ASTR 498
Problem Set 5
Due Thursday, April 17

1. [**4 points**] In accretion onto nonrelativistic and weakly magnetic nonrotating stars, half of the accretion energy is released in the inspiral to just above the surface, and half is released in the boundary layer formed when the matter slows down to a stop on the surface. What about for a neutron star? Assuming that matter (a) spirals in nearly-circular orbits to the ISCO, (b) drops to the surface without releasing further energy, then (c) releases any additional energy in a boundary layer so that it comes to rest on the surface, calculate in full general relativity the fraction of the *total* energy released in the boundary layer for neutron star radii of $R = 6M$, $R = 5M$, and $R = 4M$. Assume the star is nonrotating. These fractions suggest that high-amplitude timing phenomena in the accretion-powered X-rays (such as quasiperiodic oscillations) may have their clock set in the disk, but the energy release must primarily be on the stellar surface.

2. [**4 points**] Planetary Solutions, Inc. has hired you as a consultant. An alien race has decided that it would be chic to put their planet into a circular orbit around the Crab Pulsar. Their planet is very similar to Earth, so they want to be placed where the flux from the *total* spindown energy of the pulsar is equal to the flux from the Sun received at Earth. Using the standard dipolar spindown model, calculate the distance that is needed (and show all your work). You'll need to look up the spin rate of the Crab as well as its magnetic field (please give me the URL of any references); note that the pulse shape from the Crab tells us that it is an orthogonal rotator, with $\sin\alpha = 1$.

This isn't part of your grade on this problem, but can you think of any special difficulties about this environment? What warning might you give?

3. [**4 points**] The most rapidly rotating known pulsar spins at 716 Hz. Suppose that the magnetic dipole moment of this pulsar is 3×10^{26} G cm^3 and that the current spin rate was the equilibrium spin rate (i.e., the orbital frequency at the Alfvén radius) during accretion. As a function of the unknown mass of the pulsar (measured in solar masses), compute the average accretion rate during this phase.

4. [**4 points**] Dr. Sane has realized that there is another possible energy source for supernovae, which has been overlooked by lesser scientists. His calculations suggest that during the collapse of the core, various processes result in about 1% of the neutrons escaping from nuclei as free particles (i.e., basically in zero density environments). His proposal is that these neutrons then decay into protons, electrons, and electron antineutrinos, and that this energy release powers the supernova.

J. Craig Wheeler, president of the American Astronomical Society, is investigating this model for a possible award of a major prize to Dr. Sane. He asks you to do a calculation to see if this model works. The input: the core itself starts at $1.4M_\odot$ of mass, of which initially about half is neutrons. Supernovae produce about 10^{51} ergs of energy in photons and kinetic energy. You'll need to look up the masses of neutrons, protons, and electrons. Give Dr. Sane the maximum benefit of the doubt by assuming that *all* of the energy in his neutrinos couples to matter and helps drive the supernova.