The Physics Behind the Cosmic Microwave Background

Without question, the source of the most precise information about the universe as a whole and about its early state is the cosmic microwave background (CMB). This background is incredibly smooth, with a temperature that varies by a typical fractional amplitude of only $\sim 10^{-5}$. Nonetheless, the fluctuations themselves have now been observed so well that they have strongly supported the hot Big Bang model, provided evidence for the early rapid expansion of the universe called inflation, and along with other observations provide evidence in support of dark energy and dark matter. In this lecture we will discuss the physics behind the CMB, and in the next will discuss the implications and what awaits future instruments.

Basics

First, let us discuss some zeroth order aspects of the CMB: its existence, its spectrum, and its smoothness.

The first prediction of background radiation was made by George Gamow in the late 1940s. He reasoned that if the universe was once much hotter and denser than it is now, then it would be optically thick, meaning that a typical photon would scatter many times on a trip across the universe. This would also lead to occasional absorption and reemission, meaning that the radiation and matter would be in thermodynamic equilibrium. At this time, therefore, the radiation would have a blackbody spectrum.

As we discussed in an early lecture, redshifting preserves a blackbody spectrum, simply decreasing the temperature as $T \propto a^{-1} \propto (1+z)$. Therefore, when the radiation and matter decoupled, the radiation would be left to stream across the universe to us. The current temperature of this background radiation is $T_{\rm CMB} = 2.73$ K, and the energy in this background is greater than the energy in all other photons in the universe combined.

An interesting point about this background is that it is isotropic: the temperature is the same in all directions, to roughly a part in 10^5. At first glance this may seem unremarkable; isn't it just what we expect from the cosmological principle? Further thought, however, reveals a puzzle. Recall that in a matter-dominated universe the scale factor goes with time as $a \propto t^{2/3}$, and in a radiation-dominated universe goes as $a \propto t^{1/2}$. This means that the region of the universe in causal contact with us (i.e., that could be physically affected by things moving at light speed or slower) is constantly increasing. In turn, this implies that at the time that the CMB set on its way to us, at redshift $z \sim 1000$, only small patches of what we see could have been in causal contact. How, then, could they have known to coordinate their temperatures to such a degree? The best current answer turns out to be inflation, which we shall discuss in a later lecture.

Radiation-matter decoupling

Now, however, it is instructive to estimate the redshift from which the CMB streams. We will start as always with a simple estimate, then proceed to a more sophisticated approach that is an example of how to deal with potentially complicated equations.

(a) To make a first simple guess as to when the universe became transparent, let's do the following. The most important interaction of light with matter at that stage was Thomson scattering. The "cross section" for scattering (i.e., the effective area of an electron for scattering by a photon) was then

$$\sigma = 6.65 \times 10^{-25}\ \text{cm}^2\ . \tag{1}$$

The universe is mostly ionized at the present time (that is, most electrons are free rather than bound in atoms). The number density of electrons at the current time is $n_0 \approx 2\times 10^{-7}\ \text{cm}^{-3}$. At a redshift z, the number density is $n(z) = n_0(1+z)^3$. The age of the universe was $t(z) = 1.4 \times 10^{10}\ \text{yr}(1+z)^{-3/2}$ (actually, it's somewhat different than this, because of the dominance of dark energy in the last half of the age of the universe), hence the radius of a causally connected part of the universe was

$$R(z) = ct(z) \approx 1.4 \times 10^{28}(1+z)^{-3/2}\ \text{cm}\ . \tag{2}$$

The *mean free path* of a photon (i.e., the typical distance one would travel) is

$$L(z) = 1/[\sigma n(z)]\ . \tag{3}$$

You should find that at large redshift, $L(z) < R(z)$, so that a photon scatters before it crosses the universe, whereas at smaller redshift, $L(z) > R(z)$, so that a typical photon crosses the universe without scattering.

Using these assumptions, what is the redshift when $L(z) = R(z)$?

Answer:

(a) We find that $L(z) = 1/(n\sigma) = 7.5 \times 10^{30}(1+z)^{-3}$ cm. Equating $L(z)$ and $R(z)$ gives $(1+z)^{3/2} = 540$, or $1+z \approx 66$. As we know, this is much too low, because it assumes almost complete ionization. That's why we need to do things in a more sophisticated way.

