Core Collapse Supernovae

Supernovae, the catastrophic explosions of stars, are some of the most luminous events in the universe for the few weeks that they are at peak brightness. As we will discuss in the next two lectures, there are actually two entirely distinct categories of supernovae, plus minor subvariations. One type, core collapse supernovae, is the cause (so far as we know) of the production of any newly formed neutron stars or black holes in the current universe. The other type, white dwarf supernovae, has found profound application in the measurement of the universe at large distances, and forms a key pillar in the case for accelerated universal expansion and dark energy.

In this lecture we will discuss some of the history behind supernovae, followed by their observational classification (which is still with us but does not follow our understanding of the categories as above). We will then derive the maximum mass that can be supported by electron degeneracy pressure (the Chandrasekhar mass), which is key to both types of supernova. We will then discuss core collapse supernovae and some of the issues involved in simulating them.

Historical supernovae

On rare occasions, supernovae occur in our Galaxy close enough and unobscured enough that they can be seen with the naked eye. Pre-telescope examples include the ones in 1006, 1054, 1181, 1572, and 1604. There was also a naked-eye supernova in the Large Magellanic Cloud in 1987, for which we actually had observed the progenitor star.

Very early supernovae were recorded by various civilizations, but the one in 1572 (commonly called "Tycho's supernova") was special because Tycho Brahe observed it from different vantage points and concluded that it had to have come from outside the orbit of the Moon. This demonstration helped erase the distinction between the flawed character of things within the Earth's influence (this included the Moon) and the pristine, changeless aspect of the heavenly spheres.

It was, however, centuries before any kind of physical understanding of supernova was to come up. Part of the reason for this was that there are other things that look like supernovae if you don't know their distance, e.g., classical novae. These occur when a white dwarf accretes hydrogen and helium from a companion. The material builds up until nuclear fusion becomes unstable at the base of the accreted layer. At that point, the whole layer goes kaboom and blows out the accreted matter and more besides. By the early part of the 20th century, however, distance measurements had advanced to the point that it was clear that there was a clear distinction in energetics between the comparatively paltry classical novae and supernovae.

Observational classification

What, then, should be done to figure out what supernovae are? Spectra are a reasonable way to distinguish types, so that is what observers did. The basic classification came down to:

- Type II supernovae show hydrogen in their spectra. Their light curves are rather diverse, and their peak luminosities are around 10^{42} erg s^{-1}.
- Type I supernovae do not show hydrogen in their spectra. They are subclassified based on other considerations. If they have certain silicon lines, they are called Type Ia. Otherwise, if they have helium they are Type Ib and if they do not they are Type Ic. The Type Ia supernovae reach peak luminosities of about 2×10^{43} erg s^{-1}.

This isn't bad, but the confusing part is that in our current understanding the Type Ias are white dwarf supernovae, whereas all the rest are the result of a core collapse of a massive star. We will now discuss the core collapses for the rest of the lecture, and talk about white dwarf supernovae next time.

Evolution of a massive star

Our first step is to review the evolution of the progenitors of supernovae. Consider an isolated star that begins with at least 8 $M_\odot$. This star spends most of its life burning hydrogen to helium. When it gets through about 15% of its hydrogen (the part in the core, which is dense and hot enough to fuse), it goes through a red giant phase, then burns helium to carbon in a second "main sequence", then burns carbon to oxygen, oxygen to neon, neon to magnesium, and so on.

Ask class: what prevents it from going on indefinitely? If you think about the binding energy per nucleon, you note that hydrogen to helium liberates about 7 MeV per nucleon, but that the pickings become slimmer as the nuclei increase in complexity. In fact, there is a maximum binding energy at ^{62} Ni (it is common to indicate that the maximum occurs at ^{56} Fe, but nickel beats it out by a hair). This binding energy per nucleon is just about 8.7 MeV, meaning that after helium you only get about a quarter of the energy you got during the main sequence. In addition, the nuclei have greater electric charge, meaning that the temperature has to be higher, and thus the burn rate greater, to fuse. When the last stage of fusion happens (silicon to iron, basically), the temperature is so high that photons can split up nuclei and thus decrease the net energy production even further. At this point, the star has a shell-like structure (see Figure 1 for a not-to-scale simplified version of this).

