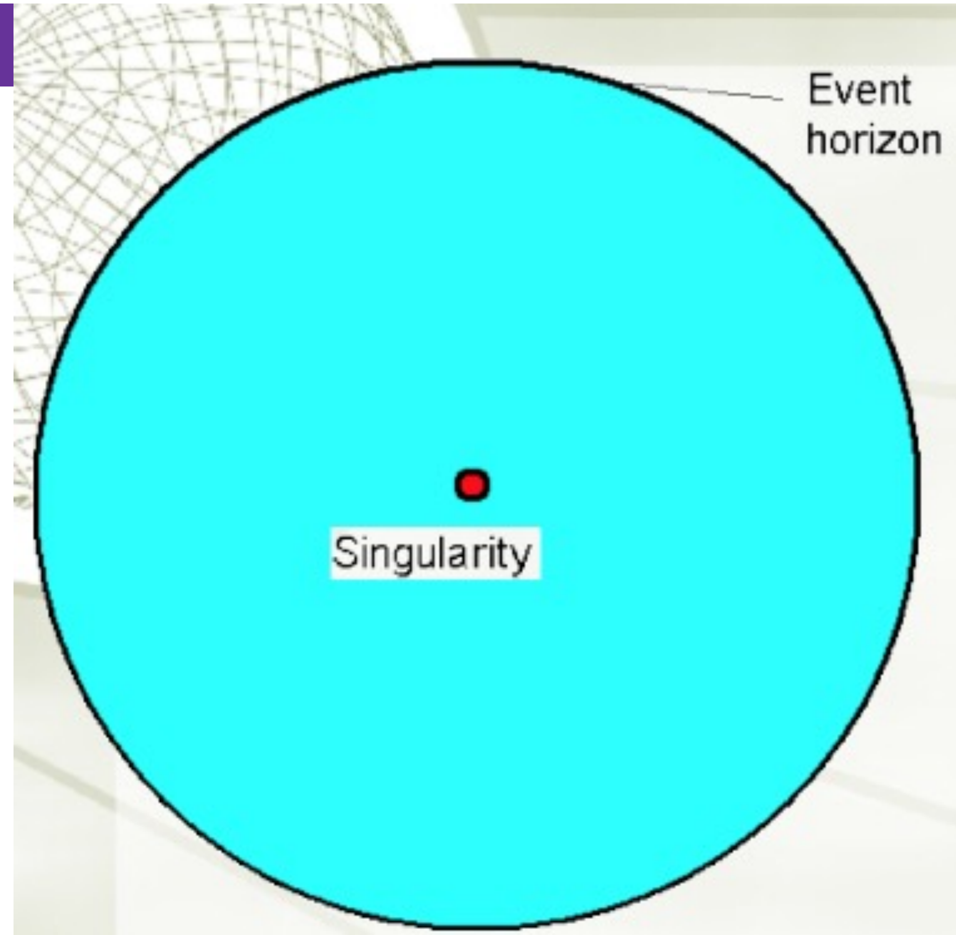
Allowing more curvature eventually leads to a black hole. The exterior of the hole, depicted above, ends in a horizon (the green circle); anything falling past the horizon cannot come back out.

https://www.asc.ohio-state.edu/mathur.16/tutorial1sep17/bhtemplate3.html

GR black holes

For a body of the Sun's mass,
Schwarzschild radius
$R_S = 2GM/c^2 \rightarrow 3km$

- **Singularity** – spacetime curvature is infinite. Everything destroyed. Laws of GR break down.
- **Event horizon (R_s)** – gravitational time-dilation is infinite as observed from large distance.
- Any light emitted at R_s would be infinitely redshifted - hence could not be observed from utside



Schwarzschild radius is NOT the singularity

At the Schwarzschild radius gravitational time dilation goes to infinity and lengths are contracted to zero

