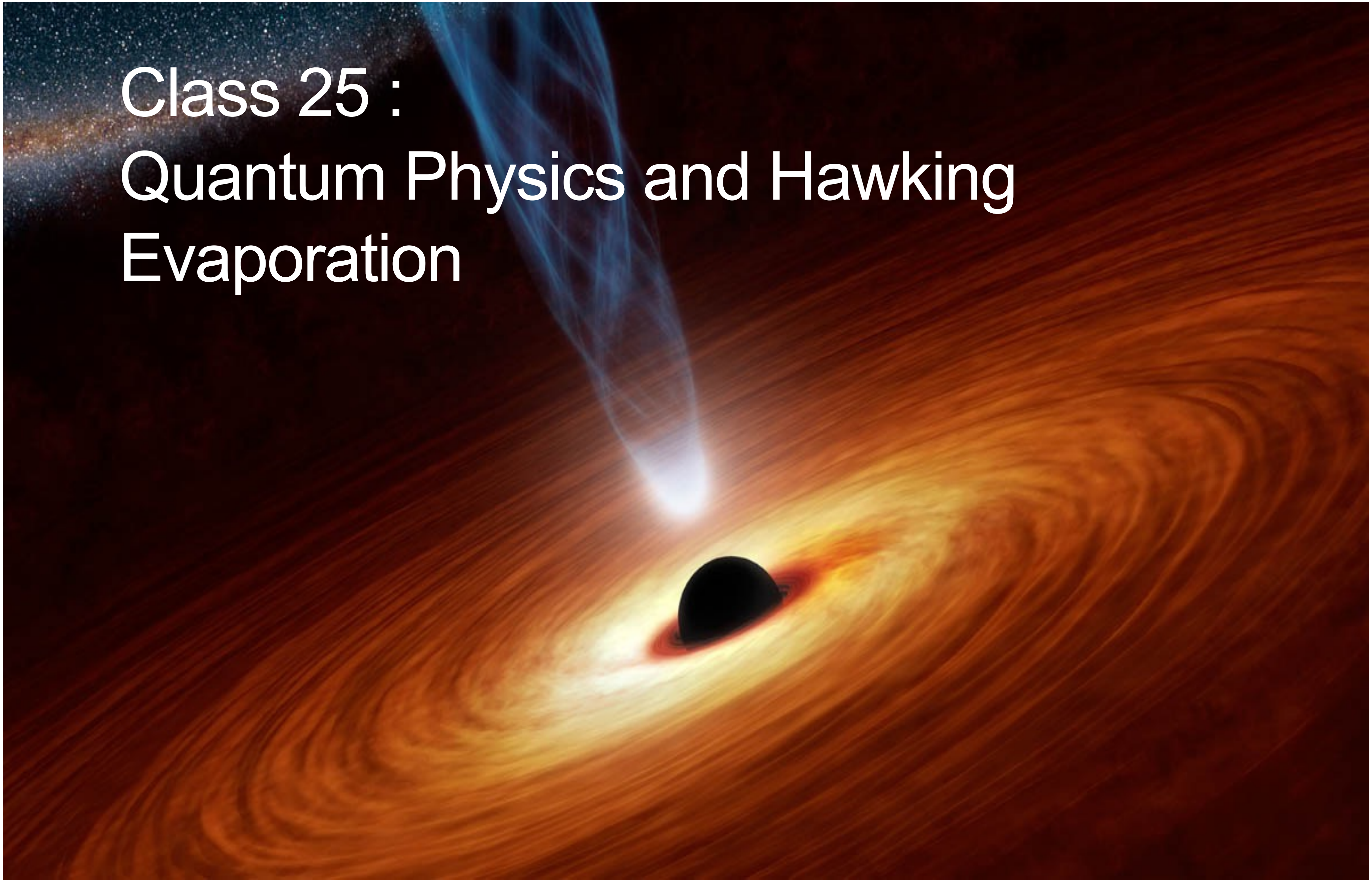


Class 25 :
Quantum Physics and Hawking Evaporation

ASTR350 Black Holes (Spring 2022)
Cole Miller

Today

- Black holes and fundamental physics
  - Start discussion of one of the hottest topics in theoretical physics…
  - Black holes as a place where GR and quantum mechanics battle it out.
- Today
  - **Brief primer on some quantum physics**
  - Heisenberg Uncertainty Principle
  - Hawking radiation and black hole evaporation

