Specific Intensity

Initial question: A number of active galactic nuclei display jets, that is, long, nearly linear, structures that can extend for hundreds of kiloparsecs. Many have two oppositely-directed jets, but some have only one. What's going on?

Ask class: what physical properties of radiation can be measured? Photon energy, flux, polarization, angular distribution. There are a lot of ways that radiation can be produced or affected. However, for a start, let's think about radiation when there is no matter present. In particular, consider a bundle of rays moving through space. We want to know what happens to those rays. For example, do they diverge or converge? What happens to the intensity that is measured by an observer? Are there conserved quantities that we can use? For completeness, let's define a few terms. Assume that over a time interval dt, a total energy dE is received by a detector with area dA perpendicular to the direction from which the radiation is coming. Then the *flux* is $F = dE/(dtdA)$. If the source subtends a solid angle $d\Omega$ as seen by the observer, then the *surface brightness* is $S = dE/(dtdAd\Omega)$. Finally, if we concentrate only on the energy received in a frequency range $d\nu$ from ν to $\nu + d\nu$, then the *specific intensity* is defined as

$$I_\nu = \frac{dE}{dA\,dt\,d\Omega\,d\nu}\ . \tag{1}$$

This can be remembered as "energy per everything", and it is a fundamental quantity in understanding radiation. But how does this quantity change for radiation moving through free space?

Let's begin with a simple situation: an idealized model of the Sun that has constant brightness across its surface and is a perfect sphere that radiates isotropically. **Ask class:** how does the flux received depend on the distance r of the observer from the Sun? This is an inverse square law, so $F \propto r^{-2}$. **Ask class:** how does the solid angle of the Sun as perceived by the observer depend on r? This is also an inverse square, so $d\Omega \propto r^{-2}$. **Ask class:** so, how does the surface brightness of the Sun depend on r? It doesn't! The r dependences cancel out. This means that the surface brightness is a constant in this simplified situation. **Ask class:** how does the specific intensity change? Again, if the frequency does not change (note the qualifier!), the specific intensity is constant.

We can immediately apply this to a number of situations. One example is gravitational lensing. Suppose you have a distant galaxy which would have a certain brightness if observed directly. Gravitational lensing, which does not change the frequency, splits the image into two images. One of those images has twice the flux of the unlensed galaxy. Assume no absorption or scattering. **Ask class:** how large would that image appear to be compared to the unlensed image? Surface brightness is conserved, meaning that to have twice the flux it must appear twice as large. This is one way that people get more detailed glimpses of distant objects. Lensing magnifies the image, so more structure can be resolved.

Okay, but what happens if the photon frequency *does* change? A highly important simplification to remember is that all photons do the same thing redshift-wise, meaning that the fractional change in frequency is independent of the initial photon frequency. Let's think about a quasar at cosmological distances, where redshifts can be important. Treat it as a point (i.e., not resolved) and assume that it radiates isotropically. Assuming we've already taken out the direct r^{-2} distance dependence, then **Ask class:** how should the flux we see depend on the quasar's redshift $1+z$, where $\nu_r = \nu_e/(1+z)$? The energy of each photon is decreased by a factor of $1+z$, and the time interval between photons is increased by a factor of $1+z$, so the flux is down by a factor of $(1+z)^2$.

Ask class: now suppose the redshift is gravitational instead. For example, you've got a source deep in the gravitational well of a black hole (but outside the event horizon!). How does the flux you observe depend on the gravitational redshift? In the same way: like $(1+z)^2$. The source of the redshift doesn't matter, just the fact that the photon frequency and time between photons is altered.

It's clear, then, that we need to modify our conservation law if the photon energies change during propagation. Later in the course we'll talk about Liouville's theorem, which says that the phase space density, that is, the number per (distance-momentum)^3 (e.g., the distribution function), is conserved. For photons, this means that the quantity I_ν/ν^3 is conserved in free space. The source of the possible frequency change could be anything: cosmological expansion, gravitational redshift, Doppler shifts, or whatever. The integral of the specific intensity over frequency, $I = \int I_\nu \, d\nu$, is proportional to ν^4.

This can be applied in many ways. Consider, for example, surface brightness. **Ask class:** how does the surface brightness of a galaxy at a redshift z compare with that of a similar galaxy nearby, assuming no absorption or scattering along the way? The frequency drops by a factor $1+z$, so the surface brightness drops by $(1+z)^4$. Since galaxies are detected based on surface brightness contrast (**Ask class:** why is this? Because one always makes a detection compared to a background, and for an extended object it's the surface brightness that matters), this means that it becomes extremely difficult rather quickly to detect galaxies at high redshifts.

