ASTR 680 Practice questions for lecture 14: Observational signatures of strong gravity

1. In the notes we mention that it is difficult to grow a $10^9\ M_\odot$ black hole by $z = 7.1$. Let's say that you start with a $10\ M_\odot$ black hole, and accrete with an efficiency $\eta \equiv L/\dot{M}c^2 = 0.1$ at the Eddington luminosity for 600 million years (roughly the time from the formation of the first stars to $z = 7.1$). How high a mass do you reach? What are some ways to get around these restrictions?

2. Suppose that gravitons have rest mass-energies of 10^{-20} eV. Assuming that a graviton with a frequency ν has a total energy $h\nu$ (and that special relativity applies as normal), compute the arrival time difference between two gravitons at Earth assuming that one has a frequency $\nu = 50$ Hz and the other has a frequency $\nu = 200$ Hz, and that they were produced at the same time, a billion light years from us. Compare that time difference with the ~ 0.2 second duration of GW150914.