(b) Now let's do things more carefully, distinguishing between cross section and opacity. For simplicity, we will pretend that the universe was pure hydrogen instead of about 25% helium by mass. The Saha equation then tells us that the fractional ionization $y \equiv n_e/n$ (i.e., the number density of free electrons divided by the total number density) is given by

$$\frac{y^2}{1-y} = \frac{4 \times 10^{-9}}{\rho} T^{3/2} e^{-1.579\times 10^5/T}\ , \tag{4}$$

where ρ is the *total* mass density (including protons) in g cm^{-3} and T is the temperature in K: $T = 2.725(1+z)$.

With these assumptions, what is the redshift at which $L(z) = R(z)$?

Answer:

(b) The Saha equation is moderately complicated, so we need to be able to approach it carefully. Suppose that the ionization fraction at the epoch of transparency is y. We then find a redshift that is given by $(1+z)^{3/2} = 540/y$, or $1+z \approx 66y^{-2/3}$. We note that $\rho = 2 \times 10^{-7}(1+z)^3 \times 1.7 \times 10^{-24}$ g cm^{-3}, where the second factor is the mass of a proton, meaning that we can write the Saha equation as

$$\frac{y^2}{1-y} = 5.3 \times 10^{22}(1+z)^{-3/2}e^{-5.794\times10^4/(1+z)} \ , \tag{5}$$

where we have also used $T = 2.725(1+z)$. Substituting $1+z = 66y^{-2/3}$ gives

$$\frac{y}{1-y} \approx 10^{20}e^{-878y^{2/3}} \ . \tag{6}$$

We need to solve this for y. It looks ghastly; how do we do it? The key is in the exponential. It is clear that if y is anywhere close to unity, the exponential factor will be vanishingly small, so we will not get close to the answer. Therefore, y must be much less than unity. As a result $1-y$ is close to 1, so we get

$$y \approx 10^{20}e^{-878y^{2/3}} \ , \tag{7}$$

where $y \ll 1$. This is still a transcendental equation, but since the right hand side varies wildly with different choices of y, we can converge fairly rapidly on the solution. For example, $y = 10^{-3}$ gives a right hand side of 1.5×10^{16}, which is much too large. $y = 10^{-2}$ gives a right hand side of 425, which is better but still too big. $y = 3 \times 10^{-2}$ gives 1.5×10^{-17}(!!), which is much too small. Eventually, we find that $y = 0.01373$ does the trick, giving $1+z = 1151$. The actual answer is $1+z = 1089$. What is the cause of the discrepancy?

The main problem is that in fact the ionization fraction is not quite what the Saha equation would say, because the ionization is not in equilibrium. When an electron and a proton combine to form hydrogen, a photon is emitted that has an outstanding chance to ionize another hydrogen atom. This would leave no net change in the ionization. Other processes are necessary, e.g., that the photon redshift enough before absorption that it cannot ionize the atom, or that two-stage recombination happens (i.e., the electron and proton first form an excited state of hydrogen, then radiate to the ground state; neither of those photons could ionize hydrogen in its ground state). Consideration of the rate equations means that there is residual ionization left as the universe expands. This higher ionization fraction pushes the redshift of transparency to a lower value than it would otherwise have.

By the way, a secondary contributor is that, given the recent rapid expansion of the universe, the density at a given cosmic time is slightly different than we assumed. By itself, however, this would only change the redshift to about 1140 instead of 1150.

Acoustic Oscillations

Typically, in space no one can hear you scream. However, if you scream loudly enough then even the relatively low density in space will react with acoustic oscillations. In fact, quantum theory predicts that very early in the universe fluctuations would have introduced disturbances at all scales. Because of this generic prediction, we expect that the universe is "ringing" at various special frequencies. In turn, this implies special angular scales at which ripples in the CMB will be especially prominent. The precise angular scales of those ripples, and their relative amplitudes, contain the information for which the CMB is justifiably famous.

