ASTR 498
Problem Set 3
Due Thursday, March 13

1. **4 points** Suppose that matter in a circular orbit at the ISCO around a nonrotating black hole spirals into the black hole without any further loss of energy or angular momentum. Derive the angular velocity Ω of the matter as seen at infinity, as a function of the radius r of the matter, where $r < 6M$ as the matter falls. Check in particular what the angular velocity is when $r = 2M$, i.e., when the matter is just about to enter the event horizon. Discuss whether the prediction of your expression make sense in this case.

2. **4 points** Is the surface of a cylinder flat or curved, in a geometric sense? To determine this, consider the outside of the cylinder (not the flat caps), and think about two initially parallel geodesics. Either prove that they always remain parallel regardless of their initial angle to the axis of the cylinder, or exhibit a specific case in which they converge or diverge. **Hint:** set up a coordinate system on the surface (only the two spatial components, not the time component), and write an expression for ds^2. If you can do a global transformation to something that is obviously flat, you're done; if you can prove it is impossible, you are also done.

By the way, you won't be graded on this, but you might want to try the same thing for the surface of a sphere.

3. **4 points** You observe an accreting black hole with a maximum disk temperature of 10^6 K and a luminosity of 10^{40} erg s^{-1}. To within a factor of 10, derive the mass of the black hole assuming emission from a geometrically thin disk, that the hole is nonrotating, and that there is no emission inside the ISCO.

4. **4 points** Dr. Sane is under consideration for a position at Goddard Space Flight Center. The idea he has presented to them involves a remarkable new type of object: neutrino disks. He points out that neutrinos have mass, so they can move nonrelativistically. He suggests that they can cluster around black holes in the early universe and accrete onto them through a disk, in just the same way that normal accretion disks work. Dr. Sane says this will produce bright X-ray sources that could be seen by the planned Constellation-X mission, and that indeed this would be the most important discovery of that mission. Dr. Nick White of Goddard has consulted you about this potential hire. What is your response?