Binaries, Continued

We will now discuss various categories of binary sources and their properties. We will focus on compact objects: white dwarfs, neutron stars, and black holes.

Double white dwarf systems (WD-WD) should be extraordinarily plentiful in the Milky Way and other galaxies. In fact, they should be so common that they will provide a limiting noise for low-frequency detectors such as LISA. We can understand this at a crude level as follows. Suppose that LISA, with its frequency range of $\sim 10^{-5} - 10^{-1}$ Hz, observes for three years (about 10^8 s). Its frequency resolution is therefore 10^{-8} Hz. It can observe both polarization modes, but if more than two WD-WD sources are in the same frequency bin they can't be distinguished, and in fact they act as unresolvable noise.

If there are 10^8 WD-WD binaries in our Milky Way, this implies that, assuming a maximum frequency of ~ 1 Hz, there is on average one binary per bin. However, the strong increase of energy loss to gravitational radiation as the orbit shrinks means that binaries spend most of their time at low frequencies. The net result is that low-frequency bins are buried in unresolvable WD-WD binaries, whereas at high frequencies there is on average less than one binary per 10^{-8} Hz frequency bin, meaning that it will be possible to identify them individually and model them out of the data stream. The frequency at which one can start to resolve individual WD-WD binaries has been variously calculated to be in the 2-3 mHz range (see Farmer & Phinney 2003 for a recent discussion). In the $\sim 10^{-4} - 10^{-3}$ Hz range, it is expected that this background will be more important than the LISA instrumental background for determining sensitivities. The extragalactic WD-WD background, although smaller in amplitude, involves so many sources that it will produce an unresolvable background all the way up to $\sim$1 Hz, but at a level far below the current LISA background (see Farmer & Phinney 2003).

What about neutron stars? There are fewer than ten double neutron star binaries (all in or near our galaxy, of course), and about half of them will merge within a Hubble time (i.e., the current age of the universe). In one of them, J0737, we observe both neutron stars as pulsars. It is interesting to note that NS-NS mergers are the only high-frequency gravitational wave source *known* to exist (see Figure 1 for a classic demonstration of gravitational waves taking away orbital energy). Other sources are extremely likely (e.g., NS-BH or BH-BH binaries) and many suggestions have been made for sources whose strength is uncertain (e.g., continuous or burst sources). Discovery of any such source would yield important astrophysical information. However, when making the case for ground-based gravitational wave detectors, it is necessary to estimate the rate of detection of sources we can project with some confidence.

This is typically done (e.g., see papers by Kalogera and colleagues) by using population synthesis codes (in which large populations of stars, including binaries, are evolved with certain assumptions and the result is a simulated population of compact binaries). These results are calibrated statistically by comparison with the observed population of NS-NS binaries that will merge in a