Quantum Physics and GR

- Remember from lecture 21 that
  - GR applies to the physics of the very big (gravity and the curvature of spacetime).
  - While quantum mechanics governs the realm of the very small (subatomic particles and their components)

Black holes are both.

Intro to Quantum Mechanics

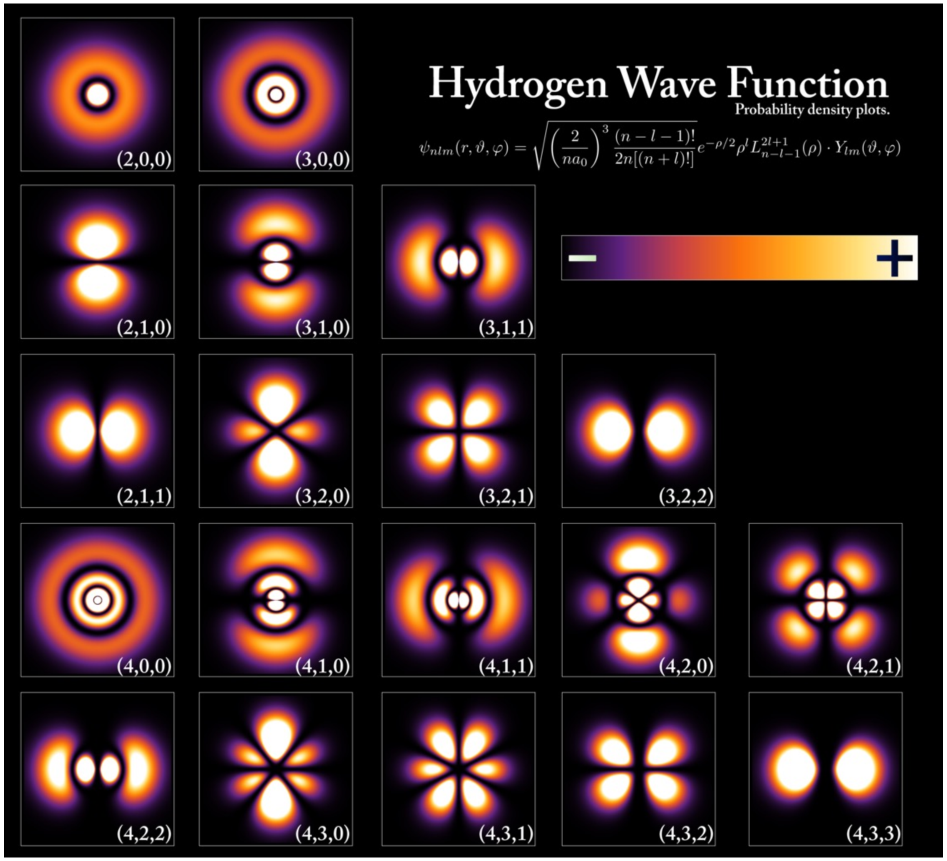
- **This is a very rich and complex field and we will only touch on a tiny bit of it...**

First: philosophical issues

- Quantum mechanics is crazy from our everyday standpoint! Lots of people want to discuss big philosophical issues based on the theory; okay as long as ridiculous claims aren't made (quantum healing???)
- For me: I don't care about the philosophy. I care about whether the theory describes the real universe
- It does! It is tremendously successful
- So whether you use the Copenhagen interpretation, the many-worlds hypothesis, or anything else, my question is: Does your interpretation make different predictions?
- If so, let's hear it and (crucially) how to measure it
- If not, enjoy your philosophizing!
- And now back to the science of quantum mechanics...

