ASTR 498
Problem Set 4
Due Thursday, April 3

1. Dimensional analysis and Hawking radiation. Consider a nonrotating black hole of mass M. Hawking radiation is thermal radiation whose wavelength is of order the radius of the event horizon. Based purely on dimensional analysis:

(a) [**3 points**] Use this to derive the effective temperature of the black hole, as well as its luminosity and lifetime. All factors of G, c, etc., must be in your expressions, but factors such as 2, π, and so on don't need to be because this is a dimensional analysis.

(b) [**1 point**] How massive, in grams, would a black hole have to be at the beginning of the universe (about 14 billion years ago) so that it evaporates to nothingness now? An answer to within a factor of 100 is fine.

2. [**4 points**] The most convincing argument in favor of black holes powering AGN comes from their short variability timescales. Is it possible, though, that the total energetics can be explained by something else? Consider a starburst, for example. Suppose that 10^{11} stars formed all at the same time. Assume that their mass distribution is given by the Salpeter form $dN/dM \propto M^{-2.35}$ all the way from $M_{\rm low} = 0.2\ M_\odot$ to $M_{\rm high} = 20\ M_\odot$. Let their luminosity be given by $L = (M/M_\odot)^{3.5} L_\odot$, where $L_\odot = 3.8 \times 10^{33}$ erg s^{-1} is a solar luminosity and $M_\odot = 2 \times 10^{33}$ g is a solar mass. Calculate the total luminosity of this extreme starburst. Then, do web research to find the most luminous quasar (this is the *intrinsic* luminosity, not how bright it appears to us) and see if your starburst can account for that luminosity. Please give me the URL to the website that discusses the high-luminosity quasar.

3. [**4 points**] Spin of stellar-mass black holes. Consider a stellar-mass black hole (we assume $M = 10\,M_\odot$) accreting from a binary companion. There are, generically, two types. Low mass X-ray binaries (LMXBs) have a companion that is perhaps $0.5\,M_\odot$ and accretes via Roche lobe overflow. The accretion can last hundreds of millions of years, and proceeds until the companion is virtually exhausted. In contrast, high mass X-ray binaries (HMXBs) have a companion that is massive, perhaps $10\,M_\odot$, and accretion occurs via a stellar wind. When the wind is strong, accretion can occur at the Eddington rate, which we will estimate as $\dot{M}_E \approx 10^{18} (M_{\rm BH}/M_\odot)$ g s^{-1}. However, this phase of a strong wind doesn't last long before the companion star evolves. For our purposes, we will assume it lasts 10^6 yr.

With these assumptions, estimate to within a factor of 2 the maximum final spin of a black hole in a binary system, assuming it is initially nonspinning and that the matter gets all the way to the ISCO before plunging. Therefore, argue that in an isolated binary, a black hole has quite close to its initial mass and spin parameter, meaning that if we can determine those values we

have a record of the result of the supernova that created it. **Hint:** you should be able to do this by hand; no complicated calculations needed!

4. There is an opening in the UMd astronomy colloquium series, and Dr. Sane has volunteered his name. Stuart Vogel, our department chair, is consulting with you about whether to have Dr. Sane give the talk. The proposed talk title and abstract:

A Disproof of the Existence of Black Holes

Dr. I. M. N. Sane, private theorist

The existence of black holes has long been disputed by the great minds of our day. Finally, I, and only I, have been able to show that they do not exist! A binary system has been observed with thermal emission that has a typical temperature of $T = 10^7$ K and a luminosity of $L = 4 \times 10^{37}$ erg s^{-1} around a mass $M > 5\,M_\odot$. I will prove once and for all that this is best explained by a "Sane shell" of dark matter that is concentrated at $r = 50M$ in geometrized units, with nothing inside. Matter spirals to the edge of the Sane shell, spreads over the entire spherical surface and emits its energy as a blackbody, then floats around the inside of the shell with nothing else to do. After proving this in the first ten minutes and accepting your applause, I will then describe a number of my other wonderful ideas.

Ignoring the odd phrasing, do a calculation to evaluate Dr. Sane's claim and then make your recommendation to Stuart.