ASTR 680 Problem Set 4: Due Thursday, November 3, 2022

1. **4 pts** Dr. I. M. N. Sane has been applying his deep intellect to the study of gravitational lensing. He is especially interested in strong lensing (i.e., multiple imaging) of background quasars by the supermassive black holes found in the centers of essentially all big galaxies. His calculations suggest that there are hundreds of such lensing events in the Sloan Digital Sky Survey (undoubtedly missed by establishment scientists), and he has issued a press release announcing the revolution in cosmology and black hole astrophysics that will be precipitated by his discovery once it is confirmed observationally.

Ron Cowen, an outstanding freelance writer, has called you to determine whether he should run with this story. Following Dr. Sane's calculations, we assume that the number density of big galaxies is 0.003 Mpc^{-3} and that the relevant volume of the universe is a 4 Gpc radius sphere centered on us. We also assume that each galaxy has a $10^7\,M_\odot$ black hole. In addition, Dr. Sane assumes (and you should as well) that for these lensing situations $D_L = D_{LS} = 2$ Gpc, and $D_S = 4$ Gpc. The Sloan Survey has roughly 10^6 quasars. Given this, calculate the expected number of multiple images by supermassive black holes, and write your response to Ron Cowen based on this.

In the three problems below, you will consider a cluster with a temperature of $T = 10^8$ K. Assume that the cluster is composed of fully ionized hydrogen, 20% by mass, with the rest of the mass being dark matter. Assume that the dark matter and the gas are both distributed as singular isothermal spheres ($\rho \propto r^{-2}$, so that inside a radius r the mass is $M(<r) \propto r$), with a cutoff radius of 1 Mpc.

2. **4 pts** Derive the total mass of the cluster, and the velocity dispersion of the galaxies in the cluster.

3. **4 pts** Suppose that a photon from a distant background galaxy has an impact parameter of 1 Mpc relative to the cluster. Derive the total angle by which it will be deflected.

4. **4 pts** Write a program to compute the cooling time as a function of radius. We will define this as the time over which the *current* bremsstrahlung luminosity per volume would reduce the local temperature by a factor of two. Specifically, inside of what radius is this time less than 10^{10} yr? As usual, please e-mail me your code and hand in a hardcopy of the cooling time vs. radius.