

Absorption and Addition of Opacities

Initial questions: Why does the spectrum of the Sun show dark lines? Are there circumstances in which the spectrum of something should be a featureless continuum? What can we learn from spectra?

If the only way photons could interact was via simple scattering, there would be no blackbodies. We'll go into that in much more detail in the next lecture, but the basic reason is that in simple scattering there is no creation or destruction of photons. For that we have to treat emission and absorption, respectively. We'll start with absorption, and talk about the important principles of how opacities are combined.

To be ridiculously basic, absorption happens when a photon gets eaten by something. It could be by an atom or molecule, in which case the atom or molecule changes atomic state or is even ionized or dissociated. It could be by a free electron moving near an ion, in a process inverse to bremsstrahlung. Whatever the detailed process, though, it is rarely frequency-independent (unlike scattering, which is often nearly independent of photon frequency). In addition, multiple different sources of opacity are present simultaneously, meaning that to get a qualitative handle on how spectra will look, how radiation will move through a medium, etc., we need to know how to add opacities.

For the following, it is useful to start by imagining a hypothetical situation in which all energy transport is done with photons of a fixed energy and polarization. Of course, Doppler shifts mean that the photon energy "seen" by an interacting particle (an electron, an atom, a molecule, ...) could vary depending on the motion of the particle, but we'll ignore that. Thus we'll suppose that after scatterings, and even after absorption and re-emission, the photon energy and polarization are the same as they were before. This would be an example of a *fixed* channel of energy transfer. For a fixed channel, the opacities add linearly (highest opacity dominates). Even though we have imagined photons forced to retain their energy and polarization, for our analysis a given energy and polarization counts as a fixed channel; that is, if we consider photons that currently have a given energy and polarization, that's the channel.

Now, in reality there will always be processes that can change the energy of a photon or even its polarization. Doppler shifts will do it, for example, and absorption and re-emission will certainly do it. Thus the photon energies and polarizations with low total opacity will be more effective at transporting energy than will the photon energies and polarization with high total opacity. For the discussion below, different energies and polarizations count as *independent* channels of energy transfer. Between independent channels, therefore, it is the lowest opacity that matters most.

We can even go beyond photons. Consider the interior of a star, in which the temperature varies as a function of position and thus energy is transported. At a given location the energy will be shared by photons, electrons, and ions (and occasionally more exotic particles). If you consider, say, the photons and electrons, they can both transport energy (by radiation and by conduction, respectively). If the effective opacity to photons is a lot less than the effective opacity to electron conduction, then most of the energy is transported via radiation. The reverse can happen if the effective opacity to conduction is much less than that to radiation.

Because these concepts have posed great difficulties throughout many classes I have taught, I will now present them again in a slightly different way:

- The *cross section* is the “effective area” of a single particle/atom/molecule/whatever. Equivalently, it’s the number of interactions of that particle per second divided by the flux of incoming photons (number per area per second).
- The *opacity* is the “effective area” per gram of material. This is *not* equivalent to cross section. For example, the ionization cross section can be much larger than the Thomson cross section for some photon energies. However, the ionization opacity also depends on the number of atoms with electrons; the ionization opacity of fully ionized material is zero, even though the ionization cross section is still the same as it was.
- For a fixed channel of energy transfer, *highest* opacity dominates. For example, suppose we are interested in the opacity to photons of a given frequency. The photons will interact first via the highest opacity process. If, e.g., there is a line at that frequency, the photons will interact quickly. This means that for a fixed channel, opacities add *linearly*. **Example:** suppose we construct a half-sphere that is mirrored on the inside, and put a light source in the middle. We poke some holes in the sphere, and surround the sphere with transparent glass. **Ask class:** In a given direction, what determines whether light shows through? If the sphere does not have a hole (the opacity is high), then it is dark in that direction. The presence of a low opacity along that line of sight (the transparent glass) doesn’t make a difference. For a fixed channel, opacities add like resistors in series.
- For independent channels of energy transfer, *lowest* opacity dominates. For example, consider the “average” opacity over a full blackbody spectrum. Photon energies with low opacity are easy for photons to travel in. So, photons tend to diffuse into those energies away from energies where the opacity is high. That’s why in optically thick material, lines are dark. **Example:** in the half-sphere example above, **Ask class:** where does the light come out? It comes out where there are holes (i.e., low opacity). In directions where there are no holes, the light reflects back and eventually escapes via a hole. Another example is conduction versus radiative energy transport. When energy is easily transferred via conduction, that’s what determines the overall energy

transfer rate. When energy is easily transferred via radiation, that's what dominates. **Example:** if you're standing in cold air, you lose heat at a certain rate. If you put your hand on a piece of metal at the same temperature as the air, you lose heat a lot more rapidly! It's an independent channel of energy loss. **Ask class:** mathematically, how would one weight the smaller opacities more? For independent channels, opacities add *harmonically*, that is, $1/\kappa = \sum(1/\kappa_i)$ for different channels i , weighted appropriately. For independent channels, opacities add like resistors in parallel.