But is this the whole story? Suppose we are observing galaxies in a particular waveband, for example the R band. **Ask class:** if we look at two identical galaxies, one close ($z_0 \ll 1$) and one at a significant redshift z, will the surface brightness we observe from the more distant galaxy be $(1+z)^{-4}$ times that of the closer galaxy? No! The problem is that we aren't looking at the same portion of the spectrum. What we see as the R band may have been B band when it was emitted. This adds significant complications in cosmological observations; the correction for the shifting of wavelength is called K-correction. The thing to remember is that if one could do *bolometric* observations (over all wavelengths) then the frequency-integrated specific intensity would transform like ν^4.

This is an *extremely* powerful way to figure out what is happening to light as it goes every which way. The specific intensity is all you need to figure out lots of important things, like the flux or the surface brightness, and in apparently complicated situations you just follow how the frequency behaves. Many people don't use this, and their derivations are often overly complicated and subject to error as a result. One example relates to gamma-ray bursts. The model for the "afterglow" in X-rays, optical, IR, and radio is that there is a blast wave produced by a central explosion, and what we are seeing is radiation from the surface of this highly relativistic blast wave, which in addition could be a jet instead of being spherically symmetric. The quantities of interest (e.g., the light curve of the burst, the flux, etc.) are all derivable from the specific intensity. Given that there is a cosmological redshift ($z \sim 1$ in many cases) and a strong Doppler shift (Lorentz factors $\gamma \sim 300$ are inferred), this can be a very tough road to hoe otherwise. I've also used this extensively in computations of ray tracing around rotating neutron stars, where in general the spacetime is quite complicated.

Let's go through a few of the quantities that can be derived from the specific intensity, then we'll consider a couple of examples of how to use it. Suppose we use a detector of area dA and want to know the differential flux dF_ν observed from a source in the solid angle $d\Omega$. Assume that $d\Omega$ is at an angle θ with respect to the normal to dA. **Ask class:** how does dF_ν relate to I_ν? It's simply

$$dF_\nu = I_\nu \cos\theta d\Omega \ . \tag{2}$$

The net flux is the integral of this quantity over all solid angles:

$$F_\nu = \int I_\nu \cos\theta d\Omega \ . \tag{3}$$

Note that we can check whether this is reasonable: the units agree and the limits agree (e.g., when $\theta = \pi/2$ the flux is zero, as it should be). But wait: what if I_ν is isotropic, i.e., there is an equal amount of radiation coming from all directions? Then $F_\nu = 0$. Is this reasonable? Yes, because we want the *net* flux. If the radiation field is isotropic (or indeed back-front symmetric), then there is no net flux because as much flux is coming through the $-\mathbf{n}$ direction as the $\mathbf{n}$ direction.

How about the pressure? **Ask class:** before calculating, do we expect that in an isotropic radiation field the pressure will vanish, as the flux did? No, we don't. Just as in a uniform gas, the pressure is nonzero even if there is no net motion. The pressure is the momentum per unit time per unit area. The momentum of a photon of energy E is E/c, so the momentum flux is dF_ν/c. The component of the momentum flux normal to the area element dA (i.e., the pressure) needs to be multiplied by another factor of $\cos\theta$, so

$$p_\nu = \frac{1}{c}\int I_\nu \cos^2\theta d\Omega\ . \tag{4}$$

As expected, this does *not* vanish for an isotropic radiation field. In fact, if I_ν is nonzero anywhere then p_ν is positive, as it must be.

As we'll discuss later, radiation can exert a net force (by scattering, for example), which has many applications. Lots of people (including the authors of our textbook!) loosely talk about radiation *pressure* when they mean radiation *force*. Don't make that mistake! As you see, the pressure can be nonzero when the flux is zero, and similarly the pressure can be nonzero when the force is zero. If all the radiation is moving in the same direction (e.g., the radiation field outside a point source) then the distinction is blurred, but in general it can be complicated.

Obviously, you can integrate any of these quantities with respect to ν to get the frequency-integrated flux, pressure, etc.

Ask class: can they think of a way that an object could experience a net radiation force in an isotropic radiation field?