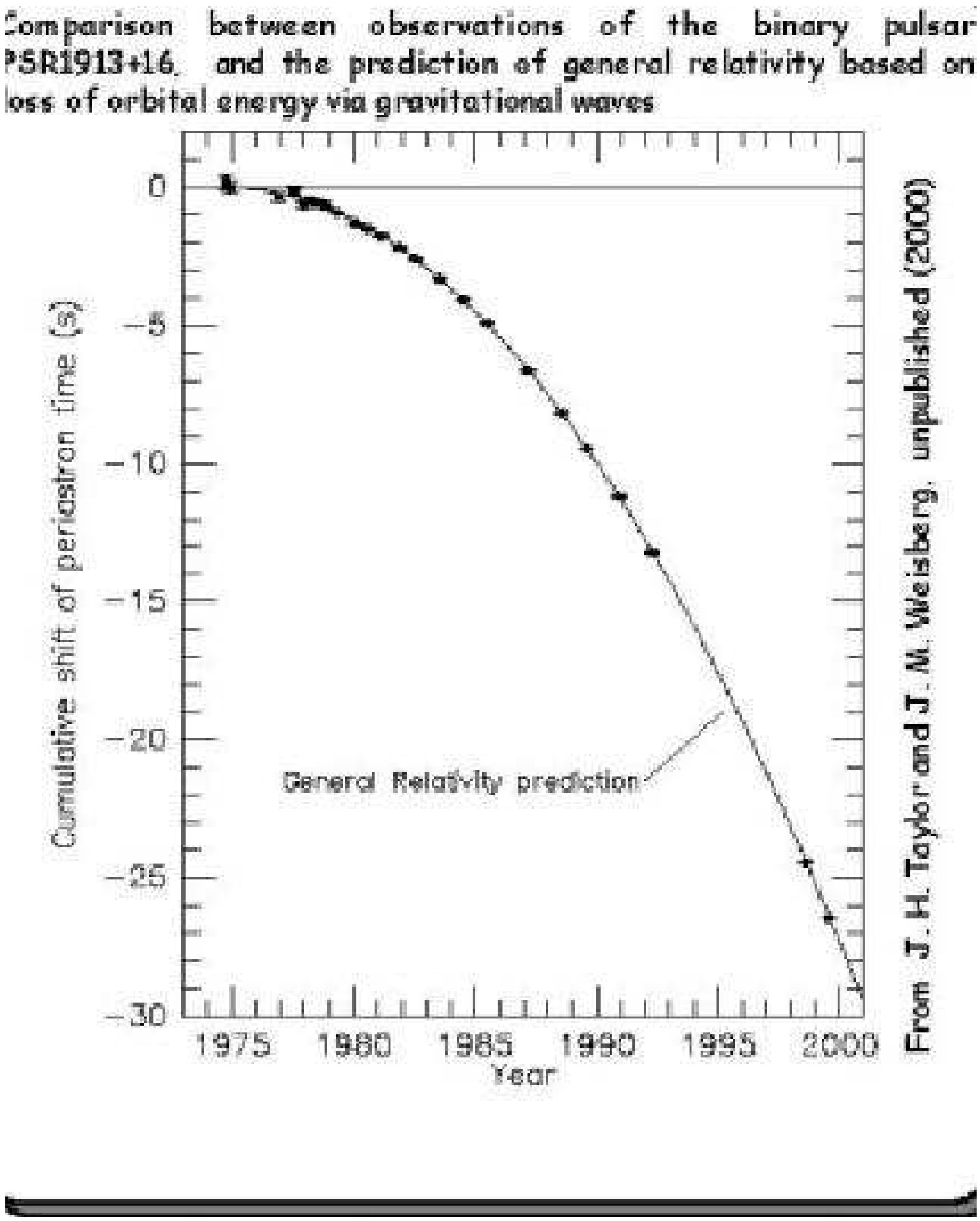

Fig. 1.— Predicted and inferred change in the orbital phase of the first binary pulsar. This figure shows the accuracy of the prediction of general relativity. Original figure from an unpublished work by Taylor and Weisberg.

Hubble time. One of the NS-NS binaries is in a globular cluster. There are ~ 100 globulars around the Milky Way galaxy, and in each there might be a few hundred neutron stars. Even if there are then a hundred NS-NS binaries per cluster, and they *all* merge within a $\sim 10^{10}$ yr Hubble time, this gives a total rate of only $\sim 10^{-6}$ yr^{-1} per Milky Way Equivalent Galaxy (or MWEG, as it is known in the business). The rates estimated from the binaries in the disk of our Galaxy are larger than this, hence only the disk binaries are usually included in such analyses. The best current estimates are that the rate is about $10^{-5} - 10^{-4}$ per year per MWEG. At the high end there are tremendous uncertainties, because with the small number of sources a source that is dim (hence can only be seen nearby) and has a short time to merge (hence can only be seen for a short time) has a large statistical weight. This is in fact the case for the double pulsar J0737. At the low rate end of the estimates, though, multiple sources contribute and the uncertainties are not as great. The net result is that, with high confidence, one expects the next generation of ground-based instruments (such as Advanced LIGO) to see at least a few NS-NS mergers per year.