View of an External Observer

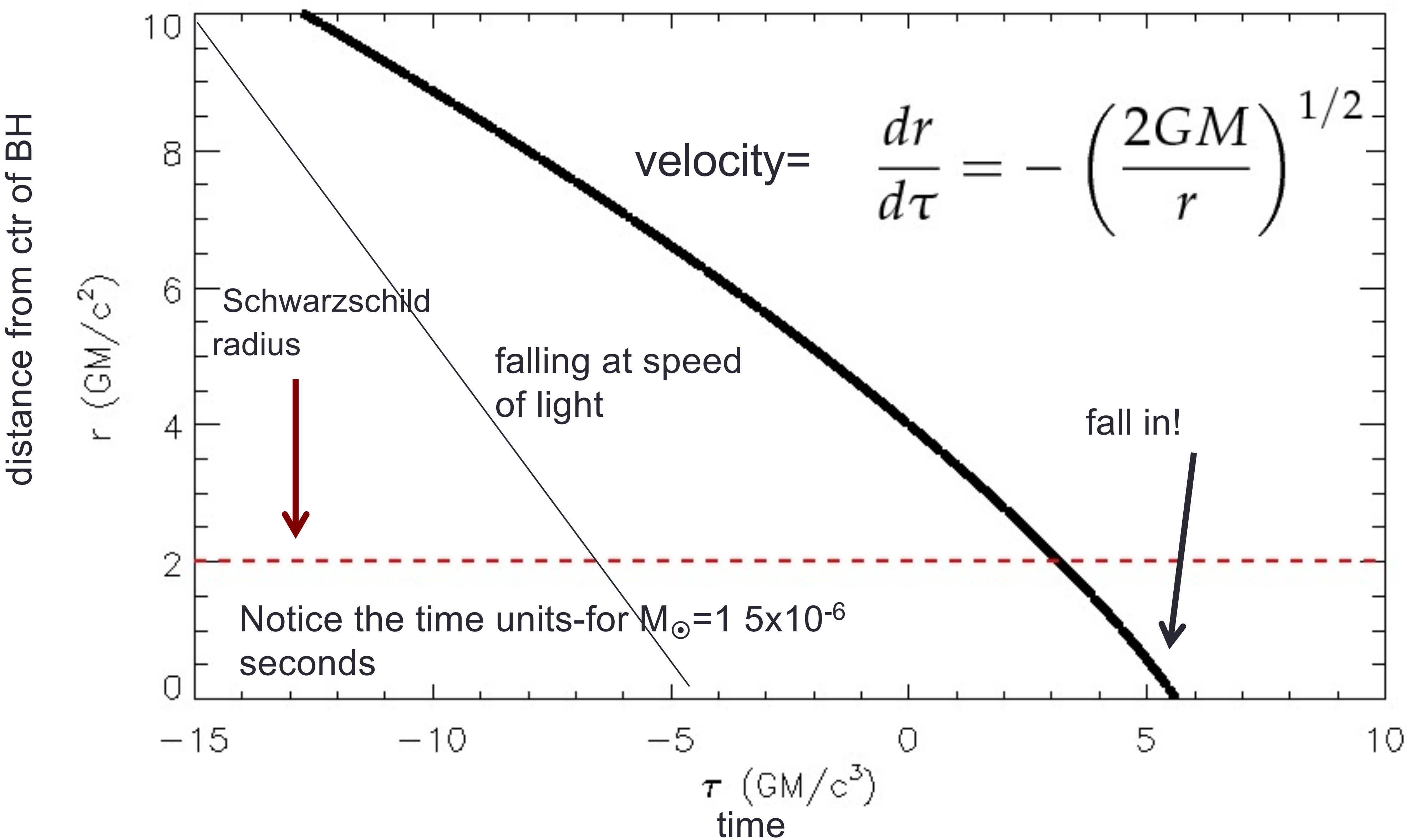
- The event horizon
  - Surface of infinite gravitational redshift
  - *From point of view of distant observer*, infalling objects will appear to freeze at event horizon
  - Old name for black holes was “Frozen Star” (referring to the star that collapsed to create the black hole)
  - Infalling object will also appear to fade away as it freezes (why?)
    Thus, no, you don’t expect to see frozen surprised aliens at the edge of a black hole!