Wave functions

- First weirdness: things aren’t localized perfectly in quantum mechanics
- In classical physics, something is in a specific place
- But in quantum mechanics, a function called a ***wave function***, when squared, gives you the probability of something being at any given spot
- Can extend infinitely far, but almost all probability is fairly well localized



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Why do we care?

- Why not just jump straight to Hawking radiation?
- The answer is that quantum mechanics provides some context; in particular, we will eventually conclude that a fundamental principle of QM (the uncertainty principle) allows ghostly "virtual particles" to pop in and out of the vacuum, and that if this happens near a black hole it can cause the black hole to radiate and lose mass(!!!)
- But let's lead into that gradually by looking into wave-particle duality

I : Wave-Particle *Duality*

- Einstein (1905)
  - Proposed that light is emitted and absorbed in discrete packets (“quanta”), what are called photons.
  - These packets behave as particles with energy E and momentum p, given by

$$\boxed{E = \frac{hc}{\lambda} \qquad\qquad p = \frac{h}{\lambda}}$$

  - Here, E is energy, *p* is momentum, h is “Planck’s constant” and λ is the wavelength of the light.

Matter has Wavelike Properties Also

- de Broglie (1924)
  - Proposed that matter particles (e.g. electrons, protons) also have *wavelike* properties, with a wavelength given by

$$\lambda = \frac{h}{p}$$

  - This naturally explained the curious behavior of electrons in atoms previously noted by Bohr… that only certain energy levels are allowed.
  - Verified with interference experiments with electrons. One even gets interference with only 1 electron at a time (!!)

Matter has Wavelike Properties Also

- This phenomenon has been verified for all matter: elementary particles, atoms and even molecules.
- For non-relativistic particles, p=mv
- For macroscopic particles, because of their extremely short wavelengths, wave properties are NOT noticed in everyday life.
- For example, a flea crawling at 1 mm/sec has $\lambda \sim 10^{-22}$ cm
  For comparison: a proton has a radius $\sim 10^{-13}$ cm
- You'd never detect this in a classical experiment

Matter has Wavelike Properties Also

- But at the atomic scale, it matters
- Wavelength for a 10 eV (that's a typical energy value) electron is ~4x10^{-10}m. This is comparable to the space between atoms
- Thus crystals can act as a diffraction grating for electrons

Matter has Wavelike Properties Also

- We can tell this by looking at the interference pattern using a diffraction grating using molecules traveling at ~200m/sec rather than light.