With these firmly in mind, let's quickly explore several different types of absorption. Each of these will be treated in greater detail later on, so this is just a survey.

Radiative opacity sources: Ask class: what are the ways in which a photon can interact? Can be done off of free electrons, atoms, molecules, or dust. Can also interact with protons/nuclei, magnetic fields, and other photons, but in this class we'll ignore those for the most part. For free electrons, have electron scattering (not an absorption) and free-free (inverse bremsstrahlung, which is an absorption). Scattering means that the photon retains its identity (in a somewhat ill-determined way), whereas absorption means that the photon is destroyed and later another one is emitted.

For atoms, have transitions between atomic levels (bound-bound absorption) and ionizations (bound-free absorption). **Ask class:** when do they expect opacities involving free electrons to be most important? When ionization fraction is high. Molecules and dust are more important for lower-energy photons. Same with H^- opacity, which is important for optical opacity in some stars.

Bound-bound absorption This is when a bound electron absorbs a photon and consequently makes a transition to a higher bound level. As an example, take the Ly α resonance of hydrogen, where an electron in the 1s state is absorbed into the 2p state. This has an oscillator strength of 0.42 and a peak cross section of $7.2 \times 10^{-11} \text{ cm}^2$. This is *enormous*, a good 10^{14} times the Thomson cross section. **Ask class:** do they expect this to be important in the overall transport of energy? It's a really high cross section, but over only a very small range in energy, so it produces a dark line but the energy easily moves to neighboring frequencies and escapes where the opacity is lower. The lifetime is about 10^{-9} s , so the energy width is $\hbar/10^{-9} \approx 10^{-6} \text{ eV}$. Generally, therefore, bound-bound transitions have extremely high cross sections over extremely small regions of the spectrum. There *are* cases in which there can be so many lines that their wings overlap, leading to "line blanketing" and significant effects on opacities. **Ask class:** would they expect that this would happen in stars with low fractions of elements heavier than helium ("metals" in astronomical nomenclature) or high fractions?

Bound-free absorption The process here is that a photon ejects an electron from a bound state inside an atom. **Ask class:** let's say you have a photon whose energy is much larger than this threshold energy. Do you expect the cross section (i.e., strength of

interaction) to increase or decrease with increasing energy? Decrease, because it's farther from the resonance energy of ionization.

Free-free absorption In this process a free electron passes by an ion of charge Z , scatters off of it, and absorbs a photon. This is the inverse process to bremsstrahlung. Conservation of energy means that

$$\frac{p_k^2}{2m_e} = \frac{p_s^2}{2m_e} + \hbar\omega . \quad (1)$$

Here, of course, we're assuming nonrelativistic electrons. The presence of an ion is very important. For example, suppose we just want an electron to emit a photon. Then it is impossible to conserve both energy and momentum.

Rosseland mean opacities: When the optical depth is extremely high, one is well justified in assuming that the local radiation is that of a blackbody. We then want to know an "average" opacity such that if we assume that the opacity at all frequencies is this average, we'll get the radiative transfer about right. The way to do this is to weight the average harmonically by the Planck (blackbody) function:

$$\frac{1}{\kappa} = \left[\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T} d\nu \right] \left[\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu \right]^{-1} \quad (2)$$

where $B_\nu = (2h\nu^3/c^2)/(e^{h\nu/kT} - 1)$ is the Planck function. This is the Rosseland mean opacity. **Ask class:** at a given frequency ν , how should we combine opacities from different sources, e.g., scattering or absorption? Linearly (highest opacity is most important for a single channel). **Ask class:** why is the Rosseland mean opacity harmonically weighted? Because the *low* opacities will dominate the transfer, since photons at those energies will go farther and sample larger gradients in temperature.

The Total Radiative Opacity

At this point, let's step back and think about what this means. In principle, a photon propagating through gas will have some probability of interacting in all of the ways above (plus others we haven't discussed as much, like pair production for high energy photons, and so on). We need a way to determine what dominates when.

