Photons

1. Why photons?

Ask class: most of our information about the universe comes from photons. What are the reasons for this? Let's compare them with other possible messengers, specifically massive particles, neutrinos, and gravitational waves.

- Photons have a small cross section, but not too small. Neutrinos and gravitational waves sail through the universe with almost no interactions. That means that if we could detect them, they would give good directional information about their sources, which combined with energy/frequency resolution could potentially tell us quite a lot. However, they also sail through detectors for the most part, so only exceptionally energetic events can carry information via these channels. Massive particles have the opposite problem. Electrons, protons, and nuclei can be accelerated to high energies, but they are curved by the Galactic magnetic field and slam into air molecules (or go all the way through detectors), so some information is lost. Again, the best observations can come only from highly energetic sources.
- All kinds of objects can emit photons. Heat is all that is needed, but many other processes produce photons as well (this is fundamentally because the electromagnetic interaction is pervasive and relatively strong). In contrast, significant production of gravitational waves requires fast acceleration of large masses, and production of high energy particles needs large potential drops or other acceleration mechanisms. Neutrinos are actually produced pretty commonly (hydrogen fusing into helium generates them), but not enough to compensate for their extremely low cross section.
- Detectors can measure with precision many aspects of photons. These include energy, direction, time of arrival, and polarization. In principle these quantities can also be measured for the other messengers, but in practice such measurements are at much worse precision than is usually available for photons.

2. Photons in a vacuum

Of course, there are some phenomena that are easiest to characterize using gravitational waves, neutrinos, or massive particles, but for the above reasons we will focus first on photons. We will start by considering photons in a vacuum, then recall interactions with matter at low energies before considering high-energy interactions specifically.

Radiation in vacuum: Consider radiation when there is no matter present. In particular, consider a bundle of rays moving through space. **Ask class:** what can happen to those rays in vacuum? They can be bent gravitationally, or redshifted/blueshifted in various ways (Doppler, gravitational, cosmological). In this circumstance, it is useful to recall Liouville's theorem, which says that the phase space density, that is, the number per (distance-momentum)^3 (i.e., the distribution function), is conserved. For photons, this means that if we define the "specific intensity" I_ν as energy per everything:

$$I_\nu = \frac{dE}{dA\,dt\,d\Omega\,d\nu}\ , \tag{1}$$

then the quantity I_ν/ν^3 is conserved in free space. The source of the possible frequency change could be anything: cosmological expansion, gravitational redshift, Doppler shifts, or whatever. The integral of the specific intensity over frequency, $I = \int I_\nu\,d\nu$, is proportional to ν^4.

One application is to the surface brightness. This is defined as flux per solid angle, so if we use S for the surface brightness, then $S = I$. **Ask class:** how does surface brightness depend on distance from the source, if ν is constant? It is independent of distance (can also show this geometrically). However, **Ask class:** how does the surface brightness of a galaxy at a redshift z compare with that of a similar galaxy nearby, assuming no absorption or scattering along the way? The frequency drops by a factor $1+z$, so the surface brightness drops by $(1+z)^4$. This is why it is so challenging to observe galaxies at high redshift. Note that in a given waveband, the observed surface brightness also depends on the spectrum, because what you see in a given band will have been emitted in a different band (these are called K-corrections, because why should anything in astronomy be named in a clear way?).

Another application is to gravitational lensing. Suppose you have a distant galaxy which would have a certain brightness. Gravitational lensing, which does not change the frequency, splits the image into two images. One of those images has twice the flux of the unlensed galaxy. Assume no absorption or scattering. **Ask class:** how large would that image appear to be compared to the unlensed image? Surface brightness is conserved, meaning that to have twice the flux it must appear twice as large. This is one way that people get more detailed glimpses of distant objects. Lensing magnifies the image, so more structure can be resolved.

This is an *extremely* powerful way to figure out what is happening to light as it goes every which way. The specific intensity is all you need to learn lots of important things, such as the flux or the surface brightness, and in apparently complicated situations you just follow how the frequency behaves.

3. Low-energy photons

Now we need to consider how low-energy (say, UV and longward) photons can interact.

Radiative opacity sources: Ask class: what are the ways in which a photon can interact? Can be done off of free electrons, atoms, molecules, or dust. Specific examples include:

- Scattering off of free electrons. At low energy, this process is elastic (the photon energy after scattering equals the photon energy before scattering), and is called Thomson scattering. This cross section is useful to remember: $\sigma_T = 6.65 \times 10^{-25}$ cm^2.
- Free-free absorption. A photon can be absorbed by a free electron (i.e., one not in an atom) moving past a more massive charge (such as a proton or other nucleus). The inverse process, in which a photon is emitted by an accelerating charge, is called bremsstrahlung.
- Atomic absorption. The two main types are bound-free (in which an electron is kicked completely out of an atom by a photon) and bound-bound (in which an electron goes from one bound state to another). Free-free and bound-free absorption cross sections tend to decrease with frequency roughly like ω^{-3} (in the bound-free case this of course applies only above the ionization threshold). Bound-bound absorption is peaked strongly around the energy difference between the two bound states.
- Molecular absorption. The extra degree of freedom associated with multiple atoms in a molecule allows for vibrational and rotational transitions (for example: for rotation the angular momentum is quantized in units of $\hbar$, so a transition could be from angular momentum $\hbar$ to angular momentum $2\hbar$). For relatively simple reasons, there tends to be a strong ordering of energies: atomic$\gg$vibrational$\gg$rotational.