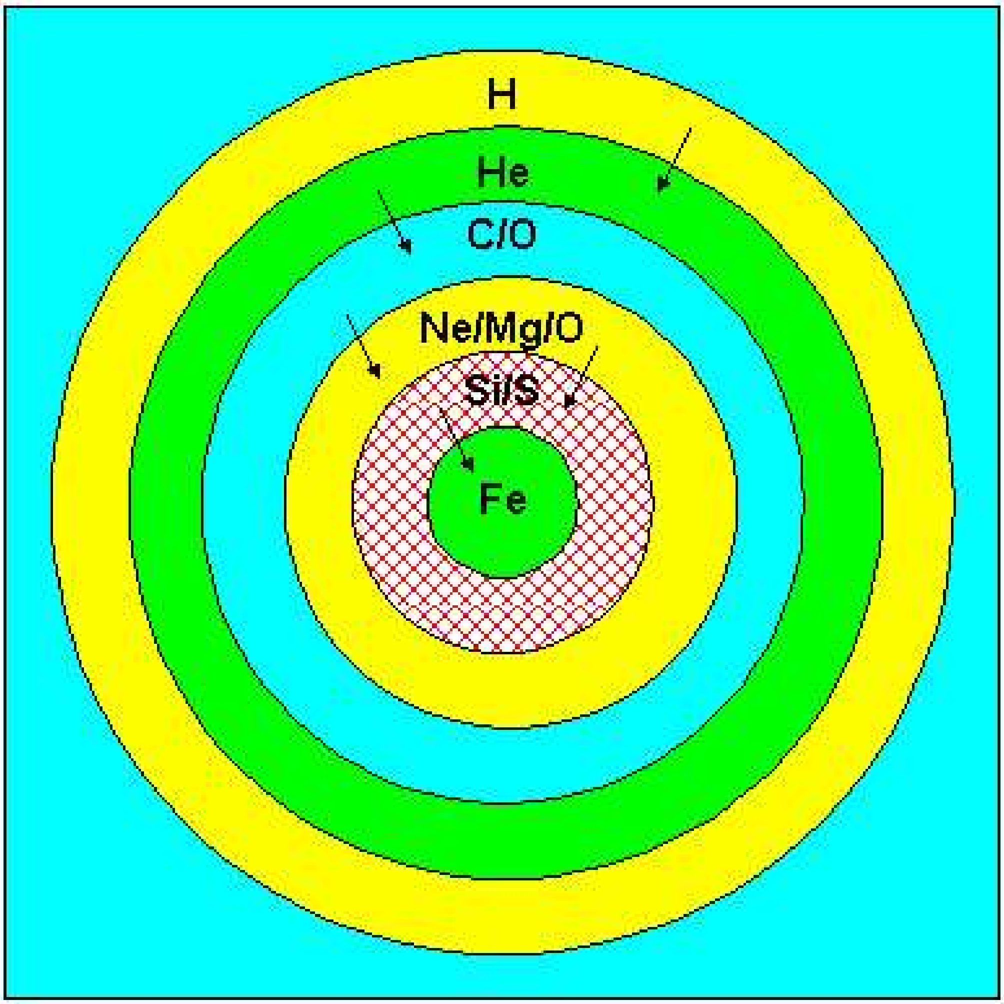

Fig. 1.— Simplified, and not to scale, picture of a star just before the core collapses. In reality, most of the star is still hydrogen at this stage, and each successive shell has less and less mass. From http://zebu.uoregon.edu/2002/ph123/snonion.gif

With lower-mass stars, the temperature never gets high enough to take the final leap, but for stars that begin above 8 $M_\odot$ the finale of the sequence is that a large, inert core of iron builds up. One might have imagined that such a core could support itself to arbitrary masses, but as Chandrasekhar showed, there is a maximum mass beyond which such support fails. We now go through this derivation, using the Landau version.

The Chandrasekhar mass

We'll do a squiggle derivation, meaning that the proper factors of π and (importantly) electrons per baryon are not included. By good luck it turns out that we actually get the right answer this way, but for precision applications we'd need to do it properly.

To start, let's review degeneracy pressure. If you have taken quantum mechanics, you recall the uncertainty principle: $\Delta x \Delta p \gtrsim \hbar$. In more detail, if you've ever done a square well calculation, you know that a particle that is confined within a well of size Δx actually does have a momentum of the order of $\hbar/\Delta x$. Now consider a white dwarf, which might typically have a radius of 10^{3-4} km and a mass of $0.5-1.4\ M_\odot$. The electrons are all squeezed together much more tightly than in atoms (you can figure out the typical separations), so they are free. Therefore, each electron feels the influence of neighboring electrons, and this acts as a confining square well. As a result, even at zero temperature, the electrons have a *Fermi momentum*

$$p_F \sim \hbar/\Delta x \sim \hbar n^{1/3} \tag{1}$$

where n is the number density. There is a corresponding Fermi energy, which is $E_F \approx p_F^2/2m_e$ in the nonrelativistic limit and $E_F \approx p_F c$ in the relativistic limit.