The view of an infalling observer

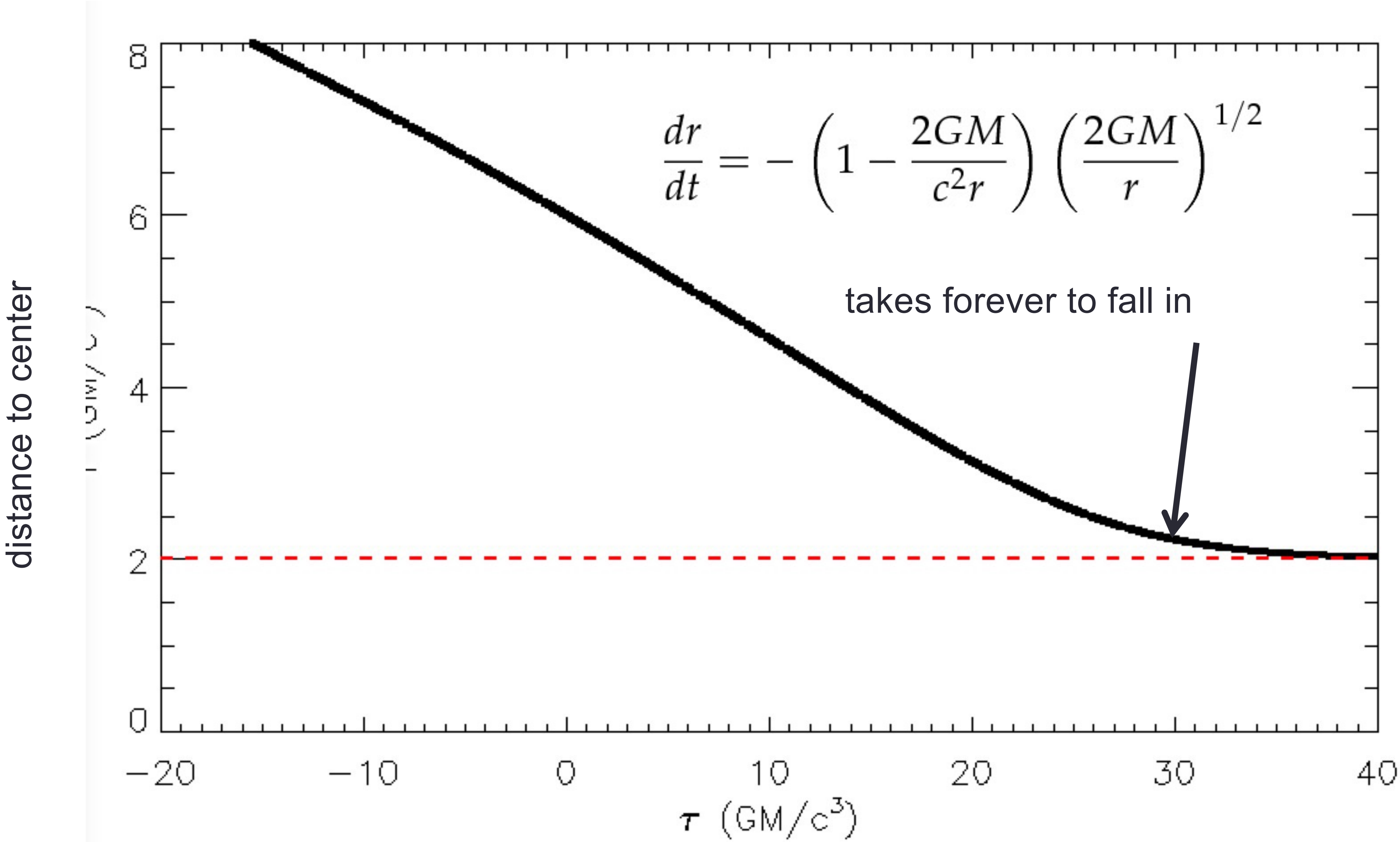
- Very different view by infalling observer
  - Pass through the event horizon without "fuss" – **see later**
  - Eventually reach the center (r=0)

- What happens at the center?
  - Equations of General Relativity break down (predict infinite space-time curvature, corresponding to the infinite density of matter that has been crushed there).
    - Called a **spacetime singularity**
  - Means that GR is invalid and some other, deeper, laws of physics are needed to describe this location (Quantum Gravity)

Falling radially into a black hole – victim's view



Falling radially into a black hole – **external** view



Units

- In previous graphs we had time in units of GM/c^3- unit is seconds
  - G is $6.674\times10^{-11}\ m^3\cdot kg^{-1}\cdot s^{-2}$
    - mass in units of kg, c in units of ms^{-1}
- and distance in units of GM/c^{2-} unit is kg
- These are all normalized to the "gravitational" radius GM/c^2, which is half the Schwarzschild radius

- The Schwarzschild metric is singular at the horizon ($r=2GM/c^2$) but this is only a coordinate artifact.
  - *A free falling observer feels no drama going through the horizon.*

