ASTR 498
Problem Set 2
Due Thursday, February 28

1. Low-energy cosmic rays. The radius of curvature R (in centimeters) of a particle of energy E (in ergs) and nuclear charge Ze (where $e = 4.8 \times 10^{-10}$ esu) in a magnetic field of strength B (in Gauss) is $R = E/(ZeB)$. Consider a supernova that occurs a distance $d = 1000$ pc from us, and assume that the average strength of the interstellar magnetic field is $B = 3 \times 10^{-6}$ Gauss.

(a) **2 points** What is the energy in GeV needed for an iron nucleus so that $R = d$? As this energy and above, the nucleus will basically come directly at us.

(b) **2 points** If $R \ll d$, the tangled nature of the interstellar magnetic field means that the nucleus will basically undergo a random walk. Note, though, that the nucleus will not lose significant energy from the magnetic deflections. In this random walk, the nucleus will effectively take $(d/R)^2$ steps of length R each to go a net distance d. Use this, plus special relativistic principles, to compute the energy $E_{\rm min}$ (in GeV) below which most ^{10}Be nuclei (with rest-frame half-life of 3.9×10^6 yr) will decay in transit from the distance $d = 1000$ pc.

2. **4 points** Dr. I. M. N. Sane has resolved the origin of cosmic rays with energies above the "ankle" at 10^{19} eV. Like many fundamental advances in human thought, the basic idea is simple: ultra-high-energy cosmic rays (UHECRs) are neutrons. Dr. Sane argues that these are produced in active galactic nuclei (AGN); the acceleration to such high energies actually happens to protons, but these convert to neutrons (which have a rest-frame half-life to decay of about 1000 seconds) before moving on. Dr. Sane points out that neutrons aren't deflected, and that this is consistent with directions of UHECRs being close to many known AGN, the closest of which is about 15 Mpc away. Your favorite candidate for President wants to discuss this discovery at a fundraiser as an example of American ingenuity, but is consulting you first to determine your thoughts. What is your evaluation of the idea?

3. Gravitational redshift.

(a) **2 points** Use a thought experiment to derive the gravitational redshift of photons one would expect in weak (i.e., Newtonian) gravity. To start, suppose you are at a radius R from the center of a spherically symmetric mass M, where $GM/(Rc^2) \ll 1$ for weak gravity. You take two photons of identical energy $E_{\rm init}$ and let them go to some radius $r > R$. At this radius r the photon energies are now each $E_{\rm red} = m_e c^2$, where m_e is the mass of an electron. You then cause the photons to produce an electron and positron with zero kinetic energy. The electron and positron are then dropped back to radius R, where they annihilate to form two photons, and each of the two has energy $E_{\rm final}$. Use the energy conservation condition $E_{\rm final} = E_{\rm init}$ to derive the gravitational redshift factor $E_{\rm red}/E_{\rm init}$ as a function of M, R, and r. Use Newtonian potential energies.

(b) **2 points** Suppose that you can measure the frequency of laser light to one part in 10^{15}. If you shine light straight upwards from the surface of the earth (mass $M = 6 \times 10^{27}$ g, radius $R = 6400$ km), how high does the light have to get so that you can measure the change in frequency?

4. **4 points** Properties of the event horizon.

In the notes we showed that a particle dropped from rest at infinity will approach the event horizon at a speed that approaches the speed of light as seen by a local static observer very close to but outside the horizon. But what if the particle is dropped from a closer radius?

Demonstrate that *regardless* of the initial radius $r > 2M$, at the horizon the particle would appear to go at the speed of light. Do this in the following way. Consider a particle of nonzero rest mass in a Schwarzschild spacetime. Release it from rest at some radius $r > 2M$. Then, prove that for any $0 < \epsilon \ll 1$ there is a static observer at some radius $R = 2M(1 + \delta)$ with $0 < \delta \ll 1$ who measures the radial speed of the particle to be $c(1 - \epsilon)$. **Hint:** remember that the specific energy $-u_t$ is conserved for the particle; that's $-u_t$ in the *global* frame, of course, not the locally measured specific energy $-u_{\hat{t}}$.