Practice Problems Related to Statistics in Astronomy

1. Show that $\sum_{d=0}^{\infty} \frac{m^d}{d!} e^{-m} = 1$ for any value of $m > 0$.

2. Show that $\int_0^{\infty} \frac{m^d}{d!} e^{-m} dm = 1$ for any value of d.

3. For a fixed d, determine the value of m that maximizes $\frac{m^d}{d!} e^{-m}$.

4. Plot $\frac{m^d}{d!} e^{-m}$ as a function of d for various values of m. Show, visually, that this distribution approaches a Gaussian near the peak when $m \gg 1$.

In the coin-flipping example (8 heads and 12 tails in 20 flips) we determined the best estimate for the probability of heads (0.4) and estimated the 1σ credible region using Wilks' theorem (which gives $\Delta \ln \mathcal{L} = 0.5$ for one parameter at 1σ). But Wilks' theorem is only an approximation.

5. If we started with a uniform prior (all values of a are equally probable from $a = 0$ through $a = 1$) then the posterior probability density is simply proportional to $\mathcal{L}$ (*not* $\ln \mathcal{L}$). Plot $\mathcal{L}$ as a function of a. Note that you do not have to include factors that are independent of a, such as $1/(8!12!)$ or e^{-20}.

6. A correctly normalized probability must integrate (or sum) to 1 over all possibilities. Determine the normalizing constant for this case.

7. Suppose that we want to establish a "1σ" uncertainty region for a. There is, in fact, no unique way to define this uncertainty region. For example, for our case, is it the smallest continguous range in a that has 68.3% of the total probability? Is it the smallest symmetric range in a (i.e., $0.4 - a_{\min} = a_{\max} - 0.4$) that has 68.3% of the total probability? Calculate both for this case (you may wish to use numerical integration), and compare the answers with the range of $a = 0.3$ to $a = 0.51$ that we got using $\Delta \ln \mathcal{L} = 0.5$.