Mergers of black holes with neutron stars have not received as much attention, but in most binary evolutionary models they are anticipated to be prominent sources for ground-based detectors. Although probably less common than NS-NS binaries (of course, none have been seen!), the higher mass of the black hole implies a larger distance out to which the merger could be seen. It is possible that this could more than compensate for the smaller number density and make NS-BH mergers prominent sources at high frequencies.

If such mergers do occur, what could they tell us? One hope has been that the tidal disruption of the neutron star would show up in the gravitational wave train. If so, this would carry information about the radius of the neutron star. Combined with the mass, this would be informative about the nature of the high-density matter in the core of the stars. The information would be easiest to extract if the disruption occurred outside the orbital radius of dynamical instability, because the stellar radius would be more easily determined over many orbits. It seems likely, however, that disruption will occur during the plunge part of the merger. It may therefore be difficult to extract information about neutron star radii from NS-BH mergers.

The final category of compact object coalescences involves two black holes. The black holes we know about are in the stellar-mass range (roughly $5 - 20\,M_\odot$), formed from the collapsed cores of massive stars, and in the supermassive range ($\sim 10^6 - 10^{10}\,M_\odot$) in the centers of galaxies. There is also growing but not conclusive evidence for intermediate-mass black holes in the $10^2 - 10^5\,M_\odot$ range in some galaxies.

BH-BH coalescences tell us different things depending on their mass and mass ratio. Mergers of two stellar-mass black holes, which would be observed at high frequencies with ground-based detectors, contain information about stellar populations and evolution. Mergers of supermassive or intermediate-mass black holes, especially at high redshift, may contain keys to understanding galaxy evolution and hierarchical structure formation. From the standpoint of gravitational physics, though, the distinction is between mergers of comparable-mass black holes, and mergers of black holes with an extreme mass ratio. In the former category, numerical relativity is required to treat the coalescence. At this stage, one can with justification say that this problem has been solved, although there are still issues with grafting the late numerical relativity waveform onto the earlier post-Newtonian waveform. If the mass ratio is extremely large, then the smaller black hole effectively traces the spacetime of the larger, hence observation of the gravitational waves from the system could be used to test detailed predictions of general relativity in strong gravity. In particular, mergers of stellar-mass (or possibly intermediate-mass) black holes with supermassive black holes are expected to be an important source for LISA. The astrophysics of such sources, and of mergers involving two supermassive black holes, introduces many concepts from N-body dynamics. We will now discuss these, starting with those relevant to two supermassive black holes.

The basic question is: since most or all galaxies have massive black holes in their centers, and galaxies often collide, will the black holes merge with each other? If so, this will provide a strong source of gravitational waves.

Stars, with typical radii of $\sim 10^{11}$ cm, are much smaller than their typical separation ($\sim 10^{18-19}$ cm in the disks of galaxies). Therefore, in the bulk of galaxies there are rarely star-star collisions. Galaxies, however, have a non-negligible size compared to their separations: a galaxy might be 10^{23} cm in radius, and 10^{23-24} cm from its nearest neighbor galaxy. In fact, galaxies tend to cluster, so that if you have one galaxy then another one is most probably nearby. Galaxies therefore often collide, but it's a "collisionless collision" meaning that the interactions are mostly gravitational (with the minor exception of the interstellar mediums interacting, which we'll ignore because that's a small fraction of the total mass).

We know in addition that most or possibly all galaxies have supermassive black holes in their centers. These black holes have a typical mass that is about 0.2% of the mass of the central bulge of the galaxy. Currently detected masses range from around $10^6\,M_\odot$ (our Milky Way has a $\sim 4\times 10^6\,M_\odot$ black hole) to several billion solar masses (the galaxy M87, in the center of the Virgo Cluster of galaxies, has a black hole of mass $3\times 10^9\,M_\odot$). If two such black holes were to collide or merge, they would produce abundant gravitational waves, easily detectable by planned space-based instruments. But does this happen?