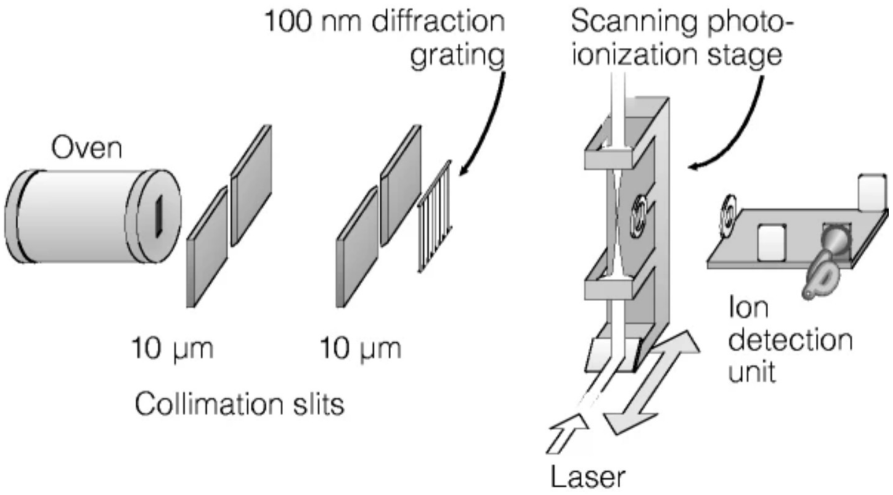
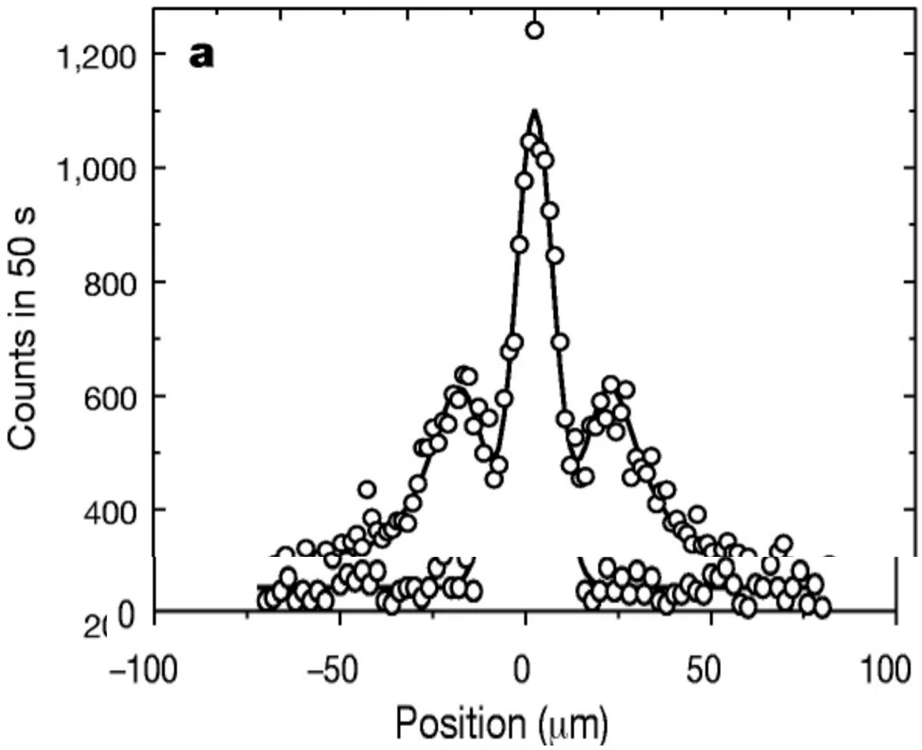


Figure 2: Interference pattern produced by C_{60} molecules.



The uncertainty principle

- One of the consequences of this new understanding is the ***uncertainty principle***, which was first articulated by Werner Heisenberg
- The idea is that you cannot simultaneously measure the position and momentum of a particle (or, more importantly for Hawking radiation, the energy and time for a particle) with perfect precision
- Unfortunately, the alien nature of this principle to our everyday life has provided a way for scammers to extract a lot of money from the unwary...
- But how do we understand the uncertainty principle?

Motivating the uncertainty principle

- Suppose that we want to know the location of a particle
- Let’s shine light on it! The shorter the wavelength of the light the more accurately we can measure the position of the particle
- BUT: the shorter the wavelength of light the more energy the photon has and when the photon hits the particle and bounces back to us it must affect the particle’s momentum!
- This is where wave-particle duality comes in: photons (and everything else) carry momentum
- You could use electrons, protons, anything else instead of photons
- Net result: better determination of the location means a larger kick and thus a worse determination of the momentum

II : Heisenberg’s Uncertainty Principle

- There are many different uncertainty principles for *different pairs of variables*
- The best known is the position-momentum principle…
  - Suppose we know the position to an accuracy of Δx and the momentum to an accuracy of Δp. Then

$$\Delta x \Delta p \geq \frac{1}{2}\hbar$$

  - Here, “h-bar” is shorthand for h/2π
  - Must be stressed that this is fundamental… not some “boring” limitation of measurement apparatus

II : Heisenberg’s Uncertainty Principle

- Thus the more precisely the position of some particle is determined, the less precisely its momentum can be predicted and vice-versa.
  - The effect is small and not readily apparent on the macroscopic scales of everyday experience.

