

Particle acceleration and generation of high-energy photons

For acceleration, see Chapter 21 of Longair

Ask class: suppose we observe a photon with an energy of 1 TeV. How could it have been produced? In particular, could it have been produced thermally? No, because the temperature equivalent is $E/k \approx 10^{16}$ K, and nothing in the universe is that hot. So, it must have been produced nonthermally. Ultimately, that means that the photon must have been produced by a high-energy particle. Let's narrow down how it could have been accelerated. **Ask class:** could the particle have been accelerated via the strong force? No, because the distance over which the strong force acts is too small. It would need to have an unbelievable acceleration over the $\sim 10^{-13}$ cm distances. Same with the weak force. **Ask class:** could the particle have been accelerated via gravity? No, but this takes a little more argument. **Ask class:** how does something get accelerated by gravity? Think in particular of boosts given to planetary probes. The boost actually comes from the velocity of the planet in its orbit, not the gravity per se. In fact, gravity is a conservative field, so in an isolated system a particle can't get any net energy from gravity alone. This means that only electromagnetism can accelerate particles.

Now let's think about how electromagnetism can accelerate particles. **Ask class:** what kind of particles are accelerated? In particular, **tell as joke** a proton, a neutron, and an iron nucleus walk into the Bar of Electromagnetic Fields. Which one gains the most energy, assuming no losses? The iron nucleus, being most charged, will. The proton is next in line. The neutron is uncharged, so it gains the least.

Ask class: what happens if a proton goes into a region with static magnetic fields but zero electric field? What happens to its speed and direction? Direction can change, but speed won't. **Ask class:** what happens to the energy of the proton in a reference frame in which the magnetic field is moving? Since the proton energy in the field frame is unchanged, you get a Doppler shift in the laboratory frame. Therefore, moving magnetic fields can also increase the energy of protons. Another way to get high-energy particles is to have acceleration of some type along an electric field. **Ask class:** how could this happen? It could be a potential drop or it could be acceleration by Compton scattering (which is indeed a form of electric field).

Therefore, to summarize, the only way to get very high energy photons is to produce them using high-energy charged particles, and to get high-energy particles either moving magnetic fields or electric fields are required. We will soon discuss some specific mechanisms of this type, but first we need to know what we have to explain. The spectra of high-energy particles (such as cosmic rays) or high-energy photons are often power laws, typically with an index of between 2 and 3 (that is, the number at a particular energy is $N(E)dE \propto E^{-\alpha}dE$,

where $\alpha=2$ to 3). These can extend up to extremely high energies; for example, cosmic rays have been seen at energies up to 3×10^{20} eV! This is an example of acceleration of a small fraction of particles to ultrahigh energies. There is also evidence in active galactic nuclei for bulk motion with Lorentz factors up to 10-30, so in some cases all the matter in a region may become significantly relativistic, but not ridiculously so!

This means we'd like to explain (1) power-law spectra and ultrahigh energies for a small fraction of particles, and (2) relativistic bulk motion. In this lecture we'll talk only about the first problem. When we get to accretion disks, we'll talk about how relativistic bulk motion may be produced. We'll consider three types of acceleration: Fermi acceleration, radiative acceleration, and acceleration by a potential drop, to see if these can do it. We will then look at energy losses, which pose serious constraints on many of the models.

Second-order Fermi acceleration

Note: this mechanism is no longer favored, but it does show the principles of collisionless plasmas and Doppler acceleration. See also http://sol.physics.usyd.edu.au/sosp3_2000/node5.html for Fermi mechanisms. Sections 21.3 and 21.4 of Longair have good descriptions of Fermi acceleration.

One mechanism, proposed by Fermi in 1949, involves motion of “clouds” of magnetic flux. You can essentially imagine these clouds moving randomly, playing tennis with high-energy charged particles. If the cloud is moving towards the particle, it will scatter the particle backwards and the Doppler shift will increase the particle's energy. If the cloud is moving away, the Doppler shift will decrease the particle's energy. **Ask class:** would they expect a net gain in energy over many collisions? If so, why? Also, what would be the net fractional energy gain in a power of v/c , where v is the cloud speed? There *is* a net gain, because head-on collisions happen more often than collisions when the particle and cloud are moving in the same direction (call these “tailgate collisions”). To first order in v/c the energy gain and loss cancel each other, so the fraction energy gain is of order $(v/c)^2$. For that reason, this is called second-order Fermi acceleration.

