**ASTR 680 Problem Set 2: Due Thursday, September 29, 2022**

1. Dr. Sane has proven that black holes cannot exist. Like many great ideas, it is profound in its simplicity and proves that the general relativistic community is just a bunch of establishment dupes. The basis for his disproof comes from our notes, in which we demonstrated clearly that the proper radial velocity for a particle initially at rest at infinity is $u^r = dr/d\tau = \sqrt{2M/r}$ (recall that we use $G = c = 1$, so this is in units of the speed of light). Therefore, if black holes exist and particles can fall to $r < 2M$, they would go faster than the speed of light and thus contradict special relativity. To demonstrate that this is not simply a pathology of the coordinates, Dr. Sane points out that the radial velocity of this particle seen by a local static observer is also $\sqrt{2M/r}$, so again we would have a contradiction if $r < 2M$. Therefore, all objects must have surfaces with $r > 2M$.

The National Science Foundation has been called in to investigate a possible case of fraud against the gravitational wave community, which has claimed that they have detected BH-BH mergers. You have been consulted as an external expert to deliver your opinion. What is your evaluation?

2. A particle with nonzero rest mass circles at the innermost stable circular orbit (ISCO) around a nonrotating black hole, then is perturbed very slightly inward and spirals to the horizon in free fall. It therefore retains its specific angular momentum $u_\phi = \sqrt{12}M$ and its specific energy $e = |u_t| = \sqrt{8/9}$, where both are as usual in units with $G = c = 1$. As a function of the Schwarzschild radius $r$, with $r$ between the ISCO and the horizon at $r = 2M$, derive the radial and azimuthal speed of the particle as measured by a local static observer. Comment in particular on the value you get as $r \to 2M^+$ (i.e., as $r$ approaches $2M$ from above); what does this mean, and does it make sense?

3. (**8 points**) This will be the first of our computational problems. Consider a neutron star, which we will model as a spherically symmetric object of gravitational mass $M$ and Schwarzschild radius $R$, measured in units of $M$ (e.g., $R = 6M$ is at the ISCO). This therefore has the Schwarzschild external spacetime. We will nonetheless assume (inconsistently) that the star rotates at an angular velocity $\Omega$ as seen at infinity. Now consider matter that eventually settles on the equator of this star, rotating with the star, from an initial state of rest at infinity.

(a) (**4 points**) Derive the expression for the specific energy release of the matter. Evaluate your expression at two limiting values of $R$ and/or $\Omega$, and judge whether your expression makes sense. Make sure that you use two *different* limits, i.e., they can't be in reality the same limit.

(b) (**4 points**) Write a program to output the specific energy release as a function of rotation rate, from $\nu = \Omega/(2\pi)$ =0 Hz to 1000 Hz, for $M = 1.4\ M_\odot$ and $R = 10$ km. For this I'll need an e-mail copy of your code, which can be in any language but must be able to compile and run on the astro machines (I'm not going to install anything!). I'll also need a hardcopy of the graph of the specific energy versus spin frequency.