II : Heisenberg's Uncertainty Principle

- There is also an uncertainty principle linking energy and time…
  - Suppose we know the energy of process to an accuracy of ΔE and the duration of that process to an accuracy of Δt. Then,

$$\Delta E \Delta t \geq \frac{1}{2}\hbar$$

  - So, for finite duration events, energy need not be strictly conserved (since it cannot be precisely defined!); since $h=6.67\times10^{-27}$ (cgs units) this is a very short time !

Stealing money from the Mafia

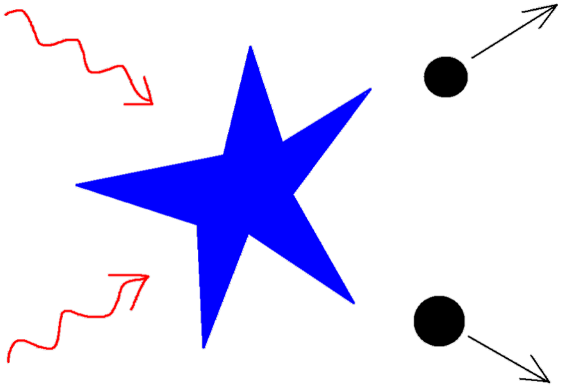
So in quantum mechanics, temporary nonconservation of energy is like stealing money from the Mafia; it’s fine as long as you give it back before anyone finds out...

Why Are We Doing This?

- What is another place where this is a lot of energy and short time scales?
- We wish to connect GR and quantum mechanics, and this (and Hawking's insight) has the lure of lots of fundamental physics
- But I also want to comment on philosophical implications
- Lots of people make a big deal about how profound the uncertainty principle is: in quantum mechanics you cannot say exactly what will happen from an initial state, whereas in classical physics you can. Amazing!
- But that's overhyped. Chaos exists in many systems. You can't predict the weather a month in advance because of chaos, and that would be true even if the universe were fully classical.

III : Pair production and vacuum fluctuations

- Remember E=mc^2…
  - Important consequence is that, if we have some energy E, then we can spontaneously create a particle/anti-particle pair



  - The shorter the time, the more energy you can borrow from the vacuum

To avoid confusion...

- Photons (and gravitons, assuming they exist) are their own antiparticles
- When we talk about an evaporating black hole, if the black hole starts with ~stellar mass or larger, the ***overwhelming majority*** of Hawking radiation comes out in photons and gravitons, not particles such as electrons or their antiparticle, positrons
- So really the picture you should have, most of the time, is that spontaneously, in the vacuum, two photons are temporarily produced
- That’s still pretty cool!