More mathematically, consider clouds with an average distance between them of L , moving at $v \ll c$. A relativistic particle moving among stationary clouds would have an interaction in a distance L . It will have head-on collisions in a distance $L/(1 + v/c)$, which change the energy of the particle by a fractional amount $+v/c$, and will have tailgate collisions in a distance $L/(1 - v/c)$, which change the energy by a fractional amount $-v/c$. The net fractional energy change is the sum of the relative rates times the energy change, or

$$\Delta E/E = (v/c)(1 + v/c) - (v/c)(1 - v/c) \sim \mathcal{O}(v^2/c^2) . \quad (1)$$

Now, in this simple argument we've neglected (1) the factor of $\gamma_v = 1/\sqrt{1 - v^2/c^2}$ in the Doppler shift, and (2) averaging over angles instead of considering just head-on and tailgate

collisions. Both of these simply change the factor in front of v^2/c^2 , and a more careful calculation gives (see Longair, §21.3)

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{8}{3} \left(\frac{v}{c} \right)^2. \quad (2)$$

Consideration of diffusion (see Longair) shows that the loss (or escape) probability is a constant in each scattering, so that the resulting spectrum of particle energies is a power law. If the escape time from a cloud is τ_{esc} and we define $\alpha = (4/3)v^2/cL$, then the spectrum is

$$N(E) \propto E^{-x} \quad (3)$$

where $x = 1 + 1/\alpha\tau_{\text{esc}}$. Getting a power-law spectrum is certainly a good thing, but there are some problems with Fermi's original suggestion, which was that interstellar clouds would play the tennis. **Ask class:** given that $v/c \sim 10^{-4}$ and $L \approx 1$ pc for interstellar clouds, how long would it take to accelerate a particle from $\sim 10^9$ eV to $\sim 10^{15}$ eV, as required for many cosmic rays produced in the Galaxy? e-folding time is about $1/[(8/3)(v/c)^2]$ times one collision time, which is about three years, so would need around a billion years. That's long enough to suffer many losses. Better to accelerate with a region of more tangled fields. However, even then, no reason why the spectral index should be in a preferred range (why would τ_{esc} and L have the same relation in all clouds?).

First-order Fermi acceleration

Fortunately, in the late 1970's several people came up with an improvement on Fermi's idea of acceleration by repeated Doppler shifts. Consider a strong shock. Fluid mechanics tells us that if the shock is traveling at a velocity U and is composed of an ideal gas, the material behind the shock (which the shock has passed through) is traveling at a velocity $3U/4$. Let us assume that both behind and ahead of the shock there are tangled magnetic fields strong enough to isotropize the distribution of particle propagation directions (this is a likely consequence of shock propagation in a plasma). Also assume $U \ll c$ and that the shock front is extremely thin, so that the particle does not interact with the shock itself. Then, as seen in the rest frame of the unshocked fluid (ahead of the shock front), what happens is (1) a particle in the unshocked fluid goes through the shock and scatters in the shocked fluid, (2) this fluid is moving at a relative velocity $3U/4$, so there is a Doppler shift of order $1 + \mathcal{O}(U/c)$, (3) after scattering, the particle makes its way back through the shock to the unshocked fluid, where it scatters again, (4) the process repeats, and every time it does a fractional energy $\Delta E/E = \mathcal{O}(U/c)$ is gained. This is first order in the velocity, so this is called first-order Fermi acceleration. Averaging over angles it turns out that it's simply $\Delta E/E = U/c$ per round trip! **Ask class:** given that we know the average energy gain per scattering, what else do we need to know to determine the spectrum? We need the probability of escape per round trip. That probability can be calculated due to a clever argument by Bell (1978). Go into the shock frame. **Ask class:** for relativistic, isotropically

moving particles of number density n , what is the flux (number per area per time) across the shock front? Flux is number density times average velocity. Half the particles are moving away from the shock front. The average velocity of the rest is $c/2$, so the flux is $\frac{1}{4}nc$. Now, what is the flux that escapes? In the shock front, the shocked fluid is moving away from it at a speed $U/4$. By the same argument, that means that the escape flux is $\frac{1}{4}nU$. Therefore, a fraction U/c escapes. As before, then, there is a constant fractional energy gain and a constant fractional number loss per scattering. In this case, though, the fraction is U/c in both cases. That means that the spectrum is $N(E)dE \propto E^{-2}$, remarkably close to what is observed.