The first thing is that, clearly, a typical galaxy-galaxy collision will not be so precisely head-on that the central black holes hit each other directly. Angular momentum guarantees that. The collisions will instead be oblique, so on the first pass the black holes will miss each other by a lot, maybe several kiloparsecs. The initial collision is obviously strongly time-dependent. Therefore, there is a phase in which the system virializes over a few orbits. This will typically take a few hundred million years. Incidentally, collisions of the interstellar mediums with each other can produce a huge rate of star formation in this period. You see that in the Antennae, a well-known pair of colliding galaxies with lots of young stars.

Now suppose that phase is over and the system has relaxed into some kind of equilibrium. The two black holes are still far away from each other, say a couple of kiloparsecs. What happens now?

We found before that more massive objects tend to sink to the center of a mass distribution. This will cause the massive black holes to get closer to each other and to the center. To understand more about this, however, we need to consider the process of *dynamical friction*. We will set this up by reviewing briefly some dynamical timescales.

Suppose that we have a gravitational potential that is produced by a large set of stars. Assume that if we smoothed out the graininess introduced by the stars, the potential would be spherically symmetric and static. This implies that the orbit of each individual star conserves its energy and angular momentum over the *crossing time* $t_{\rm cross} \equiv R/v$, where R is the characteristic size of the system interior to the orbit, and v is the characteristic speed in the orbit. In reality, the presence of a finite number of stars implies that the potential is lumpy. An individual star moving in this lumpy potential gets deflected. It turns out that the main contribution for single stars comes from two-body effects, and these are dominated by distant (hence gentle!) encounters. If you put it all together, you find that the time necessary to change the velocity (i.e., magnitude or direction) by of order itself is the *relaxation time*, which for a system of N equal-mass objects is

$$t_{\rm rlx} \approx \frac{N}{8\ln N} t_{\rm cross}\ . \tag{1}$$

For example, in the center of our Milky Way out to 100 pc, the relaxation time is roughly 10^{13} yr.

Now consider an object that is *much* more massive than other stars, as will be the case for our black holes. Then we can think of the motion of the massive object as creating a wake as it moves past the background stars. That is, the gravity of the object will focus the background stars and give them extra energy. This energy must be taken out of the kinetic energy of the massive object. This therefore leads to a drag, or a frictional effect, even though this is pure gravity and there isn't actually any net dissipation in the system.

The actual time necessary for objects to spiral to the center depends on the velocity dispersion of the surrounding stars as well as the mass M of the object and the mass density ρ in the surrounding stars. For typical values, it's about

$$t_{\rm sink} \sim 10^7\,{\rm yr}(M/10^6\,M_\odot)^{-1}(r/100\ {\rm pc})^2\ . \tag{2}$$

Here we assume that the particle is at a distance r from the center. Note that this is *much* shorter than the relaxation time, because of the high mass assumed for the object.

Before resuming our analysis of binary supermassive black holes, a couple of comments about other applications. In a cluster of galaxies, dynamical friction of large galaxies against small galaxies and dark matter can cause those galaxies to sink to the center and merge. This is thought to contribute to the formation of CD galaxies, which are massive ellipticals in the center of some galaxy clusters. In globular clusters, the sinking of massive objects can cause a "gravothermal catastrophe", in which a central minicluster of high-mass stars or stellar remnants undergoes a collapse that can lead to high densities and violent interactions.

There is an additional effect that might speed up the sinking. Since a supermassive black hole is in a bulge that has ~ 500 times the mass of the black hole, at least initially the collection of matter acts together, so M is effectively 500 times larger. This causes initially rapid sinking. Will this continue indefinitely? No, because when the two distributions star to overlap, tidal effects will strip away the stars from around the black holes. However, by that point it is likely that the holes themselves are close enough to continue sinking towards each other in a relatively short time.