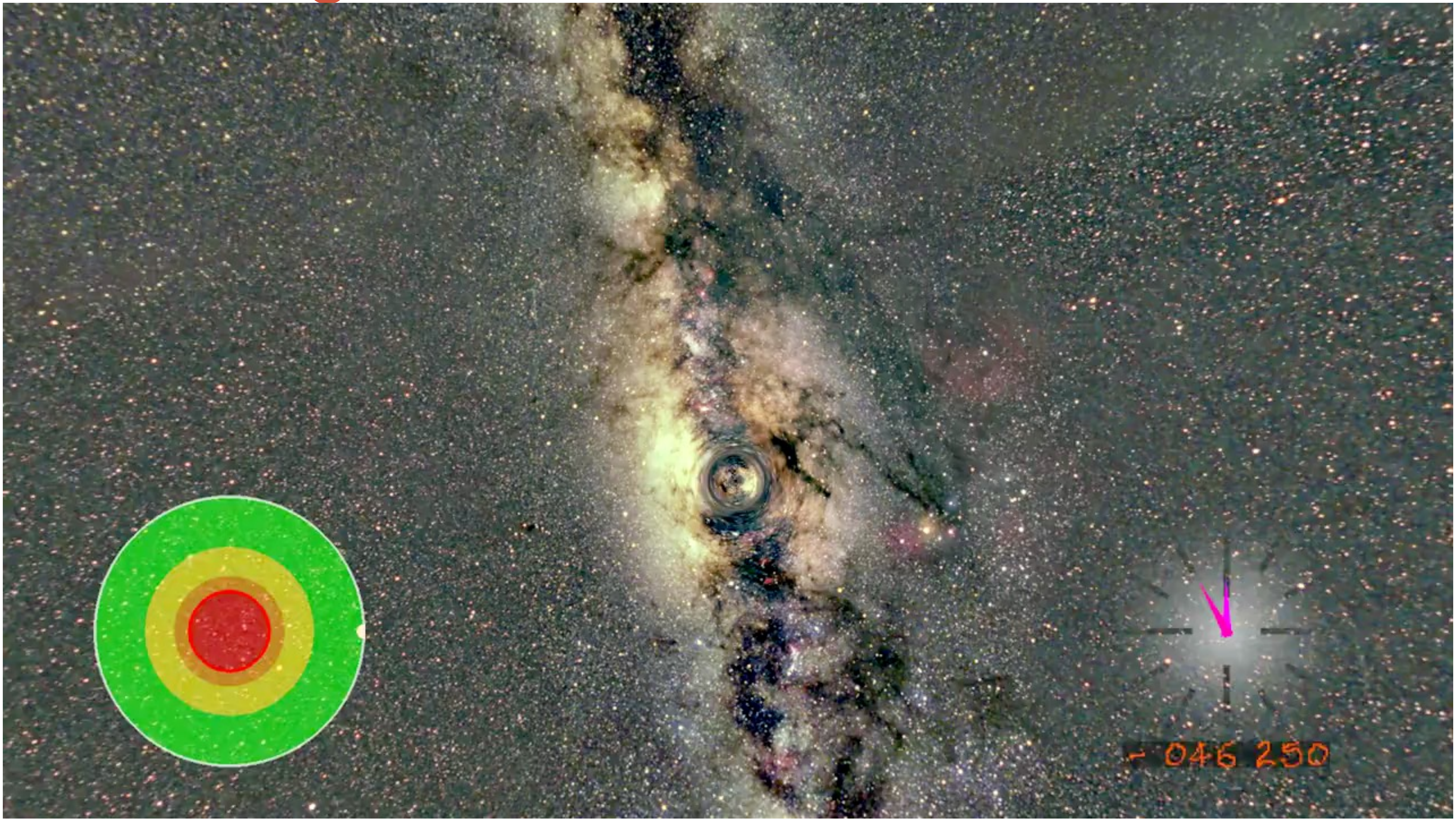
    It takes the observer a finite amount of proper time... but infinite coordinate time.

    We can remove the singularity by a proper coordinate transformation.
  - In contrast, the **origin r=0 is a singularity**. The scalar curvature is infinite and general relativity is no longer valid at this point.