This is encouraging: this method can accelerate things quickly to high energy, and it gives a spectral index close to what is seen. In general, this also depends on the compression ratio in the shock; we've considered an ideal gas, with a compression ratio of 4 (compression ratio is equal to $r = (C_V + 1)/(C_V - 1)$, where C_V is the adiabatic index; for example, 5/3 for an ideal gas, 4/3 for radiation-dominated matter). The shocked matter has a velocity $-U/r$ relative to the shock wave. **Ask class:** given this, would they expect that for $r = 7$ (for radiation-dominated matter) the spectral index would be harder (more high-energy particles) or softer? Hint: think about how the escape probability would change. It's harder, because a smaller fraction of particles escapes. The spectral index you get turns out to be $(r + 2)/(r - 1)$. The dependence on just the compression ratio, and not details like the size of the region, strength of the magnetic field, injection spectrum of particles, ... means that it is robust. The fact that the spectral index is not exactly in the 2-3 range observed suggests that some modifications would be necessary: thick or weak shocks, tangled or oblique field, energy losses, and so on. But the general idea is good.

This is an outstanding acceleration mechanism. It is thought that supernova shocks are especially promising: the shocks are strong and thin, and sweep up a lot of magnetic field that can scatter particles. However, detailed investigations show that above 10^{14-15} eV this mechanism can't produce particles via SN shocks; the SN slows down by sweeping up matter. So, something else would have to produce higher-energy particles. We'll consider one of those now.

Acceleration by potential drops

Let's turn our attention to another completely different acceleration mechanism. **Ask class:** if you have an electric field \mathcal{E} over a region of size d and put a particle of charge e in it, how much energy does the particle gain by going across the region? $E = e\mathcal{E}d$. If the potential drop $\Phi = \mathcal{E}d$ is large enough, particles can be accelerated to extremely high energies. The highest energy cosmic rays have $E \approx 10^{20}$ eV, so the potential drop needs to be $\Phi \approx 10^{20}$ V. It turns out that a good way to get large potential drops is to have a magnetic field in the same region as a spinning conductor: for example, an accretion disk or rotating neutron star. Lenz's law says that if a magnetic field of strength B is moving with

a velocity v through conducting matter, it generates an electric field $E = (v/c)B$. Over a closed circuit of length $\sim d$, this produces an electromotive force $\Phi \approx Ed = (v/c)Bd$. The net result is that a potential drop of magnitude Φ opens up along the field lines, and acceleration can take place there. Physically, what can happen is that charges are drawn apart from each other and separate, leaving a low-density region surrounded by mostly positive charges on one side and negative charges on the other.

We can calculate potential drops in some representative situations. When making conversions, we have to be a little careful: sadly, the cgs unit of potential is a “statvolt”, which is 300 Volts(!) You can also look in a book to find that $e = 4.8 \times 10^{-10}$ in cgs units and do it that way. Anyway, let’s think first of a strongly magnetized, rapidly rotating neutron star. If $B = 10^{13}$ G on the surface, $d = 10^6$ cm (the size of the star) and $v = 0.1c$ (about a 3 ms spin period) then $\Phi = 10^{17} sV = 3 \times 10^{20}$ V. If we instead are interested in a supermassive black hole, we might have $d = 10^{14}$ cm (radius near the inner edge of the disk), $B = 10^4$ G, and $v = 0.5c$. Then $\Phi = 5 \times 10^{17} sV \approx 2 \times 10^{20} V$.