Ask class: why haven't we talked about interactions of photons with protons or other nuclei? Because protons are much tougher to affect with the oscillating electomagnetic fields of photons. In particular, since they're more massive and e/m is smaller, the resulting acceleration is less and the radiation (hence cross section) is tiny by comparison to electrons. For comparison, the scattering cross section off of protons is $\approx m_e^2/m_p^2$ less than off of electrons. That's a factor of almost 4 million. So, for most purposes we can ignore photon-nucleon interactions.

At this stage it is useful to review two concepts: *opacity* versus *cross section*, and addition of opacities.

Cross section is measured in cm^2. It is the effective area of an interaction with a *single* particle, photon, or whatever, and is often indicated by the symbol σ.

Opacity is measured in cm^2 g^{-1}. It is the *total* cross section of interaction *per gram of material.* This is often indicated by the symbol κ.

In order to clarify why these two concepts, though related, are different, consider the following. Suppose we have a cloud of completely neutral hydrogen gas. **Ask class:** what is the cross section to Thomson scattering? It is just $\sigma_T = 6.65 \times 10^{-25}$ cm^2. This is *always* the Thomson cross section. However, **Ask class:** what is the *opacity* to Thomson scattering in this case, assuming a photon energy much lower than the hydrogen ionization energy? It is zero! The gas is neutral, therefore in a given gram of material there are no free electrons. Thomson scattering is scattering off of free electrons, so no go. The total opacity to *all* processes, however, is nonzero because one could have bound-bound absorption or other interactions depending on the photon energy.

This brings us to **Addition of opacities**. In many circumstances one would like to know the total effective opacity. For example, this is the relevant quantity for calculations of energy transfer. The rules are straightforward:

If the opacities operate on the **same channel**, then they add **linearly** (just like resistors in series). That is, $\kappa_{\rm tot} = \kappa_1 + \kappa_2$.

If the opacities operate on **different channels**, then they add **harmonically** (just like resistors in parallel). That is, $1/\kappa_{\rm tot} = 1/\kappa_1 + 1/\kappa_2$.

Let's work some examples. In the following cases, do the opacities add linearly or harmonically?

1. Free-free and bound-free opacity, on photons of a given energy and polarization?

2. Free-free and electron scattering opacity, on photons of a given energy and polarization?

3. Electron scattering opacity in an extremely strong magnetic field, on photons of a given energy but two different polarizations, one parallel to the field and one perpendicular?

4. Bound-free opacity on photons of different energies?

5. The total radiative opacity and the total conductive opacity?

One way to remember these rules is to realize that if energy can travel an easier path, it will. Think of an analogy. There are two roads to a given destination. One is a narrow dirt road, the other is a four-lane freeway. The "opacity" along the dirt road is larger than along the freeway, but the total traffic flow rate is *still* increased by its existence. This is consistent with adding the "opacities" harmonically. In contrast, think of a single road. Any opacity source along the way (trucks, construction, senior citizens in a parade) will stack, making the trip that much more painful!

If you are unfamiliar with any of these concepts or processes, I recommend that you read "Radiative Processes" by Rybicki and Lightman, or volume 1 (Radiation) of "The Physical Universe" by Shu. I also have online notes from when I have taught the graduate Radiative Processes class: http://www.astro.umd.edu/∼miller/teaching/astr601.

4. High-energy photons

Now, however, we need to consider extra things that can happen with photons when they have high energy. For our purposes, "high energy" means that the photon energy is comparable to or larger than the rest mass-energy of an electron. **Ask class:** what differences does this introduce?

- At these energies, the photon momentum is significant. As a result, electron recoil must be included in electron scattering. Therefore, in the reference frame in which the electron was originally at rest, the photon energy after scattering must be less than it was before scattering. In addition, it turns out that the total scattering cross section decreases at higher energies[1]. The process as a whole is called Compton scattering, and the total cross section is the Klein-Nishina cross section.
- When the photon has high enough energy, pair production is possible. For photon-photon pair production, one can verify that the condition for pair production is that in the center of momentum frame the product of photon energies exceeds $(m_e c^2)^2$, where $m_e c^2 = 511$ keV. Single-photon pair production is impossible in a vacuum, but if something else is around to absorb extra momentum (in particular, an extremely strong magnetic field), then it can happen. In the presence of a strong magnetic field, a single photon can also split into two photons[2]. Here "strong" means comparable to the quantum critical field $B_c = m_e^2 c^3/(\hbar e) = 4.414 \times 10^{13}$ G at which the electron cyclotron energy equals the electron rest mass energy.