The two views

- How do we reconcile the two views (infalling observer and distant observer)?
  - The distant observer only sees *part* of the timeline of the infalling observer… they never see the part of the timeline inside of the event horizon!
  - This is because the distant observer’s measure of time freezes at the event horizon… but the infalling observer has a perfectly well behaved measure of time as they pass across the horizon
  - We say that the event horizon is a “coordinate singularity”, not a “physical singularity” (similar to the latitude/longitude situation at the north/south pole)

Falling Into a Black Hole



Space is strongly 'distorted'

Near a black hole the curvature of space is extreme and changes rapidly with distance from the black hole

'Shape' of objects as seen by a distant observer is highly deformed

- some parts are amplified and others are
  - reduced in intensity by gravitational lensing

Objects are stretched radially and compressed transversely

Light rays can reach the distant observer from the 'back' of the black hole

More features of **Schwarzschild** black hole

- Events inside the event horizon are causally-disconnected from events outside of the event horizon (i.e. no information can be sent from inside to outside the horizon)-

- Once inside the event horizon, future light cone always points toward singularity (any motion must be inward)

- Observer who enters event horizon would only "feel" “strange” gravitational effects if the black hole mass is small.

More features of **Schwarzschild** black hole

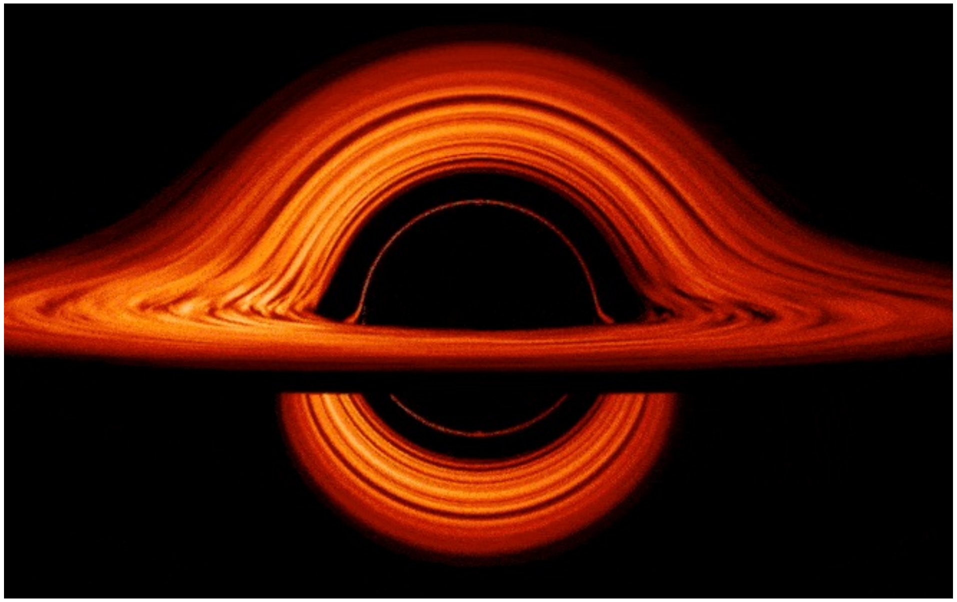
- Stable, circular orbits are not possible inside $3R_s$: inside this radius, orbit must either be inward or outward but not steady

- Light ray passing BH tangentially at distance $1.5R_s$ would be bent around into a circle
  - Thus black hole would produce "shadow" on sky (EHT picture)