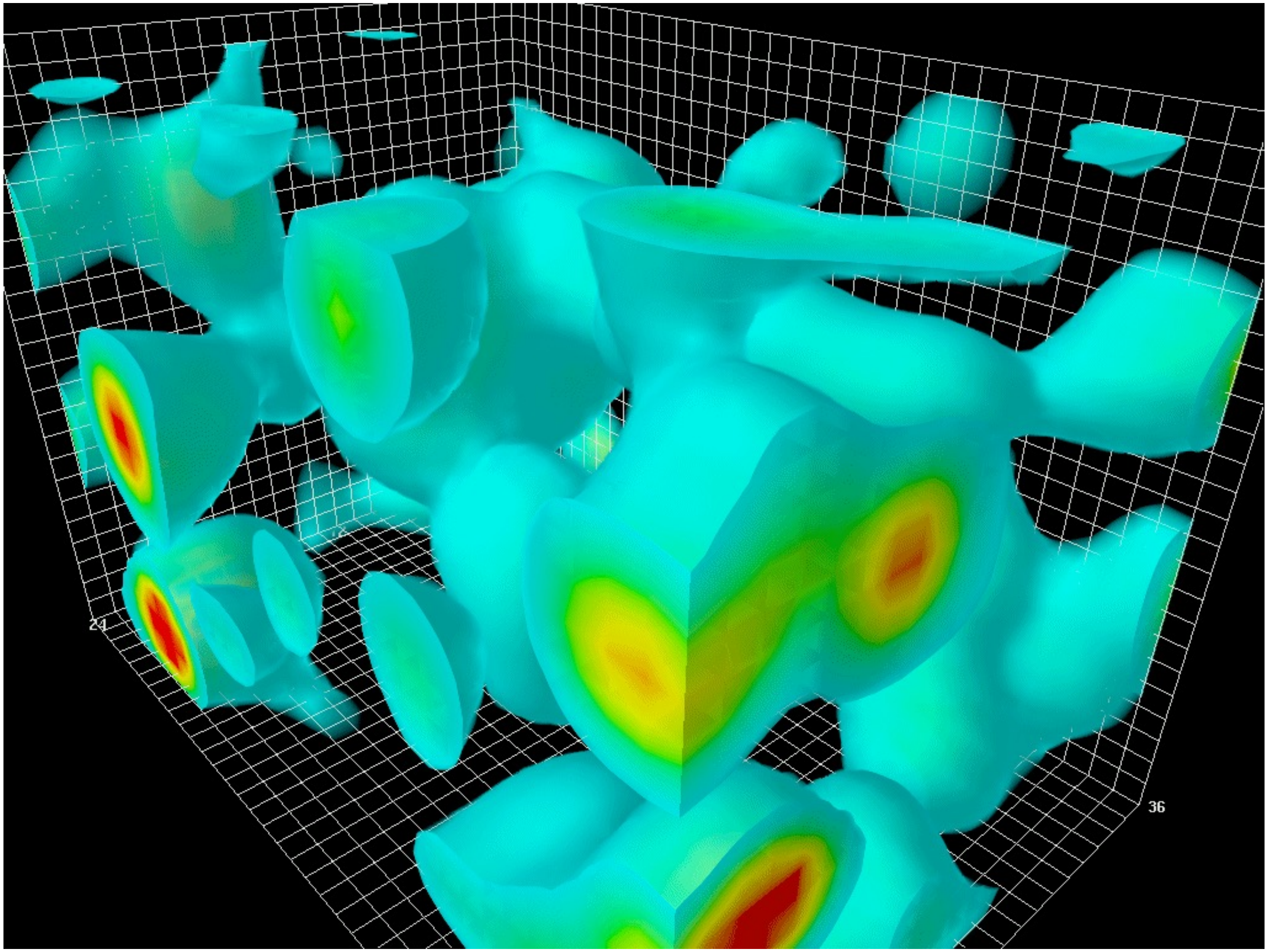
- In a vacuum, we can have particle/anti-particle pairs popping into existence spontaneously provided that they annihilate quickly enough (so that “nature doesn’t notice”)... Virtual particles.*
- New picture for the vacuum… rather than being empty, it is a sea of virtual particles of all kinds.

- *virtual particle "a transient quantum fluctuation that exhibits some of the characteristics of an ordinary particle, while having its existence limited by the uncertainty principle

How Can It Work?