To usual astrophysical accuracy, both of these seem plenty high to get particles to the $\sim 10^{20}$ eV observed for the highest-energy cosmic rays. However, as we’ll see now, it’s actually extremely tough: loss processes of many kinds come in like gangbusters at high energies. Still, for really high energies this is the best current bet.

Energy loss processes

Now, we need to consider energy loss processes. **Ask class:** what are ways in which high-energy particles can lose energy? Generically, can lose energy by interacting with photons or other particles, or by interacting with a field and radiating. Let’s break it down. First, interaction with photons (inverse Compton scattering). **Ask class:** would they expect protons or electrons to lose more energy by scattering with photons? Electrons have a much larger cross section, so they do. Now, interaction with particles. Electrons can interact with other electrons or with protons, but at ultrarelativistic energies these cross sections are relatively small. For protons, as we saw in the last lecture they can scatter off of other protons or nuclei. In such interactions the strong force is involved, and the cross section is something like $\sim 10^{-26}$ cm². **Ask class:** what does the column depth have to be for optical depth unity? The reciprocal of this, or about 10^{26} cm⁻². That’s a lot! So, in a dense environment, collisions with particles can sap energy from high-energy particles. But what if the environment is low-density? Then, as we’ve argued, acceleration of particles to ultrarelativistic energies means that magnetic fields are present. In fact, the fields need to be strong enough to confine the particles as well (otherwise they escape without further gain of energy). Therefore, interaction of the particles with the field can produce radiation, diminishing the particle energy.

Curvature radiation.—First, suppose that the particle is forced to move along magnetic

field lines with curvature radius R . As we found in the last lecture, the power radiated by a particle moving at a Lorentz factor $\gamma \gg 1$ is $P \approx (2e^2c/3R^2)\gamma^4$. A relativistic particle of charge e moving along an electric field \mathcal{E} receives a power $e\mathcal{E}c$ from the field. Equating the power gain and power loss, the limiting Lorentz factor is

$$\gamma < 2 \times 10^7 R_8^{1/2} \mathcal{E}_4^{1/4}, \quad (4)$$

where $R = 10^8 R_8$ cm and $\mathcal{E} = 10^4 \mathcal{E}_4$ sV cm⁻¹. For a supermassive black hole with $R = 10^{14}$ cm, γ might get up to 2×10^{10} (note the weak dependence on \mathcal{E} ; also, the required magnetic field for $\mathcal{E}_4 \gg 1$ would be unrealistically high for a supermassive black hole). This would give a proton an energy of 2×10^{19} eV, which sounds high but is a factor of 10 short of the highest observed energies. For a neutron star with $R = 10^6$ cm and $\mathcal{E} = 10^{12}$, $\gamma < 2 \times 10^8$. So it sounds bad. **Ask class:** but is curvature radiation always relevant? What if you have a very high-energy particle? Then, the particle doesn't follow the field lines. Remember that curvature radiation and synchrotron radiation are both just types of acceleration radiation. If a particle is moving in a straight line, it radiates very little. So, the question is whether synchrotron radiation keeps the particle along the field lines.

Synchrotron radiation.—From the last lecture, proton synchrotron radiation causes an energy decay at a rate $\dot{E}/E \approx 3 \times 10^5 \gamma B_{12}^2$ s⁻¹ if the highly relativistic protons move perpendicular to field lines. If E/\dot{E} is short compared to the time needed to travel across the acceleration region, the proton will follow field lines. Near a strongly magnetic neutron star, with $B = 10^{12}$ G, there's no chance of highly relativistic protons moving across field lines. Even for a supermassive black hole, where $B \sim 10^5$ G, a proton with $\gamma = 10^{11}$ as required will be forced to follow the field lines in a time much less than the crossing time of the region, so in both cases it is likely that protons will be forced to follow field lines.

This is a problem. My best guess is that there are AGN with field geometries such that the curvature radius is much greater (factor of >100!) than the distance from the hole. That would just barely allow the observed energies. If future detectors (such as the Pierre Auger Array) see cosmic rays of energies much greater than the current record holders, this would be a real problem. We'll get into other issues with the highest energy cosmic rays near the end of the course.