Another effect of extremely strong magnetic fields is to affect the way that photons

[1]Why does it decrease? You can think of it heuristically in the following way. A way to define the cross section of, say, scattering is as the ratio of the scattered photon energy to the incident photon energy per area. If the photon has larger energy then the intially-at-rest electron acquires a kinetic energy after scattering. This, in turn, means that there is less scattered photon energy, so the cross section decreases.

[2]How? A good way to understand this is to realize that you can think of electric and magnetic fields as comprised of virtual photons. Thus when a photon interacts with a magnetic field, there are channels by which a virtual photon can be made real. Another path to understanding is again related to momentum: for a single photon to turn into two photons in vacuum, the two photons have to travel in exactly the same direction as the original photon. Thus there is zero phase space for this to occur. But when a magnetic field is around to take up momentum, the restriction is relaxed, and the two photons can occupy a broader range of solid angles.

scatter. At first sight this might seem odd: photons aren't charged, so why should magnetic fields affect them? To understand this, consider a photon scattering off an electron. In a classical sense, what is happening is that the oscillating electric field of the photon accelerates the electron up and down. Accelerated charges radiate, thus the electron sends out a photon in some direction. The net result is that the photon hits the electron and bounces off in some other direction. **Ask class:** how would this change if it occurred in a very strong magnetic field? If the electron moves parallel to the field, there is no difference because there is no resisting force. Therefore, for photons polarized along the field, the scattering cross section is basically the same as it was before (roughly Thomson). However, for photons polarized across the field it's different. The electron has great difficulty moving in that direction, so it is tough to radiate and thus the cross section is decreased a lot. For a photon of frequency ω and an electron cyclotron frequency ω_c, the cross section for a perpendicular polarization (also called the "extraordinary mode" versus the "ordinary mode" for parallel polarization) is roughly $\sigma = \sigma_T(\omega/\omega_c)^2$. This can make a big difference for neutron stars.

Appendix: conservation of I_ν/ν^3 from Liouville's theorem

Variants of this derivation can be found in many places.

Liouville's theorem says that the phase space density f of particles (including photons) in free motion is constant. That is, if we consider a phase space volume $d^3r d^3p$ in which there are dN particles, then if at a given instant

$$dN = f d^3r d^3p \tag{2}$$

then when we check in later on that bundle of particles (again, the particles could be photons), $f = dN/(d^3r d^3p)$ will be the same. dN is just a number, so that obviously won't change, but the interesting thing is that although the volume that the particles occupy (i.e., d^3x) can change, and so can the momentum volume that they occupy, the product of the space volume and the momentum volume is constant.

Now let's see how this applies to photons and specific intensity.

When we consider specific intensity I_ν, we are thinking about the energy dE in a small frequency range $d\nu$ around ν, in a small area dA in a small solid angle $d\Omega$, and in a short interval dt. Then

$$I_\nu = \frac{dE}{dA\,dt\,d\Omega\,d\nu}\ . \tag{3}$$

If we orient our surface dA perpendicular to the direction Ω (if you follow correctly the factors of $\cos\theta$ you can show that angling the surface leads to the same conclusion), then the volume traversed in a time dt by the photons is $d^3r = dAcdt$. The momentum volume, in general, is $d^3p = p^2 dp d\Omega$. For photons, $p = E/c = h\nu/c$ for frequency ν, so $dp = dE/c = hd\nu/c$ and $p^2 dp d\Omega = (h\nu/c)^2 (hd\nu/c) d\Omega$. Therefore the phase space volume is

$$d^3r d^3p = (dAcdt)(h^3/c^3)\nu^2 d\nu d\Omega \ . \tag{4}$$

Now, if we have photons of energy $E = h\nu$, then the number of photons in our bundle is $dN = dE/(h\nu)$. As a result, Liouville's theorem becomes

$$dE/(h\nu) = f\nu^2 (h^3/c^2) dA\, dt\, d\Omega\, d\nu \tag{5}$$

where we recall that f is constant. Dividing through we get

$$\frac{dE}{dA\, dt\, d\Omega\, d\nu} = (h^4/c^2) f\nu^3 \tag{6}$$

or

$$\begin{aligned} I_\nu &= (h^4/c^2) f\nu^3 \\ I_\nu/\nu^3 &= (h^4/c^2) f \ . \end{aligned} \tag{7}$$

Because h^4/c^2 is constant and f is constant, this finally means that I_ν/ν^3 is constant for propagation in free space.