Consider first the nonrelativistic limit. If there are N electrons in a star of radius R, then the number density is $n \sim N/R^3$ (note again the lack of factors such as $4\pi/3$; we are aiming for qualitative understanding). Then the Fermi energy per electron is $E_F \sim \hbar^2 N^{2/3}/(2m_e R^2)$ and the gravitational energy per electron is about $E_G \sim -GMm_B/R$, where m_B is the mass of a baryon (note that the mass is in the baryons rather than the electrons, and we are assuming roughly one baryon per electron) and $M = Nm_B$. The total energy per electron is then

$$E_{\rm tot} = E_F + E_G \sim \frac{\hbar^2 N^{2/3}}{2m_e R^2} - \frac{GNm_B^2}{R}\ . \tag{2}$$

Nature is lazy, so the system will adjust itself to be at the lowest possible energy. The different dependences on radius of the two terms means that this is straightforward.

Now consider the relativistic limit. Then $E_F \sim \hbar n^{1/3} c \sim \hbar c N^{1/3}/R$, so

$$E_{\rm tot} \sim \frac{\hbar c N^{1/3}}{R} - \frac{GNm_B^2}{R}\ . \tag{3}$$

If this expression is positive, E can be decreased by increasing R, hence the system becomes at least partially nonrelativistic and all is well. If this expression is instead negative, then E can be decreased indefinitely by lowering R. This is unstable.

The borderline between stable and unstable comes when the number of nucleons/electrons is

$$N_{\rm max} \sim \left(\frac{\hbar c}{G m_B^2}\right)^{3/2} \sim 2\times 10^{57}\ , \tag{4}$$

which miraculously gives about the right value $M_{\rm max} \sim 1.5\ M_\odot$. Indeed, as presented this argument would give the same maximum mass for a neutron star (in which the supporting fermions are neutrons instead of electrons). In reality, this argument if done more exactly would predict a maximum mass for neutron stars that is closer to 6 $M_\odot$; the true maximum mass is much less, for reasons having to do with general relativity and the equation of state of very dense matter.

In any case, this discovery was momentous. Chandra did the derivation on a boat trip to England from India when he was only 20 years old(!!). Unfortunately for him, his mentor Eddington did not believe the derivation, thinking that it was an inappropriate marriage of quantum mechanics and relativity. Eventually, though, the community accepted Chandra's derivation.

In core collapse supernovae, this has the essential effect of limiting the mass to which the iron core can grow. Beyond that mass, the core collapses to a neutron star, at least temporarily, and the energy that is released produces a supernova. As we now discuss, however, the details of the supernova are far from certain.

Getting a supernova to explode

At first blush, getting a supernova to explode seems easy. Consider that the energy released in a collapse to a neutron star is

$$E \sim GM^2/R \sim G(1.5\ M_\odot)^2/10\ {\rm km} \sim {\rm few}\times 10^{53}{\rm erg} \tag{5}$$

whereas the binding energy of the original core, which is comparable to the binding energy of the whole star (within factors of several) is

$$E_{\rm bind} \sim G(1.5\ M_\odot)^2/1000\ {\rm km}\ , \tag{6}$$

or about 1% of the liberated energy. No problem, right?

The difficulty is that observationally, supernovae only emit about 10^{51} ergs in light and a similar amount in kinetic energy, so quite a lot is missing! The missing energy turns out to go into neutrinos. As we discussed when we talked about neutron stars, at high densities it is energetically favorable to undergo the reaction

$$e^- + p \to n + \nu_e \tag{7}$$

where ν_e is an electron neutrino. We know, though, that neutrinos have very low cross sections. Indeed, although it has now been about forty years since the realization that neutrinos coupling to matter probably drive the supernova, simulations of this effect remain elusive. It is a formidable computational problem. To do it properly, one probably needs full 3-dimensional general relativistic photomagnetohydrodynamics with three-flavor neutrino transport! Not easy at all.

In addition, it is likely that there are occasions in which core collapse does *not* cause an explosion. The way you can see this is that regardless of how massive the star is just prior to core collapse, the energy release and coupling is about the same (it is just the Chandrasekhar mass, after all, and most of the neutrino coupling occurs pretty close to the proto neutron star). However, a sufficiently massive star can have a very large binding energy. It is thought that if the mass prior to collapse is more than about 40 $M_\odot$, there may be a direct collapse into a black hole.

Finally, a couple of notes about additional effects. It would take us too far afield, and many details are uncertain, but we should mention that there is a growing feeling that there is another type of core collapse supernova that occurs in the current universe: an "electron capture" supernova. These can occur for lower-mass cores, and might lead to relatively little recoil of the resulting neutron star.

The second note has to do with binaries. Throughout our description we have assumed that the star evolves in isolation. In reality, however, massive stars are almost always found in binaries, many times with companions close to their own mass. If the companion is far enough away then the evolution is of course unaffected, but close companions are relatively common and can have a major effect. In particular, it can happen that the companion is close enough to strip off the hydrogen (and sometimes the helium) envelope of the primary star. This is thought to lead to supernovae of Types Ib and Ic. The Type Ic supernovae are of special interest because they have been associated with the long type of gamma-ray bursts.

Intuition Builder

What effects can you think of resulting from a rapid rotation of the core just prior to collapse?