We can also compute the radiation energy density. **Ask class:** for an isotropic radiation field, do we expect the radiation energy density to vanish? No! In fact, unlike the pressure in a given direction, for the energy density each photon contributes equally regardless of its direction of propagation. Define the specific energy density u_ν as the energy per volume per frequency interval. Then the energy in a volume dV is $dE = u_\nu(\Omega)dV d\nu d\Omega$. At the speed of light c, $dV = cdAdt$ in a given direction, so

$$dE = u_\nu(\Omega)cdAdtd\nu d\Omega = I_\nu dAdtd\nu d\Omega\ , \tag{5}$$

where the second equality follows from the definition of specific intensity. Therefore, $u_\nu(\Omega) = I_\nu/c$ and the energy density is

$$u_\nu = \frac{1}{c}\int I_\nu d\Omega\ . \tag{6}$$

Notice that the energy density, the flux, and the pressure are different moments of the specific intensity (weighted by different factors of $\cos\theta$). For an isotropic radiation field (I_ν independent of Ω) we have $u_\nu = \frac{1}{c}I_\nu\int d\Omega = (4\pi/c)I_\nu$ and $p_\nu = \frac{1}{c}I_\nu\int\cos^2\theta d\Omega = (4\pi/3c)I_\nu$, so that $p_\nu = u_\nu/3$. This helps in the discussion of the thermodynamics of blackbody radiation. It also leads to the definition of the *mean intensity*, which is just $J_\nu = (1/4\pi)\int I_\nu d\Omega$.

Now let's do a couple of examples.

(a) Blazars are types of active galactic nuclei that have relativistic beams pointing close to our line of sight. Suppose you have one at a redshift z with a beam that has a Lorentz factor γ and is pointed at an angle θ from your line of sight (that is, $\theta = 0$ is directly towards you). How does its surface brightness, as measured by you, depend on z, γ, and θ?

(b) Gamma-ray bursts are relatively short (milliseconds to hundreds of seconds) bursts of gamma rays that are known to come from cosmological distances. Some people believe that there are gamma-ray bursts from very high redshift, in the range of 10 to 20. For such sources, the proper distance (i.e., the distance you would measure to them with a ruler) is essentially independent of redshift. Suppose all gamma-ray bursts are intrinsically identical. Also suppose that you observe a gamma-ray burst bolometrically (over all frequencies) for its entire duration as perceived by you. The fluence is the total energy you receive per area, that is, erg cm^{-2}, after you've integrated over the entire burst. How does the fluence depend on redshift?

Answer:

(a) Remember that surface brightness scales with photon frequency in the same way as frequency-integrated specific intensity, as $I \propto \nu^4$. Thus, all we need is knowledge of how the frequency of an individual photon will change due to Doppler and cosmological redshifts. The cosmological redshift is simply $1/(1+z)$. The Doppler shift can be looked up in any of a number of textbooks, and is $[1/\gamma(1-(v/c)\cos\theta)]$. One can check this: is it maximal when $\theta = 0$ and minimal when $\theta = \pi$, as it should be? Yes, so we have a little confidence that we didn't misplace a minus sign. Anyway, combining the two, the frequency of a given photon will change by the combined factor

$$\frac{1}{1+z}\left[\frac{1}{\gamma\left(1-(v/c)\cos\theta\right)}\right] , \tag{7}$$

and therefore the surface brightness is proportional to

$$\frac{1}{(1+z)^4}\left[\frac{1}{\gamma\left(1-(v/c)\cos\theta\right)}\right]^4 . \tag{8}$$

For these sources it is usually assumed that there are two jets, one pointing basically towards us and one pointing away. You can see that for v close to c, there can be a spectacular difference in the surface brightnesses (factors of 10^3 or more!), which leads to only one jet being observable in many cases.

(b) The fluence is the energy per area. This depends on the total energy received and the total area over which it is spread. But the total area is essentially fixed (because the proper distance is roughly fixed), so the fluence in this case just depends on the total energy received. Every photon from the burst will be redshifted by the same factor, $1/(1+z)$, which means that the total energy will be decreased by that factor. Therefore, the fluence scales as $1/(1+z)$.

This is not a large factor. It means that gamma-ray bursts may be almost as easy to detect at $z = 10$ as they are at $z = 5$. If so, they may be good beacons from the early universe. Note that, for example, galaxies are detected based on their surface brightness (because it's the contrast with the background that determines detectability). This surface brightness goes as $1/(1+z)^4$, so it drops off much more rapidly with redshift. This makes early galaxies challenging to detect.

Recommended Rybicki and Lightman problem: number 1.1