ASTR 422
Problem Set 1
Due Friday, September 14, 2007

1. For each of the following measurements of types of sources, we will assume that the intrinsic source itself is the same at all redshifts. We also assume that all observations are bolometric, meaning that they are integrated over all photon energies. Your task is to indicate (and give a brief motivation for) how the quantity in question depends on the redshift factor $(1+z)$, where z is the redshift of the source. For example, if you think the quantity is independent of this factor, you would write $(1+z)^0$.

(a) The surface brightness of a galaxy (i.e., the energy received per time per area per solid angle).

(b) The flux of a galaxy integrated over a full sphere at fixed distance from the galaxy (i.e., the energy received per time, integrated over area).

(c) The fluence of a gamma-ray burst integrated over a full sphere at fixed distance from the burst (i.e., the energy received over the full area, integrated over the entire apparent duration of the burst).

(d) The frequency of the gravitational radiation from a binary supermassive black hole.

2. In practice, most surveys are flux-limited, meaning that bright things can be seen farther than dim things. Consider the following simple case. A category of source, which emits equally and steadily in all directions, has objects whose luminosity (energy per time) ranges from $L_{\rm min}$ to $L_{\rm max} \gg L_{\rm min}$. That is, an individual object has a completely constant luminosity, but different objects can have different luminosities. Intrinsically, there is an equal probability of a source having a luminosity anywhere in this range. Therefore, if you had a truly representative sample, you would find an average luminosity of $\langle L\rangle = (L_{\rm min} + L_{\rm max})/2 \approx L_{\rm max}/2$. In reality, though, you do a survey which is complete (i.e., no missed objects) for fluxes $F > F_{\rm min}$, but sees no objects at all with fluxes lower than $F_{\rm min}$. What average luminosity do you infer from the objects you detect? Discuss whether this average should be larger or smaller than $L_{\rm max}/2$.

3. A certain Dr. I. M. N. Sane has a draft of a new textbook on cosmology, and the publishers have asked you to review it. He doesn't like the standard deviation of the Friedmann equations, and has his own pseudo-Newtonian approach.

He says that we should simply consider matter as being confined to a radially thin homogeneous spherical shell that is expanding, so that the radius from the center is R

and the time derivatives are $\dot{R}$ and $\ddot{R}$. The shell thickness is comfortably larger than our observable horizon (although much less than R), so we don't see an edge. The total mass of the universe is M, and he asks us to consider an effective density $\rho \equiv M/(4\pi R^3/3)$, and to assume that the pressure is negligible ($p/c^2 \ll \rho$). As this is a Newtonian approach, we don't worry about speeds greater than the speed of light, and we assume that the effective gravity on a particle near the outer edge is just $-GM/R^2$. With this in mind:

(a) Derive the acceleration and Friedmann equations for a particle at the outer edge. Compare them to the standard equations.

(b) How could you disprove this model observationally, keeping in mind that we can't see the edge?

4. Demonstrate that isotropy and homogeneity demand Hubble's law for local motions (i.e., for small redshift $z \ll 1$). That is, show that if every observer sees the same law, the only possibility is that the apparent recession speed must be proportional to the distance. For this problem, assume that both redshift and distance can be measured with arbitrary precision, and that all objects move with the Hubble flow.