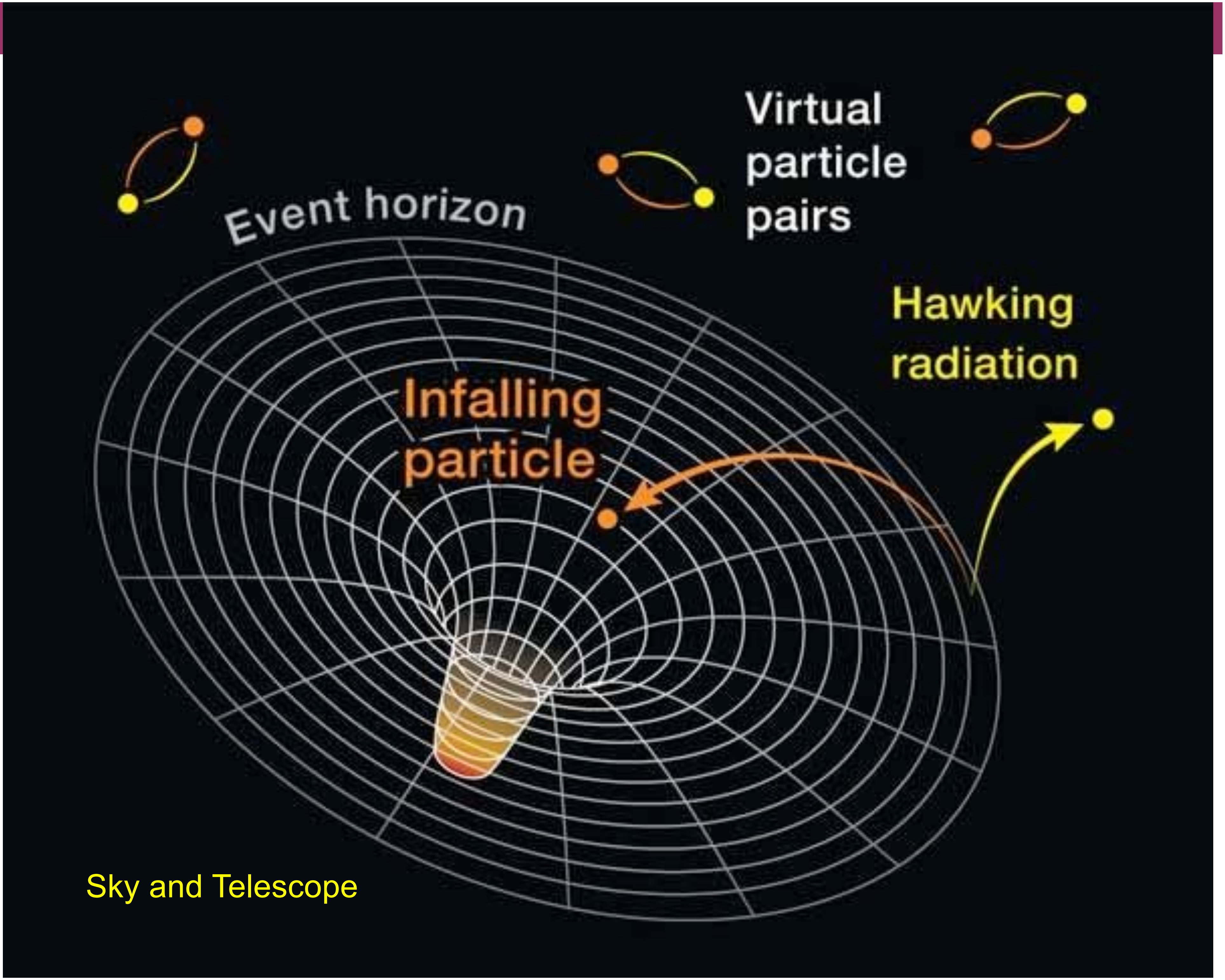
- For pair production to occur, the borrowed energy must be above a threshold of the total energy of the two virtual particles
- All other conserved quantum numbers (angular momentum, electric charge, lepton number) of the produced particles must sum to zero because they were zero in the vacuum
  - – Thus the created particles have opposite values of electric charge and other quantum variables of each other e.g. they are a particle/antiparticle pair)
  - The reverse of this process is particle/antiparticle pair annihilation.
- Mind you, the same process can happen with ***real*** particles; for example, two photons can hit each other and produce an electron/positron pair. But then you don't need to borrow energy

Visualization of Quantum Fluctuations



Hawking Radiation

- Things get interesting close to a black hole event horizon… can convert virtual particles into real particles!
- How does this happen?
  - Major discovery by Hawking, based somewhat on previous work by others (Bekenstein, Zel'dovich)
- Basic idea is that near the event horizon, the tidal gravitational field of the black hole can pull apart the pair; this injects energy and makes the pair real instead of virtual
- If one member of the pair escapes, then the black hole radiates!