Ask class: Suppose we suppress all photon interaction mechanisms except for one (scattering for definiteness). Let's consider photons of a given energy. What factors determine the mean free path of those photons? The cross section and the number density of scatterers. Specifically, $\ell = 1/n\sigma$. **Ask class:** is n the number density of gas particles in any state, or just those that can scatter? Only those that can scatter. **Ask class:** so, suppose we have a gas of 100% neutral hydrogen atoms. What is the mean free path to Thomson scattering? It's infinite, because there are no free electrons from which the photons would scatter.

This points out a difference between *cross section* and *opacity*. The cross section says (in this case): suppose you have free electrons. What is the surface density (in cm^{-2}) of those free electrons so that only a fraction e^{-1} of photons go unscattered? That's just $\Sigma = 1/\sigma$. But this does *not* tell you how important scattering is! For that, you need to know the number density of the scatterers. Opacity is expressed in terms of $\kappa = \text{cm}^2 \text{g}^{-1}$. How many scatterers (each with cross section σ) are there per gram of material? κ/σ .

So, **Ask class:** suppose you have hydrogen gas that is 100% ionized. How important do you expect bound-free ionization to be? Totally unimportant, because there are no bound atoms! How about bound-bound? Same thing. Of the processes we've considered, only free-free and scattering could possibly contribute.

Okay, now let's suppose we have photons of a given energy. We've computed the opacities for all the different processes. **Ask class:** which processes will dominate, those with high opacities or those with low opacities? High opacities, since those interactions will occur first. At a given frequency, high opacities win.

Now, suppose that we have a flat input spectrum of photon energies. We send it into a region of gas, where the total opacity varies as a function of energy. **Ask class:** do they expect most of the radiation transferred through this region to be at frequencies of high opacity or low opacity? Low opacity, because it is easier to propagate photons that way. That's why in optically thick plasmas (like the Sun!), lines tend to be dark, because there's less emission there. In fact, when one considers the "effective" opacity over a spectrum in an optically thick region, one usually takes a harmonic average (that is, weighted by the reciprocal of the opacity). That's the Rosseland mean.

Summary: opacity, not cross section, determines importance of processes. At a given photon energy, high opacities dominate. Over a spectrum, low opacities dominate.

Now let's apply these principles in a few cases. Consider a situation in which electron scattering as well as the absorption processes we considered (bound-bound, bound-free, free-free) all exist. Suppose we pick a photon frequency in the center of a bound-bound transition line. **Ask class:** will the opacity be high or low at that frequency? It will be high: for a single channel, the highest-opacity process dominates. Now suppose our photons are doing their propagating in the Sun. We see photons of a given frequency from the level at which they last interacted. The temperature of the Sun increases closer to the core. **Ask class:** given this, do we expect spectral lines in the Sun to be brighter or darker than the nearby continuum emission? Darker. The logic is that for higher opacity there are more interactions, so photons finally stream to us from a region of the atmosphere that is higher, and therefore cooler, than it would be for a photon frequency with a lower opacity.

Next example: we have a star with a fixed flux. Initially, there are no spectral lines, just

continuum opacity. We measure the specific intensity at a given frequency ν_0 . Now we add an additional opacity source in the form of a single spectral line with a frequency ν_1 that is well separated from ν_0 . The integrated flux over all photon frequencies is kept constant. **Ask class:** does the specific intensity at ν_0 go up, go down, or stay the same? It goes up. The addition of the line means that less flux is getting out at ν_1 . The total flux is kept constant by assumption, so the deficit in flux must be made up in the rest of the spectrum. Therefore, the specific intensity at ν_0 goes up by what is probably a tiny amount.

Yet another example. Focus on electron scattering. When a large magnetic field is present, scattering is modified. Qualitatively, this is because scattering involves an electromagnetic wave wiggling an electron in the direction of its polarization; the electron then radiates. If the magnetic field is strong, it's tough to wiggle the electron in that direction. The upshot is that for polarizations along the field (where the field offers no resistance) the cross section is the same as in no field, whereas for polarizations across the field the cross section is much less (qualitatively speaking). So, we have two cross sections for two independent polarizations. Consider propagation of radiation in such an environment. **Ask class:** which polarization will dominate, in the sense of carrying most of the flux? The low cross section mode, because these are two independent channels.

Recommended Rybicki and Lightman problem: 1.9