It seems, therefore, that we've answered our question: binary supermassive black holes will happily drift towards the center, where they will eventually merge with each other. Is there anything that might eventually make the process of dynamical friction less efficient? Yes! At some point, when the holes are close enough to each other, they will run out of stars with which to interact. You can see the effect by considering conservation of energy. Suppose that two black holes of mass M are a distance R from each other. In that same region is a collection of stars of total mass Nm. Imagine that the black holes eventually eject *all* the stars, with small speed at infinity. The original total orbital energy of the black holes was roughly $-GM^2/(2R)$ and of the stars was of order $-G(Nm)^2/(2R)$, so if the stars end up with zero energy then the energy of the black holes in orbit is

$$\begin{aligned} -GM^2/(2R_f) &= -GM^2/(2R) - G(Nm)^2/(2R) \\ R_f &= R[M^2/(M^2+N^2m^2)] \ . \end{aligned} \tag{3}$$

Therefore, in order to make a significant change in the orbital separation of the black holes, the holes must interact with an amount of mass roughly equal to their own mass. The central densities of galaxies can be of order $10^6\ M_\odot\ \mathrm{pc}^{-3}$, so this suggests that two $10^6\ M_\odot$ black holes can only get within about 1 pc of each other before they start to run out of stars to throw to infinity.

Whoops! Does this mean that we expect many galaxies to harbor supermassive black holes orbiting around each other at a distance of a parsec? What other effects might come in? There are several possibilities that have been discussed.

First, consider gravitational radiation. Two massive things orbiting around each other produce waves in spacetime that carry away energy. This loss of energy will cause a shrinkage of the orbit, and eventual coalescence. The problem is that the rate of inspiral depends very strongly on the semimajor axis (as a^4), and for typical supermassive black holes the separation needs to be < 0.01 pc before gravitational radiation can bring them the rest of the way in.

Second, think about the way in which the stars interact. In particular, imagine a single star of typical mass that comes near the binary black hole. It will feel a strongly nonaxisymmetric, time-dependent force, and will therefore be batted around before finally being ejected. However, when it is ejected it has an impact parameter that is comparable to the semimajor axis of the binary. Thus, unless it is thrown out with such force that it escapes the galaxy entirely, it will return with roughly the same impact parameter it had before (because its orbit will hardly be altered by the other stars). Each star therefore has more of an effect than you might have thought, and becomes negligible only when the binary has shrunk to a factor of a few less than its original semimajor axis, because then the star will completely miss the binary on its next orbit. This effect, while important, is not enough by itself.

Third, what about motion of the binary itself? All the interactions with stars will impart recoil to the binary, so it wanders around in the core region. In principle, if it wanders far enough, it can get fresh stars with which to interact, and all is well. In practice, however, supermassive black holes don't wander by more than 0.01 pc or so, which isn't enough.

Fourth, what about other stars that might come in to interact with the binary? Even if the binary acts like an eggbeater to kick out all of the stars originally within 0.1–1 pc of the core, there are other stars farther out that have orbits that bring them into the core. The problem here is that once those stars are exhausted, then in a spherically symmetric distribution it will effectively be forever until those orbits are replaced by other stars, meaning that no more stars come in to interact. Why would it take so long in a spherically symmetric potential? It's because, once the near-radial orbits are exhausted, it takes something like a relaxation time to repopulate them. That's much longer than a Hubble time. More massive objects (such as O or B stars or stellar-mass black holes) can sink more quickly, but probably not quickly enough.

Okay, so is there any other way? We assumed spherical symmetry in the distribution above; with that hint, Can we think of another possibility? My favorite idea among those I've heard is that if the central region of a galaxy is *not* spherically symmetric, but instead triaxial, then the orbits of the stars are box orbits, i.e., they can look like they are moving along the diagonals of a rectangle. Such orbits do not individually conserve angular momentum. They can therefore pass arbitrarily close to the center. This means that it's only a few orbits until the center has lots of stars to interact with the binary, instead of $0.1N/\ln N$ orbits. I like this because it seems reasonable that after a galaxy collision one wouldn't have spherical symmetry. We'll see what future calculations have to say, but I wouldn't be surprised if this is the answer. One would then find that supermassive black holes commonly merge in the universe, which would be exciting as a source of gravitational waves and as a way to learn about strong gravity and associated extreme physics.

Intuition Builder

Suppose that there is a lot of gas around instead of stars. Gas can cool; how might that affect the efficiency with which the supermassive black hole binary is brought together?