Sky and Telescope

But wait: how does the BH lose mass?

- Hold on: when we look at that picture, it seems that Hawking radiation happens when one particle falls in the black hole and the other escapes. Doesn't that mean that the BH should gain mass, because it accretes a particle?
- Perhaps surprisingly: no
- Remember that the pair of particles was virtual. To make it real, the BH's tidal field injected gravitational energy equal to the total energy of both particles. For a BH, gravity=mass, so temporarily the BH's mass drops by that much.
- If the BH only accretes one of the particles, then, it has, in net, lost on particle's worth of mass; it radiates!

Hawking Radiation

- Conclude that black holes are radiating!
  - Referred to as Hawking Radiation
  - Radiation has “black body” form with a temperature of

$$T = \frac{\hbar c^3}{8\pi G M k_B} \approx 6 \times 10^{-8} \left(\frac{M}{M_\odot}\right)^{-1} K$$

  - Radiated energy must come from the mass-energy of the black hole itself… thus the black hole’s mass must decrease over time.
  - But this is a VERY slow process unless the black hole is microscopic.
- This is starting point for the “information paradox”, which we will discuss in two lectures…

Hawking Radiation

- Notice that the less massive the black hole the hotter the radiation
- This is related to relation between BH mass and curvature of space : the smaller the event horizon is, the greater the curvature of space near the event horizon is, and thus the greater the rate of Hawking radiation which goes as T^4.
  - An order of magnitude argument: the wavelength of particles that escape is ~M. The temperature therefore is $T\sim 1/M$. In blackbody radiation, the flux is $\sim T^4$, so the flux is $\sim 1/M^4$. The area of the black hole is $\sim M^2$, so the luminosity is $L\sim 1/M^2$. The mass is ~M (obviously!), so the lifetime is $\sim M/(1/M^2)\sim M^3$.
- The Hawking radiation that leaves a black hole is almost exclusively photons, not matter or antimatter particles and is due to the annihilation of the pairs
- The explanation that I gave is extremely simplified and like most analogies is not correct in detail, but it conveys the concept.

Black Hole 'Evaporation'!

- So a black hole has a temperature and 'radiates'- so what?
- Using $E=mc^2$ the Hawking radiation carries away energy and thus mass
- As the BH gets less massive it gets 'hotter' and loses energy faster
- Thus BHs *slowly* lose their mass and 'disappear'

How slowly?

- The rate of mass loss in Hawking radiation is proportional to $\mathbf{M^{-3}}$
  - A solar-mass black hole would take 10^{67} years to evaporate but a 10^{8} **kilogram BH** would evaporate in 1 year.
  - a ~10^{11}kg black hole (the mass of Mt. Everest but the size of a proton) would evaporate in the Hubble time
    - see BH evaporation calculator https://www.vttoth.com/CMS/physics-notes/311-hawking-radiation-calculator

So What Happens?

- When a black hole decays, the last thing may be a brilliant, energetic flash of radiation and high-energy particles.
  - This might have happened for low enough mass BHs – only BHs of mass less than 1.7×10^{11}kg would have evaporated during the age of the universe !
  - Not clear how to make such an object (primordial BH ???)
  - Upper limits on γ-ray emission indicates that <1% of the dark matter could be such low mass BHs
- In next slide we'll talk about implications... but please remember that ***Hawking radiation has never been seen***, so we shouldn't get too agitated yet ☺

Some fundamental Hawking puzzles

- In Hawking's calculation, the radiation emitted from a black hole **does not depend** on how the object was created. This means that two black holes created from different initial states can end up with the identical final radiation.
- Is this a problem? Yes, it is; we'll talk about this when we get to the information paradox, but it seems to contradict an axiom of quantum mechanics
- Also, an equal number of particles and antiparticles should be radiated. For example, when we get to the point where electrons and positrons can be radiated, there should be equal numbers of both
- But it was almost all electrons going in, so that implies nonconservation of "lepton number", which is obeyed in particle accelerators; need new understanding of physical laws?