Black Hole Shadow

Real data

Simulation



The nature of the event horizon

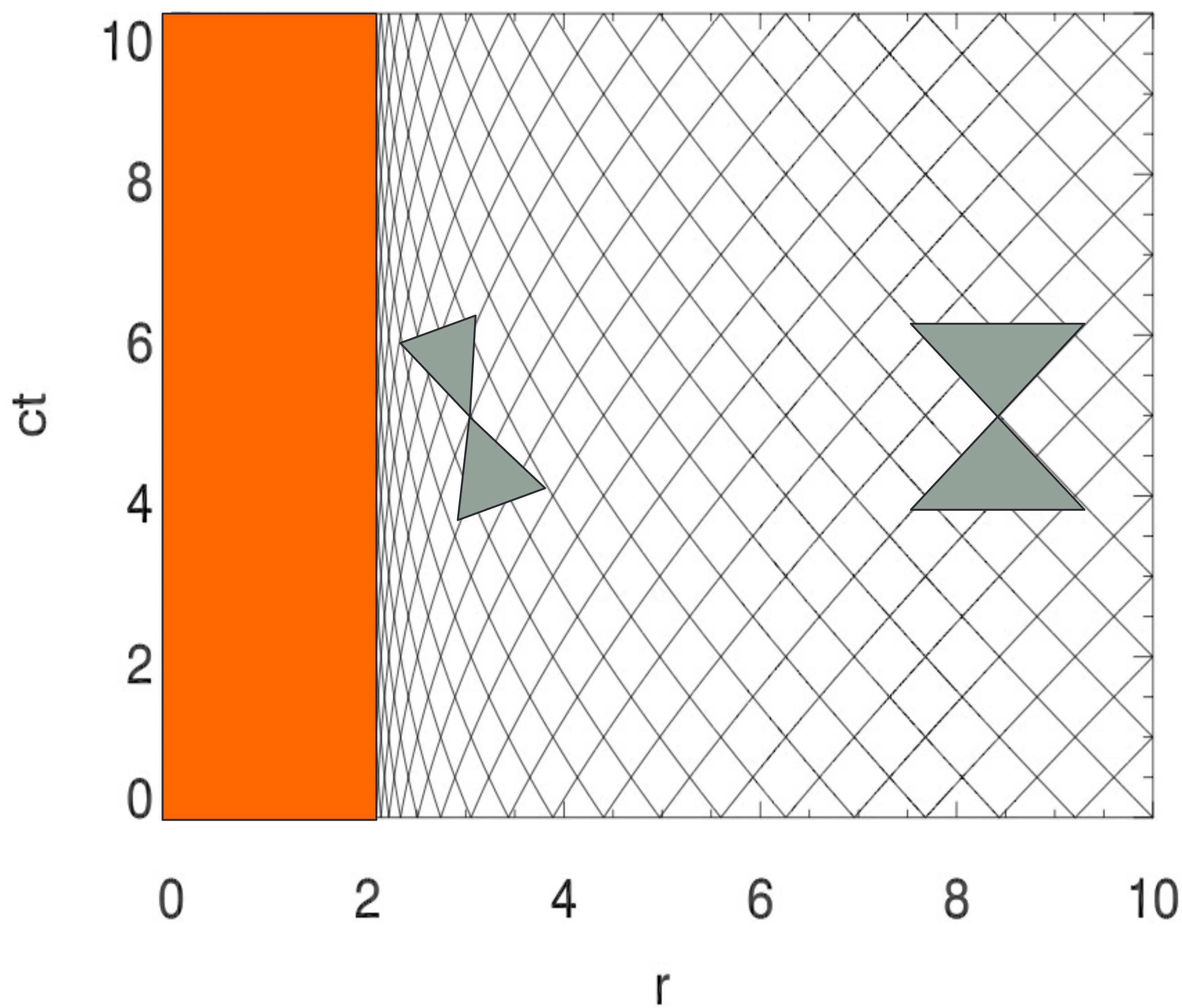
- So, we have learned important things about the nature of the ***event horizon***…
- The Event Horizon is…
  - The infinite redshift surface
  - The place where infalling objects appear to "freeze" according to external observers
  - NOT a real singularity, since infalling observers pass through it unharmed
  - The boundary of the causally disconnected region (we didn't prove this!)
- ***The real singularity is at r=0***

Spacetime diagram… path of ingoing/outgoing light rays as seen by distant observer

r is in units of $2GM/c^2$

No signal can reach a distant from observer from within $r=2GM/c^2$

this region is 'causally' disconnected



Meaning of 'grid'

the lines are the paths that photons take

the photons are emitted at ct=0 in
space-time coordinates at different values of r
– one line represents photons 'shot' outward the other photons 'shot' inward.

The lines are curved due to curvature of space time near the BH

The cones are the 'usual' ones- notice the shape of the cones change as one gets close to the BH

Fate of the photons

A photon starting on the outside of the black hole can travel through the Schwarzschild Radius.

A photon that starts on the inside of the Schwarzschild Radius, never has a path that leads outside of the black hole. It is trapped.

If we had a dense collection of mass with a physical surface boundary less than its Schwarzschild Radius, light from the surface could not escape from it to the outside.

An outside observer could detect the object's existence through its gravitational field but that observer would not be able to see it.
This is the reason why such objects came to be known as black holes.
The sphere defined by $r=2GM/c^2$ shuts off the outside world from observing what's inside.
For this reason the region is called the Event Horizon.