To understand this, let us consider the basics of acoustic oscillations. If a particular region has an excess of pressure, then it will expand or propagate. If there are no boundaries, then any frequency will do; this is the case when we talk in open air, for example. If instead there are boundaries, then special wavelengths will resonate, leading to higher amplitudes than others. This is the idea behind musical instruments. For example, a violin string is clamped at both ends, so wavelengths that fit an integer number of times are most easily excited and thus give a particular tune (determined by finger placement). Therefore, the "fundamental" (fitting just once) and the overtones (fitting two, three, four, ..., times) are evident. See Figure 1 for a mechanical analogy.

In the case of the universe as a whole, it is thought that fluctuations at all frequencies (at roughly the same amplitude) were produced at the same moment. Therefore, there were compressions and rarefactions on all scales, and thus also temperature variations on all scales. A very low frequency would have managed only a small part of a cycle by the time the universe became transparent, so the amplitude of variation would not be especially large. A very high frequency would have gone through many oscillations, and also would not be high amplitude (particularly because photons would tend to leak out of small, high-frequency regions). However, a frequency such that maximum compression was reached just as the universe became transparent would have an extra-high amplitude. So would double that frequency, for which maximum compression and then maximum rarefaction occurred, the latter just as the universe became transparent. This first overtone, however, is expected to have a lower amplitude than the fundamental because gravity and pressure gradients would then be working at cross purposes, as opposed to together at the fundamental. Extra strong oscillations would in fact be expected at all harmonics of the fundamental, with amplitudes that tend to diminish at high harmonic numbers because photons can more easily stream out of smaller regions (and thus smooth out fluctuations in temperature).

As we will discuss in the next lecture, the analysis of these acoustic peaks has brought an unprecedented level of precision to inferences about the basic cosmological parameters. The reason this is so is that, fundamentally, physics at the CMB epoch and before (back to a few microseconds after the Big Bang, or even earlier) is simple and well-understood. For example, recall that at redshifts $z \sim 1000$, the universe is to an excellent approximation flat, with no cosmological constant. In addition, this is long before the era of structure formation, so that all perturbations are linear and thus easy to treat. It is this firm grasp of the physics that makes the CMB such a reliable tool. In the next lecture, we will discuss what we have learned from it.

Intuition Builder

Suppose that matter consisted entirely of baryons, rather than being mainly dark matter that has negligible interaction with radiation. What qualitative differences might one see in terms of CMB fluctuations?

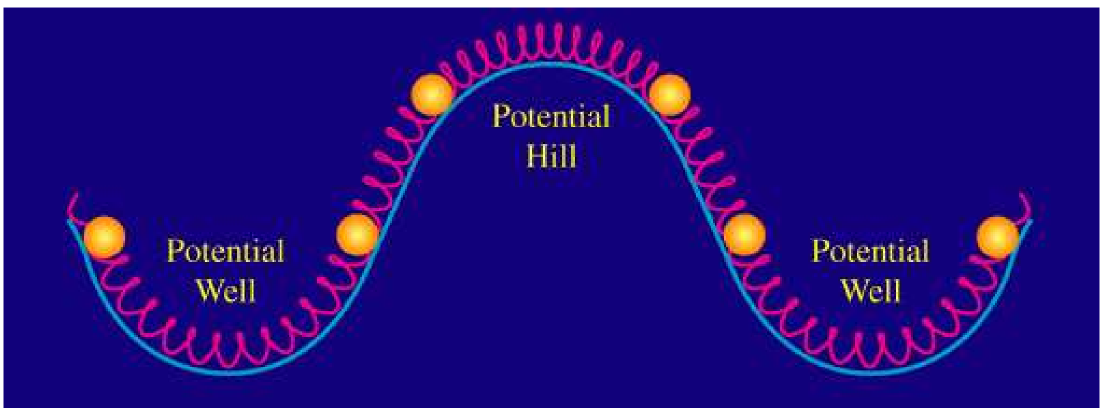

Fig. 1.— Acoustic oscillations in the early universe. When baryons are pushed together by gravity, they exert a pressure gradient that pushes them apart. This leads to periodic compression and rarefaction. For an animated GIF version of this figure, see http://background.uchicago.edu/~whu/intermediate/plane.gif