River Model of a Black Hole

https://arxiv.org/abs/gr-qc/0411060

In the river model, space itself flows like a river through a flat background, while objects move through the river according to the rules of special relativity

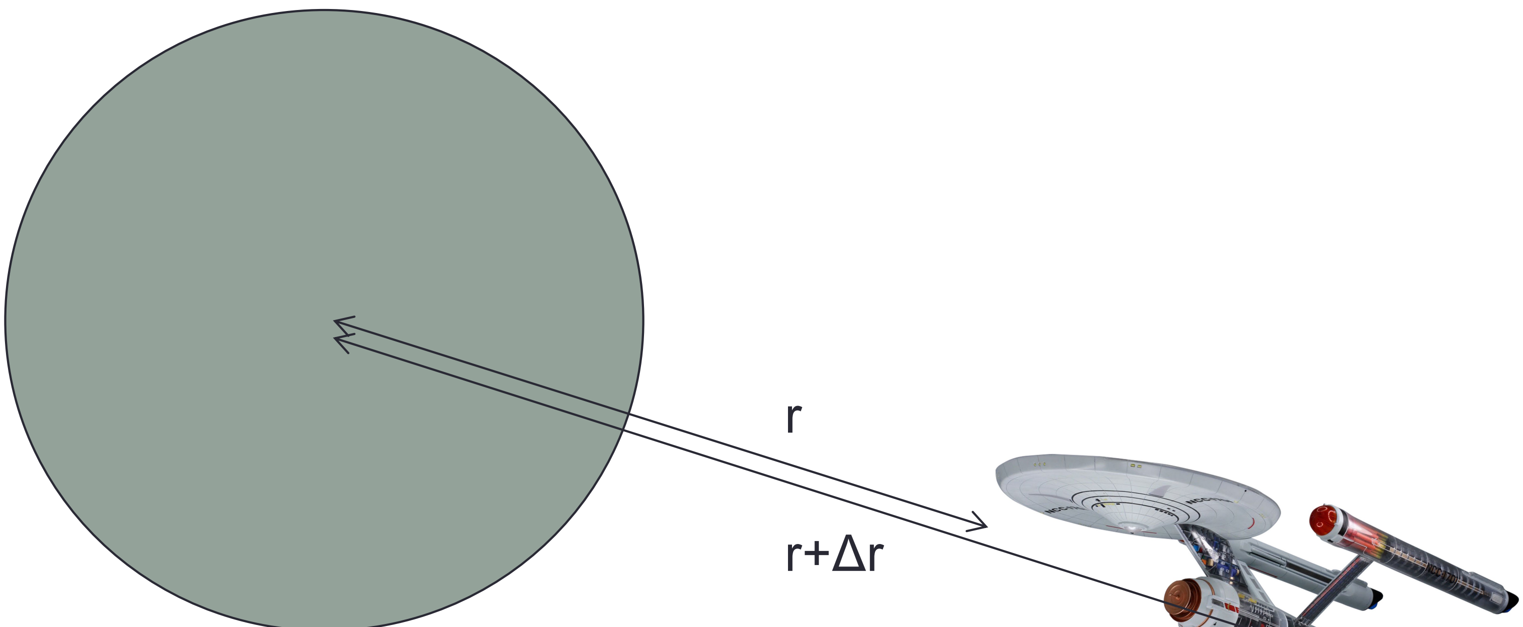
Inside the horizon, the river flows inward faster than light, carrying everything with it.

fish trying to escape

FIG. 1: (Color online) The fish upstream can make way against the current, but the fish downstream is swept to the bottom of the waterfall. F

Spaghettification

- You would might never make it to the center intact… the **gradient of gravity** would tear up an infalling observer in a low mass BH.



there is a “stretching force” known as a tidal force that is **proportional to M/r^3**. This will eventually rip the spaceship apart

$$\frac{GM}{r^2}$$

$$\frac{GM}{(r+\Delta r)^2}$$

tidal force is the difference in gravity between r and r+Δr- use Newtonian approximation

- How strong is the tidal force you feel as you fall through the event horizon? Using a Newtonian approximation, we have

$$F_{tidal} \propto \frac{GM}{R_{evt}^3} \qquad \text{with} \qquad R_{evt} = \frac{GM}{c^2}$$

- So,

$$F_{tidal,evt} \propto \frac{M}{M^3} = \frac{1}{M^2}$$

- **Thus, the more massive the black hole, the weaker is the tidal force at the event horizon!**

So where do you fall apart?

- It depends on the mass of the black holes

x= 1 is the event horizon

https://web.stanford.edu/~oas/SI/SRGR/notes/FallingIntoBH.pdf